Abstract- This paper focuses on the development of a robust Fractional Order Extremum-Seeking (FOES) scheme for controlling the ABS of vehicle motion system by continuously adjusting the brake torque. Extremum Seeking (ES) known for its properties to seek the maximum of unknown function and Fractional Order (FO) control for its robustness in the presence of parameter variations and the disturbances are employed to control the wheel slip rate in emergency braking maneuver. The aim of an ABS is to minimize brake distance while steer ability is retained even under hard braking, to understand the fundamental physical effect, which leads to wheel blocking during braking. Simulations under various road conditions are performed to demonstrate the effectiveness of the proposed control scheme.

Keywords: Extremum Seeking, Fractional Order Control, Fractional Calculus, ABS control problem, Friction force coefficient (FFC).

1. INTRODUCTION

Anti-lock braking systems (ABS) are an important tool in the automotive industry. They allow the vehicle to stop faster and make safer turns when the wheels are prevented from locking. ABS design was initially proposed to deal with braking on slippery surfaces, i.e., to prevent the wheels from locking and skidding [1, 2]. The principle aim of this research is to satisfy increasing safety, efficiency and comfort requirements. Significant progress has been achieved by the introduction and development of active safety systems like anti-lock braking systems (ABS), the anti-lock braking system (ABS) is widely used in automobiles. In an emergency braking situation the wheels of a vehicle tend to lock quickly, increasing the longitudinal slip ratio of the vehicle. The slip ratio, while braking, is defined as the difference between the speed of the vehicle and the circumferential speed of the tire, divided by the speed of the vehicle [3].

When the lock of the wheel is total (β=1), vehicle steering control and stability diminishes, and the braking distance normally increases. Therefore, the goal of the braking control system is to maintain the slip ratio within the values which obtain the maximum adherence coefficient (see Figure 4). Achieving this goal is difficult, because the maximum adherence zone varies with many parameters, for example adherence conditions between the road and
the wheel, vertical load, inflation pressure, slip angle, and soon. Therefore, the ABS control systems need to know the exact point within the adhesion curves $[\mu - s]$.

Fractional order control (FOC) is one of the fields which have attracted a lot of research efforts, with many encouraging results such as CRONE control [4], fractional PID control [5], fractional order optimal control [6], fractional adaptive control [7] ...etc. Since a decade, particularly in the adaptive control area many researchers have proposed several fractional adaptive control schemes such as the fractional model reference adaptive control [8], the fractional adaptive high gain control [9], the fractional adaptive sliding mode control [10] and the fractional adaptive IMC based control [11]. Application of these fractional order control schemes are various covering electrical machines [12], mechanical systems [13], finance and economics [14, 15], biological systems [16], signal processing [17], robotics [18] and renewable energy systems [19].

In the present work, we are interested in applying a new fractional adaptive control strategy based on extremum seeking control to optimize brake distance while [20]; steer ability is retained even under hard braking, to understand the fundamental physical effect [21, 22], which leads to wheel blocking during braking. In the last years Extremum Seeking algorithms have been applied in various research areas related to ABS [23]. A field in which the Extremum Seeking technique has proved to be extremely powerful tool is to maintain the slip ratio within the values which obtain the maximum adherence coefficient [24, 25].

The main purpose of this work is the introduction of the Fractional Order technique in the Extremum Seeking approach which presents several advantages over the conventional control techniques such as easy implementation and is expected to improve ABS system utilization efficiency under overcoming disturbances and uncertainties. The implementation of ABS using Fractional Order Extremum Seeking (FOES) achieves these two keys of vital importance. The idea of Fractional Order Extremum Seeking (FOES) algorithm has been also introduced by Malek et al. [26] with a different control configuration, based on the disturbed ES scheme, showing interesting results on a solar energy system. This study propose a robust control method for ABS which combines Fractional Order Control and Extremum Seeking for wheel slip control in emergency braking case, and will ensure the stability of the proposed closed loop control system for abrupt or fast variations of the external conditions.

