General Mathematical Solution for Selective Harmonic Elimination

Mahrous Ahmed, Mohamed Orabi, Sherif Ghoneim, Mosleh Alharthi, Farhan Salem, Bassem Alamri, Saad Mekhilef

I. NOMENCLATURE

\( h_n \) = Normalized harmonic order component

\( n = 1, 3, 5, \ldots \) harmonic orders

\( NS_{\text{min}} \) = The minimum number of switching angles per a quarter time period

\( NS \) = The number of the switching angles per a quarter time period including optional notches.

\( \alpha_1, \alpha_2, \ldots, \alpha_k \) = The switching angles

\( l \) = Number of eliminated harmonic

\( 1 + l \) = Number of equations groups

\( k = 1, 2, 3, \ldots, NS_{\text{min}} \) = switching angles order

\( S \) = Refers to the ratio among the input DC voltage sources

\( k1 \) = Refers to the number of DC sources

\( N = \) maximum available number of output voltage levels

\( M = \) Refers to the total normalized input DC voltage of the MLI

\( \text{THD} = \) Total harmonic distortion

II. INTRODUCTION

Multilevel inverters (MLIs) [1] - [4] have garnered a lot of attention due to their inherent advantages over conventional two-levels H-bridge inverters. MLIs generate a stair case of output voltage waveforms with low harmonic contents. Moreover, MLI switches are turned on/off at a low switching frequency; thus, they have low switching losses, low switching stresses, low electromagnetic interface (EMI); and high system efficiency. Both the input side boosting stage and the output side power transformer can be eliminated in MLIs resulting in reducing the system’s size and efficiency.

Recently, the concept of selective harmonic elimination (SHE) has received much attention [4] - [6]. SHE has the ability to remove the harmful lower harmonics from the output voltage waveforms. Thus, filtering higher harmonics can be easily done using a small filter. The main challenge in employing the SHE control algorithm is its nonlinear transcendental equations, which have complex solutions. In addition to their complexity, they drastically increase the number of selected harmonics; therefore, the number of equations has also increased. Recently, several trials and techniques have been introduced to simplify the transcendental equations solution with high accuracy for high levels of MLIs. These can be categorized into three main groups; (1) numerical solutions [7] – [12], (2) evolutionary solutions (EAs) [13] – [22], and (3) the mathematical solutions [23] – [32].

The first group, numerical solutions, includes the well-known Newton-Raphson (NR), gradient optimization, and sequential quadratic programming in which the solution is fast iterative [6]. The solutions converge of the above techniques mainly depends on the initial guesses for the values of the unknowns. Therefore, choosing the proper initial values of the unknowns is the main challenge of these algorithms. The second group, EAs includes particle swarm optimization (PSO), genetic algorithms (GAs), and bee algorithms (BAs). EAs transform the transcendental equations of the fundamental and lower order harmonics into a fitness function in order to fetch the optimum firing angles by optimizing the fitness function. The main advantage of EAs over numerical methods is that EAs are less influenced by the initial guesses of the unknowns [5].

The third group, mathematical solutions [23]-[32] include
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/JESTPE.2019.2932933, IEEE Journal of Emerging and Selected Topics in Power Electronics

many trials to solve the non-linear transcendental equations using algebraic algorithms. In [23]-[26], the transcendental equations are transformed into polynomial equations and the resultant new system is solved mathematically without the need of the initial guesses for the values of the unknowns. The method suffers from the computational complexity of the solutions; thus, it is not applicable in real-time and it is limited for low-level inverters. In [27], [28], simple closed form analytical solutions were proposed where the nonlinear transcendental equations are converted into a nonlinear system that are solved analytically. The solution is very fast and very accurate. Moreover, while it guarantees harmonic cancellation, as stipulated, the solution is limited for two or three switching angles; thus, they are not applicable to higher levels. In [29], a novel analytical method, called selective harmonic elimination pulse amplitude modulation (SHE PAM), was presented to calculate the switching angles for all levels. However, that method mainly depends on equating the voltage second area under the reference sine signal and the synthesized multilevel waveform, thus both the magnitude of the voltage and the switching angles must be selected at different levels. Other analytical techniques have been presented in [30]–[31]; those techniques are simple, real-time analytical, and cost effective, and they are succeeding in reducing the computational complexity. However, the studied methods are limited to five level inverter topologies. In [32], a simple mathematical solution to solve transcendental nonlinear equations was obtained by finding the direct relationships among the switching angles instead of their cosines. The solution is simple and accurate, and it has a low computation time; however, it is limited to a specific even number of switching angles, and it failed to control the fundamental component.

This paper presents a new on line analysis and generalized method to calculate the switching angles for both symmetrical and unsymmetrical MLI configurations regardless to the MLI output waveforms number of levels. The proposed algorithm advantages are: it is a mathematical solution; very accurate, very simple, valid for a wide range of modulation index, and provides voltage control. The remainder of this paper is organized as follows. Section III presents a detailed explanation of the proposed analytical procedure such as the assumptions and development of the equations. Section IV presents the general solution for the switching angles algorithm, and construction of the main matrices are demonstrated with examples. Section V presents the results and discusses of the suggested simulations and experiments used to verify and validate the performance of the proposed technique. Section VI gives numerical calculated examples for selected cases of studied. Section VII gives a detailed comparison between the proposed algorithm and its existence and recent counterpart algorithms to highlight the performance of the proposed algorithm. Finally, section VIII presents a summary of the main remarks on the proposed technique, and presents the study’s conclusion.

III. MATHEMATICAL PROCEDURE

Figure 1 (a) shows the general N-levels single phase cascaded H-bridge inverter. Each unit has single DC source and four switches. The general ratio among MLI input DC sources is $S$ ($S = 1$ for symmetrical MLI). Any DC sources ratio yields inverter output voltage waveforms as shown in figure 1(b) with $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$ as MLI switching angles. The total normalized input DC voltage of the MLI ($M$) can be calculated from the following equation

$$M = 1 + S^1 + S^2 + \ldots + S^{k-1-1}$$

(1)

$k_1$ is refers to the number of DC sources. Therefore the maximum number of available levels is

$$N = 2M + 1$$

(2)

Using Fourier series of the stair case function is shown in figure 1(b), the general normalized harmonic component ($h_n$) is normalized to $V_{DC1}$ as follows

$$h_n = \frac{4}{n\pi} \sum_{n=3,5,\ldots} \cos(n \alpha_k), k = 1, 2, 3, \ldots, N_{S_{min}}$$

(3)

Where $\alpha_1 < \alpha_2 < \ldots \alpha_N < \frac{\pi}{2}$, $n = 1, 3, 5, \ldots$ harmonic number

Where the minimum required number of switching angles, $N_S$, is defined as the ceiling of the natural logarithm (ln) of $M$ as follows

$$NS_{min} = 2^{cei(ln(M))}$$

(4)

It is clear from (4) that the minimum required number of switching angles will be equal or greater than $M$. The number $k$ is the switching angles orders. Equation (3) can be applied for voltage waveform that was given in figure 1(c) and (d) [28] as follows;

$$h_n = \frac{4}{n\pi} [\cos(n \alpha_1) + \cos(n \alpha_2) - \cos(n \alpha_3)]$$

(5.a)

$$h_n = \frac{4}{n\pi} [\cos(n \alpha_4) - \cos(n \alpha_2) + \cos(n \alpha_3)]$$

(5.b)

It could be noticed from (5.a) and (5.b) that, the rising edge of the step voltage at the switching angle gives a positive sign for the corresponding cosine term, but the falling edge of the step voltage at the switching angle produces a negative sign for the corresponding cosine term. Thus, (3) can be written in a general form as follows [6], [8], and [26];

$$h_n = \frac{4}{n\pi} \sum_{n=3,5,\ldots} \cos(n \alpha_k), k = 1, 2, 3,\ldots, N_{S_{min}}$$