The paper is organized in five sections. Section 2 presents an introduction to fractional order operators and systems with the approximation method used for their implementation. In section 3, the proposed FOES novel technique is presented and compared to the classical approaches. A feed-back scheme ABS system is described in section 4, using the proposed ESC method. Simulation results of the ABS system are discussed. Finally, some concluding remarks and future work are presented in section 5.
2. FRACTIONAL ORDER BASIC CONCEPTS

2.1 Fractional derivatives and integrals

Fractional calculus is a generalization of the integration and differentiation to non-integer order fundamental operator $aD_t^\mu$ where $a$ and $t$ are the bounds of the operation. The fractional-order differentiator can be presented by a general operator given by ([4]):

$$aD_t^\mu = \begin{cases} \frac{d^\mu}{dt^\mu} & \Re(\mu)>0 \\ 1 & \Re(\mu) = 0 \\ \int_a^t (d\tau)^{-\mu} & \Re(\mu)<0 \end{cases}$$

(1)

Where $\mu$ is the order of derivative or integral $\Re(\mu)$ is the real part of $\mu$.

The mathematical definition of fractional derivatives and integrals has been the subject of several descriptions. The three most frequently used definitions for the general fractional differintegral are: the Grunwald-Letnikov (GL) definition, the Riemann-Liouville (RL) and the Caputo definition [27, 28].

One of the most used definitions of the general fractional integro-differential operator is the Riemann-Liouville (RL) fractional order integral of order is defined as:

$$aD_t^{-\mu} f(t) = \frac{1}{\Gamma(\mu)} \int_a^t (t-\tau)^{\mu-1} f(\tau) d\tau$$

(2)

while the definition of fractional-order derivatives is:

$$aD_t^\mu f(t) = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \left[ \int_a^t (t-\tau)^{n-\mu-1} d\tau \right]$$

(3)

where:

$$\Gamma(x) = \int_0^\infty y^{x-1} e^{-y} dy$$

(4)

$\Gamma(.)$ is the Euler's Gamma function, $a$ and $t$ are the limits of the operation, and $\mu$ is the number identifying the fractional order. In this paper, $\mu$ is assumed as a real number that satisfies the restrictions $0<\mu<1$. Also, it is assumed that $a=0$. The following convention is used: $aD_t^\mu = D_t^\mu$.

The other approach is Grünwald-Letnikov definition of fractional order integral, given by:

$$aD_t^{-\mu} f(t) = \lim_{h \to 0} \left[ \frac{t-a}{h} \right] \sum_{r=0}^{\left[ \frac{t-a}{h} \right]} \frac{\Gamma(\eta-\mu+1)}{r!\Gamma(\mu)} f(t-rh)$$

(5)
where \([x]\) stands for the greatest integer not exceeding \(x\), while the definition of fractional-order derivatives is:

\[
\dot{a}D_1^\mu f(t) = \lim_{h \to 0} h^{-\mu} \sum_{r=0}^{[t-a]/h} (-1)^r \frac{\Gamma(\mu+1)}{r!\Gamma(\mu-r+1)} f(t-rh)
\]  

(6)

where the binomial coefficients \((r > 0)\) are given by:

\[
\binom{\mu}{0} = 1, \quad \binom{\mu}{r} = \frac{\mu(\mu-1)...(\mu-r+1)}{r!}
\]

(7)

Many approaches have been proposed to implement fractional order systems. The numerical simulation of such systems depends on the way to modelize the fractional derivative operator. A first approach consists on a discretizing the derivative operator according to the Grünwald method. This technique is very simple to use, but the simulation requires, for each step, the computation of sums of increasing dimension with time. Another approach needs an approximation method in order to obtain an equivalent rational transfer function, by using a specific representation in the frequency domain, and for the present work we will use the so called singularity function method proposed by Charef et al. [29], see the next section.

2.2 Singularity Function Approximation Method

In order to implement fractional order operators we shall need an approximation method in order to obtain equivalent rational transfer functions. There are many methods both in time-domain and frequency domain, and for the present work we will use the so called singularity function proposed by [29], which is very close to Oustaloup’s method ([4]), allowing the approximation of fractional order transfer function with rational ones. This method is very easy to implement and is based on the approximation of a function of the form:

\[
H(s) = s^\mu, \quad \mu \in \mathbb{R}^+
\]

(8)

by a quotient of polynomials in \(s\) in a factorized form:

\[
\hat{H}(s) = K_D \prod_{i=0}^{N} \frac{(1+\frac{s}{z_i})}{(1+\frac{s}{p_i})}
\]

(9)

Computed on the frequency interval \(\omega \in [\omega_h, \omega_c]\) such that:

\[
K_D = \omega_c^\mu,
\]

where \(\omega_c\) is the cutting frequency computed as:

\[
\omega_c = \omega_h \sqrt{10 (\frac{10}{\zeta})^{1/(\mu+1)}}
\]

and the coefficients are computed for obtaining a maximum deviation from the original magnitude response in the frequency domain of \(\zeta\) dB. Defining:
The poles and zeros of the approximated rational function are obtained by applying the following formulas:

\[ z_0 = \omega_k \sqrt{b}, \quad z_i = z_0(ab)^i, \quad p_i = a z_0(ab)^i \]

The number of poles and zeros is related to the desired band-width and the error criteria used by the expression:

\[ N = \left\lfloor \frac{\log(w_{\text{max}})}{\log(ab)} \right\rfloor + 1 \]

### 3. FRACTIONAL EXTREMUM SEEKING CONTROL

The foundations of Extremum-seeking control returns back to the early 1920s in the work of Leblanc on the search of the resonance peak of an electromechanical system [30]. In the 1960s, there were also important contributions, among which the works of Korovin and Morosanov constitute the most significant advances [30]. The nonlinear and adaptive nature of such control is clearly shown in [31]. Although there are different Extremum seeking algorithms, an important analytic effort should be made in order to establish the stability regions of a great number of reported applications [32].

In this paper, we propose a new control scheme based on Extremum-Seeking (ES) combined with Fractional-Order Systems (FOS) [33]. This auto-tuning strategy involving a fractional order integral action, is developed to optimize the system response without external dithering, exploiting disturbances already present in the control system.

#### 3.1 Control Strategy

In this section we present a new robust control scheme for the class of first order linear dynamical systems based on the approach proposed by Carnevale et al. [34] called Dynamic Extremum Seeking, which aim is to end a reference signal for a dynamical system such that an unknown cost function of its output is minimized or maximized.

We address the problem of finding the global minimum of a static unknown map \( g(.) : \mathbb{R} \to \mathbb{R} \) whose input is affected by a disturbance \( t \to d(t) \) as shown in Figure 1. As in [34], we assume that the map \( g() \) satisfies the following assumption.

**Assumption 1:** The unknown map \( g(.) : \mathbb{R} \to \mathbb{R} \) is locally Lipschitz and locally bounded.

In the considered extremum seeking synthesis problem, we assume to exploit a probing signal \( \theta \), which is constrained to act on the function \( g(.) \) through a scalar dynamical system having the following simplified form

\[ E \dot{y} = -y + \theta \quad (10) \]

where \( E \) is a positive scalar. The input and the output of the static map \( g(.) \) are assumed to be two measurable signals, as represented in Figure 1, corresponding to:

\[ u_g = y(t) + d(t) \]
\[ y_g = g(y(t) + d(t)) \quad (11) \]
As in the initial integer order control scheme proposed [26], there are some boundedness conditions on the disturbance signal \( d \) exciting the function \( g(\cdot) \) to be fulfilled, as stated in the Assumption 2.

**Assumption 2:** The disturbance \( d() \) is bounded and has bounded time derivative, namely there exist positive numbers \( \overline{d} \) and \( \overline{d}_\dot{d} \) such that \( |d(t)| \leq \overline{d} , |\dot{d}| \leq \overline{d}_\dot{d} \) for all \( t \geq 0 \).

The control strategy implies also two extremum seeking laws assigning \( q \), assuming that the signals in (11) are available. Moreover, one has to make a strong assumption on the ability to obtain an ideal derivative of the input and the output of the unknown map \( g(\cdot) \), i.e. we assume to know

\[
\begin{align*}
\dot{z}_1(t) &= \dot{u}_x(t) = \dot{y}(t) + \dot{d}(t) \\
\dot{z}_2(t) &= \dot{y}_x(t) = \frac{\partial g(y(t) + d(t))}{\partial y}(\dot{y}(t) + \dot{d}(t))
\end{align*}
\]  

(12)

It is clear that for actual implementation we will use approximations of the signals in (12).