(6)

Equation (6) is the final general harmonic equation that depends on the switching angles and their number ($N_{S_{min}}$) and so it is dependent on the normalized input voltage ($M$). The proposed technique will be explained in details for 8 switching angles. In this technique, the fundamental voltage is controlled and the lower orders of harmonics were removed. Equation (6) can be written for lower order of harmonics as follows;

$$h_3 = 0 = \frac{4}{3\pi} \sum_{k=1,2,3,\ldots} \cos(3 \alpha_k) = \frac{4}{3\pi} \sum_{k=1,3,5,7} [\cos(3 \alpha_k) + \cos(3\alpha_{k+1})]$$

(7)

Using algebra formulas of cosines, Eq. (7) can be rewritten as follows

$$h_3 = 0 = \frac{4}{3\pi} \sum_{k=1,3,5,7} 2\cos[\frac{3}{2}(\alpha_k + \alpha_{k+1})]\cos[\frac{3}{2}(\alpha_k - \alpha_{k+1})]$$

(8)
From Eq. (8) and in order to cancel \( h_3 \) and its multiples, the following condition must be satisfied

\[
2\cos\left(\frac{3}{2}\right)(\alpha_k + \alpha_{k+1}) = 0 \quad \Rightarrow \\
\left(\frac{3}{2}\right)(\alpha_k + \alpha_{k+1}) = \frac{\pi}{2}, \quad k = 1, 3, 5, 7 \tag{9}
\]

Decomposing Eq. (9) yields

\[
\begin{align*}
\alpha_1 + \alpha_2 &= \frac{\pi}{2} \quad \tag{10.a} \\
\alpha_3 + \alpha_4 &= \frac{\pi}{2} \quad \tag{10.b} \\
\alpha_5 + \alpha_6 &= \frac{\pi}{2} \quad \tag{10.c} \\
\alpha_7 + \alpha_8 &= \frac{\pi}{2} \quad \tag{10.d}
\end{align*}
\]

To cancel out the fifth harmonic, Eq. (8) is rewritten once more for the \( h_5 = 0 \) as follows

\[
h_5 = 0 = \frac{4}{5\pi} \sum_{k=1,3,5,7} 2\cos\left(\frac{5}{2}\right)(\alpha_k + \alpha_{k+1}) \cos\left(\frac{5}{2}\right)(\alpha_k - \alpha_{k+1}) \tag{11}
\]

Substituting from Eq. (9) and Eq. (10) into Eq. (11) to omit the first term, then Eq. (11) become

\[
h_5 = 0 = \frac{4}{5\pi} \sum_{k=1,3,5,7} 2\cos\left(\frac{5}{2}\right)(\cos(\alpha_k - \alpha_{k+1})) \tag{12}
\]

Applying the cosines formulas, Eq. (12) can be rewritten as follows

\[
h_5 = 0 = \frac{4}{5\pi} \sum_{k=1,5} 2\cos\left(\frac{5}{4}\right)(\cos(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3})) \cos\left(\frac{5}{4}\right)(\alpha_k - \alpha_{k+1} - \alpha_{k+2} + \alpha_{k+3}) \tag{13}
\]

Similarly, from Eq. (13) and in order to cancel \( h_5 \) and its multiples, the following condition must be satisfied

\[
h_5 = 0 = \frac{4M}{5\pi} \sum_{k=1,5} 2\cos\left(\frac{5}{4}\right)(\cos(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3})) \tag{14}
\]

Decomposing Eq. (13) yields

\[
\begin{align*}
\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 &= \frac{2\pi}{5} \quad \tag{15.a} \\
\alpha_5 - \alpha_6 + \alpha_7 - \alpha_8 &= \frac{2\pi}{5} \quad \tag{15.b}
\end{align*}
\]

Equation (14) is rewritten to cancel out the seven harmonic \( (h_7 = 0) \) as follows

\[
h_7 = 0 = \frac{4}{7\pi} \sum_{k=1,5} 2\cos\left(\frac{7}{4}\right)(\cos(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3} + \alpha_{k+4} - \alpha_{k+5} + \alpha_{k+6} - \alpha_{k+7})) \cos\left(\frac{7}{4}\right)(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3} + \alpha_{k+4} - \alpha_{k+5} + \alpha_{k+6} - \alpha_{k+7}) \tag{16}
\]

By applying the cosine formulas, Eq. (16) can be rewritten as follows

\[
h_7 = 0 = \frac{4}{7\pi} \sum_{k=1} 2\cos\left(\frac{7}{4}\right)(\cos(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3} + \alpha_{k+4} - \alpha_{k+5} + \alpha_{k+6} - \alpha_{k+7})) \cos\left(\frac{7}{4}\right)(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3} + \alpha_{k+4} - \alpha_{k+5} + \alpha_{k+6} - \alpha_{k+7}) \tag{17}
\]

Similarly, from Eq. (17) and in order to cancel \( h_7 \) and its multiples, the following condition must be satisfied

\[
\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \alpha_5 - \alpha_6 + \alpha_7 - \alpha_8 = \frac{4\pi}{7} \tag{18}
\]

To control the fundamental value, Eq. (7) is rewritten as follows

\[
h_1 = \frac{4}{\pi} \sum_{k=1,3,5,7} 2\cos\left(\frac{1}{2}\right)(\alpha_k + \alpha_{k+1}) \cos\left(\frac{1}{2}\right)(\alpha_k - \alpha_{k+1}) \tag{19}
\]

Substituting from Eq. (1) into Eq. (19) and after mathematical processing it yields

\[
h_1 = \frac{4}{\pi} \cos\left(\frac{1}{2}\right)(\alpha_1 + \alpha_2) \sum_{k=1,3,5,7} 2\cos\left(\frac{1}{2}\right)(\alpha_k - \alpha_{k+1}) \tag{20}\]

Thus

\[
\sum_{k=1,3,5,7} 2\cos\left(\frac{1}{2}\right)(\alpha_k - \alpha_{k+1}) = (\pi h_1/4) / [2 \cos\left(\frac{\pi}{6}\right)] \tag{21}
\]

Substituting from Eq. (15) into Eq. (21) and after mathematical processing it yields

\[
\sum_{k=1,5} 2\cos\left(\frac{5}{4}\right)(\alpha_k - \alpha_{k+1} + \alpha_{k+2} + \alpha_{k+3}) = \left(\pi h_1/4\right) / [4 \cos\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{10}\right)] \tag{22}
\]

Then Eq. (22) become

\[
\sum_{k=1} 2\cos\left(\frac{5}{8}\right)(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3} + \alpha_{k+4} - \alpha_{k+5} + \alpha_{k+6} - \alpha_{k+7}) \cos\left(\frac{1}{8}\right)(\alpha_k - \alpha_{k+1} + \alpha_{k+2} - \alpha_{k+3} + \alpha_{k+4} - \alpha_{k+5} + \alpha_{k+6} - \alpha_{k+7}) = (\pi h_1/4) / [4 \cos\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{10}\right) \cos\left(\frac{\pi}{14}\right)] \tag{23}
\]

Substituting from Eq. (15) into eq. (21) and after mathematical processing it yields

\[
\cos\left(\frac{1}{8}\right)(\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \alpha_5 - \alpha_6 + \alpha_7 - \alpha_8) = \left(\pi h_1/4\right) / [8 \cos\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{10}\right) \cos\left(\frac{\pi}{14}\right)] \tag{24}
\]

Thus

\[
\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \alpha_5 - \alpha_6 + \alpha_7 - \alpha_8 = 8 \cos^{-1}\left(\frac{\pi h_1}{4}\right) / \left(8 \cos\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{10}\right) \cos\left(\frac{\pi}{14}\right)\right) \tag{24}
\]