The proposed control scheme is obtained by introducing a fractional order integral \( 1/s^\lambda \) to the initial control scheme of Carnevale et al. [34] as represented in Figure 2. The unknown map is \( g(\cdot) \), which inputs are the output of the first order linear dynamical system \( y \) and the disturbance signal \( d \).

![Figure 1 Block diagram of the scheme under consideration.](image1)

![Figure 2 The dynamical extremum seeking scheme using fractional order integration.](image2)
The parameter $\varepsilon > 0$ sets the convergence speed of $y$ to $\delta_k \theta$, where $\delta_k > 0$ is the static gain of the linear plant. The output of a unit saturation is fed with the signal $k_2 z_1(t) z_2(t)$ and is integrated by fractional order $1/s^\lambda$ and multiplied by $k_1$, yielding the plant reference $\theta(t)$, with positive scalars $k_1$ and $k_2$.

By introducing FO integration we get:

$$\frac{d^\lambda \theta}{dt} = -k_1 \text{sat}(k_2 z_1(t) z_2(t))$$

so

$$s^\lambda \theta = -k_1 \text{sat}(k_2 z_1(t) z_2(t))$$

and the control law becomes,

$$\theta = \frac{-k_1 \text{sat}(k_2 z_1(t) z_2(t))}{s^\lambda}$$

with $k_1 > 0$, $k_2 > 0$, and fraction order $0 \leq \lambda \leq 1$. The block diagram of the corresponding closed-loop system is represented in Figure 2.

In [8, 35] they have noticed that the introduction of fractional order integration in adaptation algorithms allows to increase the reference amplitude variation domain where the closed loop stability is maintained. In fact, this stability control objective is better achieved with a sufficiently small regulating parameter $k_2$. This is why we can stabilize the adaptive control loop by using fractional order integration where an integer control rule fails.

**Remark 1:** The selection of $\theta$ as in (15) guarantees that $\theta \leq \frac{k_1}{s^\lambda}$, an appealing property of the probing signal since it avoids exciting possible high frequency dynamics. Moreover, whenever necessary, the method allows meeting the rate saturation constraints of the physical devices [36].

### 4. APPLICATION OF FOES TO THE ABS SYSTEM

#### 4.1 Problem Description

Due to the non-linearity of the dynamics and uncertainty in the braking systems, the design of ABS is difficult. The character of the friction force acting on the tires has a maximum for allow (non zero) wheel slip and decreases as the slip increases. Standard ABS systems apply braking pressure in a rapid intermittent fashion. In some of them, the purpose of the intermittent action is to “seek” the maximum of the friction characteristic.

In this paper, we study the ABS design via extremum seeking control schemes; our goal is to design a control algorithm for the braking torque to achieve maximal friction force without prior knowledge of the optimal slip. The wheel model and the perturbation based extremum seeking design are due to Ariyur and Krstic (Chap.6 of [37]).

Consider the single wheel model depicted in Figure 3. The wheel dynamics are given by

$$m x_1 = -N\mu(\beta)$$

$$I x_2 = -B x_2 + NR\mu(\beta) - u$$

where $x_1$ is the linear velocity $v$ and $x_2$ is the angular velocity $\Omega$ of the wheel, $m$ is the mass, $N = mg$ is the weight of the wheel, $R$ is the radius of the wheel, $I$ is the moment of inertia of
the wheel, $Bx_2$ is the braking friction torque, $u$ is the braking torque, $\mu(\beta)$ is the friction force coefficient and the wheel slip $\beta$ is expressed as:

$$\beta = \frac{x_1 - Rx_2}{x_1}$$  \hspace{1cm} (17)

for $Rx_2 < x_1$.