Equations (10), (15), (18), and (24) represent a linear 8x8 equations with 8 unknowns \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \) and \( \alpha_8 \) which can be solved mathematically as follows:

\[
AX = B \quad \Rightarrow \quad X = A^{-1}B \tag{25}
\]

Matrix \( A \) and Matrix \( B \) equal

\[
A = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & 1
\end{pmatrix}
\]
The number 8 in (27) represents the switching angles number \(N_{\text{switching}}\). The modulation index \((m_i)\) is defined in terms of the normalized fundamental component to the normalized DC input as follows

\[
m_i = \frac{H_1}{M_{\text{DC1}}} = \frac{b_4}{M}
\]

Where \(H_1\) is the fundamental component of the required output voltage. Substituting from Eq. (28) in Eq. (27), Eq. (27) can be rewritten in a general form as

\[
B_4 = N_{\text{min}} \cdot \cos^{-1} \left( \frac{\pi b_4}{8 \cos \left( \frac{\pi}{10} \cos \left( \frac{\pi}{14} \right) \right)} \right)
\]

(27)

The number 8 in (27) represents the switching angles number \((N_{\text{switching}})\). The modulation index \((m_i)\) is defined in terms of the normalized fundamental component to the normalized DC input as follows

\[
m_i = \frac{H_1}{M_{\text{DC1}}} = \frac{b_4}{M}
\]

(28)

Where \(H_1\) is the fundamental component of the required output voltage. Substituting from Eq. (28) in Eq. (27), Eq. (27) can be rewritten in a general form as

\[
B_4 = N_{\text{min}} \cdot \cos^{-1} \left( \frac{\pi b_4}{8 \cos \left( \frac{\pi}{10} \cos \left( \frac{\pi}{14} \right) \right)} \right)
\]

(29)

Using the general equation for \(B_4\) to solve Eq. (25) and Eq. (24) gives the general solution for the switching angles values as follows

\[
\alpha_k = c_{1k} + c_{2k} \cos^{-1} \left[ \epsilon * N_{\text{min}} * M m_i \right]
\]

(30)

\(k = 1, 2, 3, ..., N_{\text{min}}, \ c_{1k}, c_{2k}\), and \(c_{3k}\) are certain constants.

Equation (30) gives the direct closed form relationship between the switching angles and the modulation index for the calculated \(N_{\text{min}}\) that is suitable for the on line operation.

Moreover, the proposed algorithm has the merit of cancelling more harmonic orders for the same number of inputs \((S, M, k_1\) and \(N_{\text{min}})\) by introducing extra switching angles (notches), thanks to the notches in the output voltage waveform. Then the total number of switching angles is calculated as

\[
N_S = 2^k N_{\text{min}}
\]

(31)

For example when \(S = 2\) and \(k_1 = 2\), thus from Eq. (1) \(M = 3\) and from Eq. (4) \(N_{\text{min}} = 4\), then \(N_S\) can be 4, 8, 16, ..., etc. the number of the eliminated harmonics \(l\) is defined as

\[
l = \frac{\ln(N_S)}{\ln(2)}
\]

(32)

\(l + 1\) defines the number of equation groups. For the above case Eq.(26), \(N_S = 8\), one can notice that, there are four groups as depicted in matrix \(B\) elements as \(B_1, B_2, B_3,\) and \(B_4\).

Generally, there are some observations for the proposed technique which are summarized in the following points:

- The second group of the generated equations has \(N_S/4\) equations and they cancel the fifth harmonics \(h_5\) and its multiples.
- The third group of the generated equations has \(N_S/8\) equations and they cancel the seventh harmonics \(h_7\) and its multiples.
- The overall groups of equations are \((1+1)\).
- And so on the generated equations can cancel harmonics \(h_3, h_5, h_7, h_9, ...\) and their multiples based on the number of groups.
- The last group of equation is a single equation and it is devoted to control the fundamental component \(h_1\).
- \(\alpha_1 < \alpha_2 < \alpha_3 < ... < \alpha_k < \pi/2, k = 1, 2, 3, ...
- If \(|\alpha| < \pi/2 \implies \alpha_k = |\alpha_k|\), this means the voltage step at this angle is increased from lower level to next higher level.
- If \(|\alpha| > \pi/2 \implies \alpha_k = \pi - |\alpha_k|\) and this means the voltage step at this angle is decreased from higher level to the next lower level.
- The number \((M)\) must be less or equal than the number of switching angles \((N_S)\).
- All elements of matrix \(A\) and matrix \(B\) except the last element of matrix \(B\) are independent of the modulation index. Therefore they are calculated only single time. Thus the proposed technique has reduced computation complexity because it needs only to update the last element of matrix \(B\) and this is shown by Eq. (29).

- The second group of the generated equations has \(N_S/4\) equations and they cancel the fifth harmonics \(h_5\) and its multiples.
- The third group of the generated equations has \(N_S/8\) equations and they cancel the seventh harmonics \(h_7\) and its multiples.
- The overall groups of equations are \((1+1)\).
- And so on the generated equations can cancel harmonics \(h_3, h_5, h_7, h_9, ...\) and their multiples based on the number of groups.
- The last group of equation is a single equation and it is devoted to control the fundamental component \(h_1\).
- \(\alpha_1 < \alpha_2 < \alpha_3 < ... < \alpha_k < \pi/2, k = 1, 2, 3, ...
- If \(|\alpha| < \pi/2 \implies \alpha_k = |\alpha_k|\), this means the voltage step at this angle is increased from lower level to next higher level.
- If \(|\alpha| > \pi/2 \implies \alpha_k = \pi - |\alpha_k|\) and this means the voltage step at this angle is decreased from higher level to the next lower level.
- The number \((M)\) must be less or equal than the number of switching angles \((N_S)\).
- All elements of matrix \(A\) and matrix \(B\) except the last element of matrix \(B\) are independent of the modulation index. Therefore they are calculated only single time. Thus the proposed technique has reduced computation complexity because it needs only to update the last element of matrix \(B\) and this is shown by Eq. (29).

\[
\begin{pmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4
\end{pmatrix} = \begin{pmatrix}
\pi/3 \\
\pi/3 \\
\pi/3 \\
2\pi/5 \\
2\pi/5 \\
4\pi/7 \\
B_3 \\
B_4
\end{pmatrix} \quad \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4 \\
\alpha_5 \\
\alpha_6 \\
\alpha_7 \\
\alpha_8
\end{pmatrix}
\]

(26)

\[
B_4 = 8 \cos^{-1} \left( \frac{\pi}{8 \cos \left( \frac{\pi}{10} \cos \left( \frac{\pi}{14} \right) \right)} \right)
\]

(27)

\[
4 \cos^{-1} \left( \frac{\pi}{8 \cos \left( \frac{\pi}{10} \cos \left( \frac{\pi}{14} \right) \right)} \right)
\]

(28)

where \(M_{\text{DC1}}\) represents the switching angles number \((N_{\text{switching}})\). The modulation index \((m_i)\) is defined in terms of the normalized fundamental component to the normalized DC input as follows

\[
m_i = \frac{H_1}{M_{\text{DC1}}} = \frac{b_4}{M}
\]

(29)

\[
\alpha_k = c_{1k} + c_{2k} \cos^{-1} \left[ \epsilon * N_{\text{min}} * M m_i \right]
\]

(30)

\(k = 1, 2, 3, ..., N_{\text{min}}, \ c_{1k}, c_{2k}\), and \(c_{3k}\) are certain constants.
Based on the aforementioned analysis, the proposed technique can be extended to any number of levels for symmetrical and unsymmetrical cascaded MLI. Building of matrix $A$ and matrix $B$ in a general form is the main part of the proposed analysis. The following summarizing steps explain the general algorithm:

- First, $M$ is calculated, from (1), based on the number of the available DC input sources and their ratio.
- Second, a suitable value for the switching angles are calculated based on (31).
- The generated set of groups of equations is defined by (32).
- The first group elements have either 0 or 1. Figure 2 shows how to generate the non-zero values at any row inside the first group: each row has two non-zero values, their coordinates can be found by multiplying the row number by the value of ‘2’ then subtract ‘1’ and the next element coordination is defined by multiplying the row number by the value of ‘2’ as shown in figure 2.
- All matrix $B$ elements values inside this group are equal to $b_2 = 2^1 \pi / 5$, ‘2’ represents the group number while ‘5’ represents the harmonic order that will be removed.
- All elements of matrix $B$, except the last element, can be calculated using $b_1 = 2^{(l-1)} \pi / h_n$, ‘$l$’ is the group number and ‘$h_n$’ refers to the harmonic order that will be cancelled by this group of equations.
- The last group has always single row, which responsible for controlling the fundamental component $h_1$. It is generated in a similar way as shown in figure 3.
- By following the above steps, table 1 demonstrates the above general summarizing steps of algorithm. Tables 2, 3 and 4 are demonstrated examples for building elements of both matrices $A$ and $B$ for $n = 2, 4$ and $16$ respectively.

Figure 4 shows a block diagram to build elements of both matrix $A$ and matrix $B$ for a general number of switching angles. The proposed algorithm receives the data of the input DC sources ($S$ and $K_1$ ) and the data of the outputs ($m_i$ or $h_1$ ). It could be noticed that only the last element in matrix $B$ needs to be updated when the modulation index or the fundamental ($m_i$ or $h_1$ ) changes. While all other elements of both matrix $A$ and matrix $B$ are constants. Also there are no loops or iteration with initial variables guess resulting in a reduced computation complexity.
Table 1
SHOWS THE GENERAL ALGORITHM (ROW2 (1) : '2' REFERS TO GROUP NUMBER AND '1' REFERS TO ROW NUMBER INSIDE THE GROUP)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of rows</th>
<th>Matrix A $2^l \times 2^l$</th>
<th>Matrix B $2^l \times 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group1: Removing $h_3$ and its multiples ($n = 3$)</td>
<td>Row1(1)</td>
<td>$1$ $0$ $0$  ....  $0$</td>
<td>$b_1 = 2^0\pi/3$</td>
</tr>
<tr>
<td></td>
<td>Row1(2)</td>
<td>$0$ $0$ $1$ $1$ $0$ .... $0$</td>
<td>$b_1 = 2^0\pi/3$</td>
</tr>
<tr>
<td></td>
<td>Row1(3)</td>
<td>$0$ $0$ $0$ $0$ $1$ $1$ .... $0$</td>
<td>$b_1 = 2^0\pi/3$</td>
</tr>
<tr>
<td></td>
<td>Row1($2^l-1$)</td>
<td>$0$ $0$ $0$ .... $1$ $1$</td>
<td>$b_1 = 2^0\pi/3$</td>
</tr>
<tr>
<td>Group2: Removing $h_5$ and its multiples ($n = 5$)</td>
<td>Row2(1)</td>
<td>$1$ $-1$ $1$ $-1$ $0$ .... $0$ $0$</td>
<td>$b_2 = 2^2\pi/5$</td>
</tr>
<tr>
<td></td>
<td>Row2(2)</td>
<td>$0$ $0$ $0$ $0$ $1$ $-1$ $1$ $-1$ $0$ $0$</td>
<td>$b_2 = 2^2\pi/5$</td>
</tr>
<tr>
<td></td>
<td>Row2($3 \times 2^l-2$)</td>
<td>$0$ $0$ $0$ .... $0$ $0$ $1$ $-1$ $1$ $-1$</td>
<td>$b_2 = 2^2\pi/5$</td>
</tr>
<tr>
<td>Group3: Removing $h_7$ and its multiples ($n = 7$)</td>
<td>Row3(1)</td>
<td>$1$ $-1$ $1$ $-1$ $1$ $-1$ $1$ $0$ .... $0$ $0$</td>
<td>$b_3 = 2^3\pi/7$</td>
</tr>
<tr>
<td></td>
<td>Row3(2)</td>
<td>$0$ $0$ $0$ $0$ $0$ $0$ $1$ $-1$ $1$ $-1$ $1$ $0$ $0$</td>
<td>$b_3 = 2^3\pi/7$</td>
</tr>
<tr>
<td></td>
<td>Row3($7 \times 2^l-3$)</td>
<td>$0$ $0$ $0$ $0$ $0$ $0$ $0$ $1$ $-1$ $1$ $-1$ $1$ $0$</td>
<td>$b_3 = 2^3\pi/7$</td>
</tr>
<tr>
<td>Removing: $h_{11}$</td>
<td></td>
<td>$1$ $-1$ $-1$ $1$ $1$ $1$ $1$ $1$ $1$ $-1$ $1$ $-1$ $1$ $-1$ $1$</td>
<td>$b_4 = 2^4\pi/11$</td>
</tr>
<tr>
<td>Removing: $h_{13}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Removing: $h_{17}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group l : Removing $h_n$ and its multiples</td>
<td>Row $l(1)$</td>
<td></td>
<td>$b_l = 2^{l-1}\pi/n$</td>
</tr>
<tr>
<td>Group $l+1$: controlling $h_1$</td>
<td>Row $l+1 (1)$</td>
<td></td>
<td>$b_{l+1}$</td>
</tr>
</tbody>
</table>

Total rows $= 2^l$

\[
b_{l+1} = NS \cos^{-1}\left(\frac{n\pi}{4}\right) / \left[ NS \cos\left(\frac{1}{2^l} B_1\right) \cos\left(\frac{1}{2^l} B_2\right) \cos\left(\frac{1}{2^l} B_3\right) ... \cos\left(\frac{1}{2^l} B_l\right)\right] \quad (33)
\]

\[
l = \frac{\ln(NS)}{\ln(2)} , \quad NS = \text{number of switching angles}
\]
**Table 2**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of rows</th>
<th>Matrix A (2 x 2)</th>
<th>Matrix B (2 x 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1: Removing ( h_3 ) and its multiples (n = 3)</td>
<td>Row 1(1)</td>
<td>1 1</td>
<td>( b_1 = \frac{2\pi}{3} )</td>
</tr>
<tr>
<td>Group 2: controlling ( h_1 )</td>
<td>Row 2(1)</td>
<td>1 -1</td>
<td>( b_2 )</td>
</tr>
<tr>
<td>Total rows = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
b_2 = NS \cos^{-1} \left( \frac{\pi h_1}{4} \left[ NS \cos \left( \frac{1}{2\pi} b_1 \right) \right] \right)\\
   = NS \cos^{-1} \left\{ \frac{\pi M m_i}{4} \left[ NS \cos \left( \frac{1}{2\pi} b_1 \right) \right] \right\}
\]

\[
l = \frac{\ln(4)}{\ln(2)} = 1, NS = \text{number of switching angles} = 2
\]

\[
C = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \left( \frac{\pi}{3} \right) \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}
\]

Substituting calculated elements of \( A \) and \( B \) in (22), and after manipulations yields the general closed form solutions in terms of \( m_i \) for this MLI which has \( NS = 2, l = 1, M = 4 \) as follows.

\[
X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = A^{-1} B = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} \pi/3 \\ \pi/3 \\ 2\pi/5 \\ b_3 \end{bmatrix}
\]

Therefore,

\[
\alpha_1 = \frac{\pi}{6} + \frac{\pi}{10} + \cos^{-1} \left( \frac{\pi M m_i}{16 \cos \left( \frac{\pi}{3} \right) \cos \left( \frac{\pi}{10} \right)} \right)
\]

\[
\alpha_2 = \frac{\pi}{6} - \cos^{-1} \left( \frac{\pi M m_i}{16 \cos \left( \frac{\pi}{3} \right) \cos \left( \frac{\pi}{10} \right)} \right)
\]

\[
\alpha_3 = \frac{\pi}{6} + \cos^{-1} \left( \frac{\pi M m_i}{16 \cos \left( \frac{\pi}{3} \right) \cos \left( \frac{\pi}{10} \right)} \right)
\]

\[
\alpha_4 = \frac{\pi}{6} - \cos^{-1} \left( \frac{\pi M m_i}{16 \cos \left( \frac{\pi}{3} \right) \cos \left( \frac{\pi}{10} \right)} \right)
\]

It should be noticed that \( M \) is less or equal \( NS = 4 \). Moreover (34) and (35) are matched with the general closed form solution given in (30).