Several tire friction models describing the nonlinear behavior are reported in the literature. There are static models as well as dynamic models. The most reputed tyre model is by [3] and by [38], also known as “magic formula” and it is derived heuristically from experimental data. Here we use the expression in [39] is derived with similar methodology where $\mu$ is expressed as a function of the wheel slip $\beta$, and the vehicle velocity, $v$.

$$\mu(\beta) = [C_1(1 - e^{-C_2\beta}) - C_3\beta]e^{-C_4\beta v}$$  \hspace{1cm} (18)
Where the parameters are specified for different road surfaces. See Table 1 [39].

The parameters in (18) denote the following:

\( C_1 \): maximum value of friction curve.

\( C_2 \): friction curve shape.

\( C_3 \): friction curve difference between the maximum value and the value at \( \beta = 1 \).

\( C_4 \): wetness characteristic value and is in the range 0.02-0.04s/m.

**Remark 1:** the parameters \( C_1, \ldots, C_4 \) are chosen from Table 1 for asphalt wet surface condition.

<table>
<thead>
<tr>
<th>Surface conditions</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt, dry</td>
<td>1.029</td>
<td>17.16</td>
<td>0.523</td>
</tr>
<tr>
<td>Asphalt, wet</td>
<td>0.857</td>
<td>33.822</td>
<td>0.347</td>
</tr>
<tr>
<td>Concrete, dry</td>
<td>1.1973</td>
<td>25.168</td>
<td>0.5373</td>
</tr>
<tr>
<td>Cobblestones dry</td>
<td>1.3713</td>
<td>6.4565</td>
<td>0.6691</td>
</tr>
<tr>
<td>Cobblestones wet</td>
<td>0.4004</td>
<td>33.708</td>
<td>0.1204</td>
</tr>
<tr>
<td>Snow</td>
<td>0.1946</td>
<td>94.129</td>
<td>0.0646</td>
</tr>
<tr>
<td>Ice</td>
<td>0.05</td>
<td>306.39</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4.2 Results and Discussion

The proposed method is implemented in the software environment as a controller and the obtained results of the proposed control system are compared to the ones of the control system with the conventional method.

We can also note the substantial improvement in the friction force coefficient (FFC) for different values of the integral action fractional order compared to the classical case when \( \lambda = 1 \). Another remarkable advantage of the proposes FOES strategy is that the FFC is maintained at its desired value after the convergence phase for practically all the fractional order values as it can be seen in Figure 5. Now, to get the best integral action fractional order \( \lambda \) in terms of error minimization between the power output \( y \) and the maximal power value \( y_m \), the quadratic error criteria given by:

\[
J_{\lambda} = \sum_{k=0}^{N} \left( y(k\Delta) - y_m(k\Delta) \right)^2
\]

is calculated when \( \lambda \) is varied from 0.3 to 1.3; where \( N \) is the width of the working time window and \( \Delta \) is the time sampling rate. Figure 7 shows the plot of the quadratic error \( J_{\lambda} \) as a function of the fractional order \( \lambda \). From Figure 7, the smallest quadratic error corresponds to the parameter \( \lambda^* = 0.8 \).
For the particular fractional parameter value $\lambda^*$ in the proposed FOES control scheme, FFC tracker converges in about 0.18 sec; whereas for the modified integer order ESC-based controller of convergence time is about 0.3 sec as illustrated in Figure 6. It can be noted that for most of the existing classical ESC-based approaches the attenuation of the drop occurs when FO system introduced.

![Figure 5 FFC: Braking Torque](image1)

![Figure 6 FFC: Output for different fractional integral order $\lambda$](image2)

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>Gradient update law gain 1</td>
<td>16</td>
</tr>
<tr>
<td>$K_2$</td>
<td>Gradient update law gain 2</td>
<td>1</td>
</tr>
<tr>
<td>M</td>
<td>Min/Max Saturation</td>
<td>0.01</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

This paper proposes an efficient control algorithm combining extremum seeking and fractional calculus for the braking torque to achieve maximal friction force without prior knowledge of the optimal slip. We show that the FOES algorithm converges to the global optima faster than the traditional (integer order) approach. Introducing FO operator to ES serves to improve the regulation of the plant outputs in approaching the optimal point.
The proposed FO-ES control scheme is superior to the classical ES one because it offers a supplementary tuning parameter that is the fractional order operator, enabling more behaviour performance and robustness against disturbances and noise.

REFERENCES