**Table 3**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of rows</th>
<th>Matrix A (4 x 4)</th>
<th>Matrix B (4 x 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1: Removing ( h_3 ) and its multiples (n = 3)</td>
<td>Row 1(1)</td>
<td>1 1 0 0</td>
<td>( b_1 = \frac{2\pi}{3} )</td>
</tr>
<tr>
<td></td>
<td>Row 1(2)</td>
<td>0 0 1 1</td>
<td>( b_1 = \frac{2\pi}{3} )</td>
</tr>
<tr>
<td>Group 2: Removing ( h_3 ) and its multiples (n = 5)</td>
<td>Row 2(1)</td>
<td>1 -1 1 1</td>
<td>( b_2 = \frac{2\pi}{5} )</td>
</tr>
<tr>
<td>Group 3: controlling ( h_1 )</td>
<td>Row 3(1)</td>
<td>1 -1 -1 1</td>
<td>( b_3 )</td>
</tr>
<tr>
<td>Total rows = 4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
b_3 = NS \cos^{-1} \left( \frac{\pi h_1}{4} \left[ NS \cos \left( \frac{1}{2\pi} b_1 \right) \cos \left( \frac{1}{2\pi} b_2 \right) \right] \right)
\]

\[
l = \frac{\ln(4)}{\ln(2)} = 2, NS = \text{number of switching angles} = 4
\]

Fig. 4. Flow chart for the general solution of the proposed technique.
Table 4 shows building elements of both matrix A and matrix B for \( NS = 16 \), \( l = 4 \)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of rows</th>
<th>Matrix A ( 16 \times 16 )</th>
<th>Matrix B ( 16 \times 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1: Removing ( h_3 ) and its multiples ((n = 3))</td>
<td>Row1(1)</td>
<td>1 1 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>( b_1 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(2)</td>
<td>0 0 1 1 0 0 0 0 0 0 0 0 0 0 0</td>
<td>( b_2 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(3)</td>
<td>0 0 0 0 1 1 0 0 0 0 0 0 0 0 0</td>
<td>( b_3 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(4)</td>
<td>0 0 0 0 0 0 0 1 1 0 0 0 0 0 0</td>
<td>( b_4 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(5)</td>
<td>0 0 0 0 0 0 0 0 0 0 1 1 0 0 0</td>
<td>( b_5 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(6)</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 1 1</td>
<td>( b_6 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(7)</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 1</td>
<td>( b_7 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td></td>
<td>Row1(8)</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>( b_8 = 2^0 \pi/3 )</td>
</tr>
<tr>
<td>Group 2: Removing ( h_5 ) and its multiples ((n = 5))</td>
<td>Row2(1)</td>
<td>1 -1 1 -1 0 0 0 0 0 0 0 0 0 0 0</td>
<td>( b_5 = 2^{\frac{1}{2}} \pi/5 )</td>
</tr>
<tr>
<td></td>
<td>Row2(2)</td>
<td>0 0 0 0 1 -1 1 -1 0 0 0 0 0 0 0</td>
<td>( b_6 = 2^{\frac{1}{2}} \pi/5 )</td>
</tr>
<tr>
<td></td>
<td>Row2(3)</td>
<td>0 0 0 0 0 0 0 1 -1 1 -1 0 0 0 0</td>
<td>( b_7 = 2^{\frac{1}{2}} \pi/5 )</td>
</tr>
<tr>
<td></td>
<td>Row2(4)</td>
<td>0 0 0 0 0 0 0 0 0 0 1 -1 1 -1</td>
<td>( b_8 = 2^{\frac{1}{2}} \pi/5 )</td>
</tr>
<tr>
<td>Group 3: Removing ( h_7 ) and its multiples ((n = 7))</td>
<td>Row3(1)</td>
<td>1 -1 -1 1 -1 -1 1 0 0 0 0 0 0 0 0</td>
<td>( b_5 = 2^2 \pi/7 )</td>
</tr>
<tr>
<td></td>
<td>Row3(2)</td>
<td>0 0 0 0 0 0 0 1 -1 1 -1 1 -1</td>
<td>( b_6 = 2^2 \pi/7 )</td>
</tr>
<tr>
<td>Group 4: Removing ( h_{11} ) and its multiples ((n = 11))</td>
<td>Row4(1)</td>
<td>1 -1 -1 -1 1 1 1 1 -1 -1 -1</td>
<td>( b_7 = 2^4 \pi/11 )</td>
</tr>
<tr>
<td>Group 5: controlling ( h_1 )</td>
<td>Row5(1)</td>
<td>1 -1 -1 1 -1 1 1 -1 -1 1 -1 1 1 -1 1</td>
<td>( b_5 )</td>
</tr>
</tbody>
</table>

Total rows = 16

\( b_5 = NS \cos^{-1}\left\{ \frac{\pi}{4} \left[ NS \cos\left(\frac{\pi}{2} b_1\right) \cos\left(\frac{\pi}{2} b_2\right) \cos\left(\frac{\pi}{2} b_3\right) \cos\left(\frac{\pi}{2} b_4\right) \right]\right\} \), \( l = \frac{\ln(16)}{\ln(2)} = 4 \), \( NS = 16 \)

Number of equations groups =\( 1 + l = 5 \)

V. RESULTS AND DISCUSSION

The proposed technique is validated through both simulation and experimental results. The simulation results are carried out by using MATLAB/SIMULINK® software. Experimental results are done using MOSFET IRF640 as a switch, optoisolater 6N137 in the gate drive and the code is built and run using microcontroller PIC16F917. Two main studied cases have been adopted to validate the proposed concept, they are as follows.

Case study I: Two cells \((S = 2, k_1 = 2)\) of the asymmetrical cascaded MLI shown in figure 1 are used with \( V_{DC1} = 9 \) V. Therefore this inverter can generate up to 7 levels in the output voltage. Based on the aforementioned analysis provided in section III and IV, \( N = 7 \) and \( M = 3 \) therefore \( NS = 4, 8, 16,...,etc. \) from Eq. (31). \( NS = 4 \) and \( NS = 8 \) are two values have been selected for both simulation and experimental results and these results are shown in figures 5, 6 and 7.

Figures 5(a) and (b) show the simulation results for the percentage of harmonic analysis for a wide range of modulation index for this case study I. In figure 5(a), \( NS = 4 \), thus there are three groups of equations \((l + 1 = 3)\), as explained before in section III and IV, that can cancel 3rd and 5th harmonics and their multiples, moreover another equation to control the fundamental component. It is clear in figure 5(a) that the 3rd and 5th harmonics have been cancelled for all values of \( m_i \). In figure 5(b) = 16, thus there are five groups of equations \((l + 1 = 5)\) that can cancel 3rd, 5th, 7th, and 11th harmonic orders besides another equation to control the fundamental components. Figure 5(b) indicates that the 3rd, 5th, 7th, and 11th harmonics have been vanished for all values of \( m_i \).

Figure 6 shows both simulation and experimental results of the output voltage waveforms and the corresponding harmonic spectrum using a number of switching angles \( NS = 4 \) for two cases of modulation indexes 0.25 and 1.03, respectively. These two cases of modulation indexes, 0.25 and 1.03, generate 3 levels and 7 levels output voltage waveforms respectively. From (2), \( N = 7 \) which means the output voltage has up to seven levels \( V_{DC1}, V_{DC2}, V_{DC3} + V_{DC2}, 0, -V_{DC1}, -V_{DC2}, -V_{DC1} - V_{DC2} \) based on the modulation index values. For very low value of \( m_i \leq 0.25 \), only \( V_{DC1} \) is used to generate the required output voltage. While in case of high values of \( m_i \) and over modulation; both \( V_{DC1} \) and \( V_{DC2} \) are used to generate the output voltage. When \( NS = 4 \), thus from (31), there are three groups of equations \((l + 1 = 3)\). Group 1 will cancel the 3rd and its multiples; group 2 cancels
the 5th and its multiples, in addition to group 3 to control the fundamental components. It could be noticed that in all the two cases the 3rd and the 5th are cancelled, and the harmonics starts from 7th order and this is matched with the aforementioned analysis provided in sections III and IV.

Figure 7 shows both simulation and experimental results of the output voltage waveforms and the corresponding harmonic spectrum using a number of switching angles $NS = 16$ for two cases of modulation indexes 0.25 and 0.83, respectively. These two cases of modulation indexes generate 5 levels and 7 levels output voltage waveforms respectively. When $NS = 16$, thus from (31), there are five groups of equations $(l + 1 = 5)$. Group 1 cancels the 3rd and its multiples, group 2 cancels the 5th and its multiples, group 3 cancels the 7th and its multiples, and group 4 cancels the 11th and its multiples, in addition to group 5 to control the fundamental components. It could be noticed that in all the two cases the 3rd, 5th, 7th and the 11th are cancelled. And the harmonics starts from 13th order and this is also matched with the aforementioned analysis provided in sections III and IV.

Case study II: Three cells $(S = 3, k_1 = 3)$ of the asymmetrical cascaded MLI shown in figure 1 are used with $V_{DC1} = 5$ V. Therefore, this inverter can generate up to 27 levels in the output voltage. Based on the aforementioned analysis provided in section III and IV, $N = 27$ and $M = 13$. From the constraints relating to $NS$, a reasonable values for $NS = 16, 32, 64, \ldots$ etc. The number of equation groups ‘l’ can be calculated from (31). There are two values for $NS$ are selected, they are 16 and 32 for both simulation and experimental results and these results are shown in figures 8, 9 and 10.

Figures 8(a) and (b) show the simulation results for the percentage of harmonic analysis for wide range of modulation index. In figure 8(a) $NS = 16$, thus there are five groups of equations $(l + 1 = 5)$, as explained before in section III and IV, that can cancel 3rd, 5th, 7th and 11th harmonics and their multiples in addition to the fifth group that can control the fundamental component. It is clear in figure 8(a) that the 3rd, 5th, 7th and 11th harmonics have been disappeared for all values of $m_i$. In figure 8(b) $NS = 32$, thus there are six groups of equations $(l + 1 = 6)$ that can cancel 3rd, 5th, 7th, 11th and 13th in addition to the last equation that can control the fundamental component. Figure 8(b) indicates that 3rd, 5th, 7th, 11th and 13th harmonics have been vanished for all values of $m_i$.

Figure 9 shows both simulation and experimental results of the output voltage waveforms and the corresponding harmonic spectrum using a number of switching angles $NS = 16$ for two cases of modulation indexes 0.25 and 0.95, respectively. When $NS = 16$, thus from (31), there are five groups of equations $(l + 1 = 5)$. Group 1 cancels the 3rd and its multiples, group 2 cancels the 5th and its multiples, group 3 cancels the 7th and its multiples, and group 4 cancels the 11th and its multiples, in addition to group 5 to control the fundamental components. It could be noticed that in all the two cases the 3rd, 5th, 7th and the 11th are cancelled and the harmonics starts from 13th order and this is also matched with the aforementioned analysis provided in sections III and IV.

Figure 10 shows both simulation and experimental results of the output voltage waveforms and the corresponding harmonic spectrum using a number of switching angles $NS = 32$ for two cases of modulation indexes 0.25 and 0.95, respectively. Since $NS = 32$, thus there are five groups of equations $(l + 1 = 6)$. Group 1 cancels the 3rd and its multiples, group 2 cancels the 5th and its multiples, group 3 cancels the 7th and its multiples, group 4 cancels the 11th and its multiples, group 5 cancels the 13th and its multiples in addition to group 6 to control the fundamental components. It could be noticed that in all the two cases the 3rd, 5th, 7th, 11th and the 13th are cancelled. And the harmonics starts from 17th order and this is also matched with the aforementioned analysis provided in sections III and IV.

The percentage of the total harmonic distortion (THD) is defined by the following equation:

$$THD (\%) = \frac{\sqrt{\sum_{n=3,5,7,\ldots} |h_n|^2}}{h_1}$$

(35)

Using the definition of $h_1$ as given in (25), therefore

$$THD (\%) = \frac{\sqrt{\sum_{n=3,5,7,\ldots} |h_n|^2}}{M m_i}$$

(36)

From (5), (27) and (29), the harmonic components $h_n$ have a certain function in $m_i$. Therefore all terms of Eq. (36) have certain functions in $m_i$.

The dynamic response of the proposed procedure has been verified through simulation results. Figure 11 shows the dynamic response of the proposed procedure for two cases of MLI. Figure 11(a) through 11(c) are carried out for a MLI with $k_1 = 2$, $S = 2$, and $NS = 4$ and therefore $l = 2$. The dynamic response is done in the modulation index from 0.25 to 1.03 as shown in figure 11(a). The dynamic performance of the MLI output voltage is shown in figure 11 (b) which demonstrates the computation complexity discussion in the comparison section VI. The harmonic spectrums of the output voltage for the two cases of $m_i$ are shown in figure 11 (c). Because $l = 2$, it can be noticed that two lower order harmonics, the 3rd and the 5th harmonics and their multiples, are cancelled regardless the value of $m_i$.

Figure 11(d) through 11(f) are carried out for a MLI with $k_1 = 3$, $S = 3$, and $NS = 8$ and therefore $l = 3$. The dynamic response is done in the modulation index from 0.25 to 0.95 as shown in figure 11(d). The dynamic performance of the MLI output voltage is shown in figure 11 (e) which demonstrates the computation complexity discussion in the comparison section VI. The harmonic spectrums of the output voltage for the two cases of $m_i$ are shown in figure 11 (f). Because $l = 4$, it can be noticed that three lower order harmonics, the 3rd, the 5th and the 7th harmonics and their multiples, are cancelled regardless the value of $m_i$.

Figures 8(a) and (b) show the simulation results for the percentage of harmonic analysis for wide range of modulation index. In figure 8(a) $NS = 16$, thus there are five groups of equations $(l + 1 = 5)$, as explained before in section III and IV, that can cancel 3rd, 5th, 7th and 11th harmonics and their multiples in addition to the fifth group that can control the fundamental component. It is clear in figure 8(a) that the 3rd, 5th, 7th and 11th harmonics have been disappeared for all values of $m_i$. In figure 8(b) $NS = 32$, thus there are six groups of equations $(l + 1 = 6)$ that can cancel 3rd, 5th, 7th, 11th and 13th in addition to the last equation that can control the fundamental component. Figure 8(b) indicates that 3rd, 5th, 7th, 11th and 13th harmonics have been vanished for all values of $m_i$.

Figures 8(a) and (b) show the simulation results for the percentage of harmonic analysis for wide range of modulation index. In figure 8(a) $NS = 16$, thus there are five groups of equations $(l + 1 = 5)$, as explained before in section III and IV, that can cancel 3rd, 5th, 7th and 11th harmonics and their multiples in addition to the fifth group that can control the fundamental component. It is clear in figure 8(a) that the 3rd, 5th, 7th and 11th harmonics have been disappeared for all values of $m_i$. In figure 8(b) $NS = 32$, thus there are six groups of equations $(l + 1 = 6)$ that can cancel 3rd, 5th, 7th, 11th and 13th in addition to the last equation that can control the fundamental component. Figure 8(b) indicates that 3rd, 5th, 7th, 11th and 13th harmonics have been vanished for all values of $m_i$.
Fig. 5 Harmonic analysis simulation results using two cells asymmetrical cascaded inverter with ratio $V_{DC1}:V_{DC2} = 1:2$ for (a) $NS = 4$; (b) $NS = 16$.

Fig. 6 MLI output voltage performance using two cells asymmetrical cascaded inverter with ratio $V_{DC1}:V_{DC2} = 1:2$, $NS = 4$
Fig. 7 MLI output voltage performance using two cells asymmetrical cascaded inverter with ratio $V_{DC1}:V_{DC2} = 1:2$, $NS = 16$

Fig. 8 Harmonic analysis simulation results using three cells asymmetrical cascaded inverter with ratio $V_{DC1}:V_{DC2}:V_{DC3} = 1:3:9$ for (a) $NS = 16$; (b) $NS = 32$. 
Fig. 9 MLI output voltage performance using three cells asymmetrical cascaded inverter with ratio $V_{DC1}:V_{DC2}:V_{DC3} = 1:3:9$, $NS = 16$
Fig. 10. MLI output voltage performance using three cells asymmetrical cascaded inverter with ratio $V_{\text{DC1}}:V_{\text{DC2}}:V_{\text{DC3}} = 1:3:9$, $NS = 32$.

Fig. 11 Simulation results of the dynamic response due to step changes in $mi$ at step times are 25ms and 55ms; (a) MLI with $k_1 = 2$, $S = 2$, $NS = 4$ and $l = 2$; (b) MLI with $k_1 = 3$, $S = 3$, $NS = 16$ and $l = 4$. 
VI. NUMERICAL EXAMPLES

In these two numerical examples, the output voltage waveform in figure 6(a) and figure 6(d) will be explained in detail, moreover parameters related to these waveforms will be explained in detailed too.

Example 1: This case is related to input parameters; $S = 2$ and $k_1 = 2, m_i = 0.25$ (calculated based on $h_1$).

Then using (1), (2) and (4) yields $M = 3, N = 2, M + 1 = 7, and NS_{\min} = 4 = (k = 1, 2, 3, 4)$ respectively.

From (32), $l = 2$, and the number of group of equations is $l + 1 = 3$. Therefore the 3rd and 5th harmonics are cancelled in addition to controlling $h_1$. Matrix $A$ and Matrix $B$ can be calculated by following the general procedure explained in section IV or from table 3 as follows:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 1 & 1 \ 1 & 1 & -1 & -1 & 1 \ 1 & -1 & 1 & -1 & 1 \ 1 & -1 & -1 & 1 & -1 \ \end{bmatrix} ; \quad B = \begin{bmatrix} \pi/3 & \pi/3 & \pi/3 & \pi/3 \ 0 & 0 & 1/3 & 1/3 \ 1/3 & 1/3 & 0 & 0 \ 1/3 & 1/3 & 0 & 0 \ \end{bmatrix} ; \quad b3$$

From (27) and (28) (NS=4); $b3$ can be calculated and it is found to be, $b3 = 5.5641$

Thus values of matrix ‘$X$’ from (25) can be found by multiplying the inversion of matrix $A$ in matrix $B$, thus the theoretical solutions of $X = [2.2288 \ -1.1816 \ -0.5533] = 1.6005$

The same results of the theoretical solutions of ‘$X$’ can also be found by direct substituting in (35), $m_i = 0.25$ and $M = 3$, which results in: $X = [2.2288 \ -1.1816 \ -0.5533] = 1.6005$

Apply the last decision on the flow chart of figure 4 (taking the absolute values of $X$ and checking whether the value is within the correct bound or not ($|\alpha_k| < \pi / 2$)) yields

$$X = [\pi - 2.2288 \ 1.1816 \ 0.5533 \ -1.6005] = [0.9128 \ (C_k = 1) \ 1.1816(C_k = 1) \ 0.5533(C_k = 1) \ 1.5411(C_k = 1)]$$

Taking the absolute values of ‘$X$’ and rearranging them yields; $X = [0.9128 \ (C_k = 1) \ 1.1816(C_k = 1) \ 0.5533(C_k = 1)]$

It could be noticed that $C_k = 1$ means that the output voltage has a falling edge at this switching angle while $C_k = 1$ refers to a rising edge output voltage. From the numerical results, it is clear that the output voltage in this case has a falling edge at $\alpha_1$, rising edge at $\alpha_2$, rising edge at $\alpha_3$ and falling edge at $\alpha_4$ as shown in figure 6(a).

Example 2:

This case is related to input parameters; $S = 2$ and $k_1 = 2, m_i = 1.03$ (calculated based on $h_1$).

Then using (1), (2) and (4) yields $M = 3, N = 2, M + 1 = 7, and$ NS_{\min} = 4 = (k = 1, 2, 3, 4)$ respectively.

From (32), $l = 2$, and the number of group of equations is $l + 1 = 3$. Therefore the 3rd and 5th harmonics are cancelled in addition to controlling $h_1$. Matrix $A$ and Matrix $B$ can be calculated by following the general procedure explained in section IV or from table 3 as follows:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \ 0 & 0 & 1 & 1 & 1 \ 1 & -1 & 1 & -1 & 1 \ 1 & 1 & -1 & -1 & 1 \ 1 & -1 & -1 & 1 & -1 \ \end{bmatrix} ; \quad B = \begin{bmatrix} \pi/3 & \pi/3 & \pi/3 & \pi/3 \ 0 & 0 & 1/3 & 1/3 \ 1/3 & 1/3 & 0 & 0 \ 1/3 & 1/3 & 0 & 0 \ \end{bmatrix} ; \quad b3$$

From (27) and (28) ($SN = 4$); $b4 = 4$ acos ($\pi/4$ $M$ $m_i$ / 4 cos($\pi/6$) cos($\pi/10$)) = 2.9709

Thus theoretical solutions of the switching angles are: $X = [1.5805 \ -0.5333 \ 0.0950 \ 0.9522]$

The same results of the theoretical solution of ‘$X$’ for this case study can also be found by direct substituting in (35), $m_i =$1.03 and $M = 3$, which results in: $X = [1.5805 \ -0.5333 \ 0.0950 \ 0.9522]$

By applying the same procedure above, $X = [\pi - 1.5805 \ -0.5333 \ 0.0950 \ 0.9522] = [1.5611 (C_k = 1) \ -0.5333(C_k = 1) \ 0.0950(C_k = 1) \ 0.9522(C_k = 1)]$

Rearranging the absolute values of ‘$X$’ yields, $X = [\alpha_1 = 0.0950(C_k = 1) \ \alpha_2 = 0.5333(C_k = 1) \ \alpha_3 = 0.9522(C_k = 1) \ \alpha_4 = 1.5611(C_k = 1)]$

From the numerical results, it is clear that the output voltage in this case has rising edges at $\alpha_1, \alpha_2$ and $\alpha_3$ then falling edge at $\alpha_4$ as shown in figure 6(d).

VII. COMPARISON AMONG THE HARMONIC ELIMINATION PROCEDURES

To highlight the performance of the proposed procedure, it is compared to its main counterpart existence procedures based on generalized methods, MLI configurations, computation complexity, continuity of modulation index, and initial guess of variables as shown in table 5. In [24], it used the theory of resultants based on trigonometric identities to develop an equivalent set of polynomial equations from the transcendental equations. As the number of harmonics to be eliminated is increased, the method suffers from the computation complexity due to the increasing number of polynomial degree. In [28], it solved the harmonic equations using the classical PI control technique. The PI is used to tune the values of switching bounds on an initial guess to a single angle value and finally the stipulated harmonics are diminished. The initial guess for the angle leads to generate a single iteration loop that increases the computation required. Moreover the method is limited only to three switching angles. In [29], it proposed a solution of the harmonic equations using univariate linear equations. The proposed method generates switching angles that has a ratio of 0.5 between any switching angle and the next one. The outputs of this procedure are the DC sources values. The method is simple with reduced computational complexity. However, the method is very sensitive to the DC sources variations and also it is not valid for symmetrical MLI. The algorithm in [31] proposed a simple, cost-effective and real-time implementation method, to solve the harmonic equations. This method is limited to a five level inverter, moreover the procedure has four calculations loops resulting in large computation complexity. The proposed method in [32] introduced a simple, cost-effective, real-time implementation, and reduced computation complexity method, to solve the harmonic equations. The method received the number of MLI to generate the switching angles initial guess.

The drawbacks explained in this comparison discussion. The proposed method in this article proposed an extension procedure to the method proposed in [32] that overcomes all the drawbacks explained in this comparison discussion. The proposed method is simple, cost effective, real-time implementation, reduced computational complexity, and generalized for symmetrical or unsymmetrical cascaded MLI at any levels without the need for using loops of calculation or angles initial guess.
VIII. CONCLUSION

In this article, a general mathematical solution using SHE for symmetrical and unsymmetrical MLI with switching angles $N_S$, $N$ = maximum available number of output voltage levels, and $k$ = number of eliminated harmonics; has been introduced to generate MLI switching angles $\theta_1, \theta_2, \ldots, \theta_k$. General linear harmonic equations’ system has been formulated in the form $(A \times = B)$ which has easy and simple mathematical solution. A general algorithm was introduced and solved which enables the elimination of $k$ ($N_S = 2^k$) harmonics and their respective odd multiples in addition to control the fundamental component. Only a single element in the $B$ matrix is needed to be updated as the modulation index changed which has a reduced computational time. Since the system was linear, the resulted solution was very accurate. The algorithm was validated through both simulation and experimental results.

The proposed control algorithm has the following advantages as it is:
- Generalized for symmetrical or unsymmetrical cascaded MLI
- Ability to operate at any required levels’ number (even or odd) for the MLI
- Simple, cost effective, reduced computational complexity as it use single loop of calculation and does not require angles initial guesses.
- Real time implementation.
- Ability to control the fundamental voltage by having the modulation index in the developed equations/blocks.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial support received from the Scientific Research Deanship, Taif University, KSA, through Grant No. 1-439 – 6072.

REFERENCES

[14] F. Filho, L. Tolbert, C. Yue and B. Ozpineci, "Real-time selective harmonic minimization for multilevel inverters connected to solar panels..."


**Biographies**

Mahrous Ahmed received the B.S. and M.Sc. degrees in electrical engineering from Assiut University, Assiut, Egypt, in 1996 and 2000, respectively, and the Ph.D. degree in electrical engineering from University of Malaya, Kuala Lumpur, Malaysia, in 2007. Since 2007, he has been an assistant professor with the Aswan Faculty of Engineering, Aswan University, Aswan, Egypt. In 2014 he become an associate professor. Currently, he is an associate professor at faculty of engineering, Taif University, KSA. Dr Ahmed has awarded more than 10 research funded projects in the field of power electronics applications. He has published more than 70 papers in an international sited journals and conferences. His research interests are power conversion techniques and power electronics applications.

Mohamed Orabi (SM’08) received the Ph.D. degree from Kyushu University, Fukuoka, Japan, in 2004. He is currently a Professor at Aswan University, Aswan, Egypt. He is the Founder and the Director of the Aswan Power Electronics Application Research Center, Aswan University. He was with Empirion Inc. and Altera Corp. (June 2011–July 2014). He has published about 200 papers in international conference proceedings and journals. His research interests include dc–dc and PFC converters, integrated power management, nonlinear circuits, and inverter design for renewable energy applications. Dr. Orabi is an Associate Editor of IET Power Electronics and a Guest Editor for the IEEE JOURNAL OF EMERGING AND SELECTED TOPICS IN POWER ELECTRONICS. He received the 2002 Excellent Student Award of the IEEE Fukuoka Section and the Best Young Research Award from the IEICE Society, Japan, in 2004. He also received the SVU Encouragement Award for 2009 and the National Encouragement Award in 2010.

Sherif S. M. Ghoneim Received B.Sc. and M.Sc. degrees from the Faculty of Engineering at Shoubra, Zagazig University, Egypt, in 1994 and 2000, respectively. Starting from 1996 he was a teaching staff at the Faculty of Industrial Education, Suez Canal University, Egypt. Since end of 2005 to end of 2007, he was a guest researcher at the Institute of Energy Transport and Storage (ETS) of the University of Duisburg-Essen-Germany. In 2008, he earned Ph.D. Degree in Electrical power and machines, Faculty of Engineering-Cairo University (2008). He joins the Taif University as an assistant professor in the Electrical Engineering Department, Faculty of Engineering. His research focuses in the area of Grounding systems, Dissolved gas analysis, Breakdown in SF6 gas and artificial intelligent technique applications.

Mosleh M. Al-Harthi was born in Taif, Saudi Arabia, on October 15, 1966. He received the B.Sc. degree in electronics technology and engineering from Indiana State University, Terre Haute, USA, in 1996 and the M.S. degree in electronics technology and engineering from Indiana State University, Terre Haute, USA, in 1997. He received the Ph.D. degree in electrical engineering from Arkansas University, Fayetteville, USA 2001. He was an assistant professor at College of Technology in Jeddah, Saudi Arabia form 2001 till 2009. He is currently working as a professor at the Electrical Engineering Department, Taif University, Saudi Arabia. Now he works as a dean of college of Engineering, Taif University. His research interests are in the areas of control engineering, electronics, and signal processing.
Dr. Basem Alamri currently works as assistant professor of Electrical Engineering, College of Engineering, Taif University. He did his B.Sc degree (with 1st honor) in electrical engineering from KFUPM. Then, he received two M.Sc. degrees (with distinction) in Electrical Power Systems and Sustainable Electrical Power from King Abdulaziz University, Jeddah, KSA and Brunel University, London, UK 2007, 2008 respectively. Dr. Basem got the PhD in Electrical Power Engineering from the Brunel University, London, 2017. His research interests focus on power systems, power quality, power filter design and smart grids with particular emphasis on the integration of renewable energy sources with power grids. Dr. Basem is a member of many international and local professional organizations. He is also a Certified Energy Auditor (CEA®) and a Certified Energy Manager (CEM®) by the Association of Energy Engineers (AEE), USA. Dr. Basem has received many awards and prizes, including a certificate from Advance Electronics Company (AEC) in recognition of outstanding academic achievement during the BSc program at KFUPM. He also received the National Grid (NG) prize, the power grid operator in the UK, for being the top distinction student for the MSc of SEP program, Brunel University, London, UK.

Farhan A. Salem was born in Jordan. He received his M.Sc in Mechatronics of production systems, from Moscow state academy of instrument making and information technology from Moscow-1996. And he received his Ph.D. in Production and Manufacturing/Mechanical and physico-technical processing machines and tools from university of Russia/Moscow – July/2000. Now he is an associate Professor in Taif University, industrial engineering program, Dept. of Mechanical Engineering.

Prof. Dr. Saad Mekhilef is an IET Fellow and IEEE senior member. He is the associate editor of IEEE Transaction on Power Electronics and Journal of Power Electronics. He is a Professor at the Department of Electrical Engineering, University of Malaya since June 1999. He is currently the Director of Power Electronics and Renewable Energy Research Laboratory-PEARL. He is the author and coauthor of more than 300 publications in international journals and proceedings. He is actively involved in industrial consultancy for major corporations in the power electronics projects. His research interests include power conversion techniques, control of power converters, renewable energy, and energy efficiency.