Effect of solid volume fraction and tilt angle in a quarter circular solar thermal collectors filled with CNT–water nanofluid

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A B S T R A C T
Solar thermal collectors have significant importance due to its wide use in solar thermal technology. Augmentation of heat transfer is a key challenge for solar thermal technology. A quarter circular solar thermal collectors is investigated throughout the paper introducing carbon nanotube (CNT)–water nanofluid in the cavity. Tilt angle of this type of collector plays a vital role and heat transfer can be maximized for a particular tilt angle and solid volume fraction of the nanofluid. Galerkin weighted residual of FEM has been applied for the numerical solution of the problem. Grid independency test and code validation have been assessed for the accuracy of numerical solution. In this paper a wide range of solid volume fraction (δ = 0 to δ = 0.12) and tilt angle (ϕ = 0 to ϕ = 60°) has been investigated for Rayleigh number (Ra = 105–108) with varying dimensionless times. It has been found that both solid volume fraction and tilt angle play vital roles for the augmentation of heat transfer and a good heat transfer characteristic can be obtained by compromising between these two parameters. The results are shown using streamline, isotherm contour and related graph and chart.

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1. Introduction

For the past few decades solar thermal collector has been given a vigilant attention due to its importance in using solar energy which is a major source of renewable energy. Solar thermal collector has a wide range of application as it is used in solar water heating system, solar cooling system, solar dryer, solar desalination, solar home system and so on [1–10]. Different types of solar thermal collector are available commercially nowadays and different studies have been carried out to understand the heat transfer which is the major aspects of solar thermal technology. Basically convection and radiation heat transfer are responsible for the heat transfer in solar thermal technology.

Natural convection heat transfer has been studied to a great extent which has a certain limitation of using conventional fluid such as air and water for the heat transferring medium. To augment the heat transfer of nanofluid is justified in recent years [11–15]. Solid suspension of metal, metal oxide and carbon based material which have a very high thermal conductivity are suspended in the base fluid with a view to increasing the thermal conductivity of the base fluid. Here base fluid works as the carrier medium. Various nanofluids are used to enhance heat transfer rate in practical applications. Among them CuO–water, TiO2–water, and Al2O3–water nanofluid are very common and have been used commercially. Besides conventional nanofluids, AgO-water, diamond-water and nanodiamon-mineral oil nanofluids have gained much attention. However, CuO-water nanofluid has shown a tremendous potential to be the best of the lot due to its better performance and availability. Related literatures can be found in [16–20]. In recent years carbon nanotube (CNT), and graphene are being studied to investigate the heat transfer having the suspension of CNT and graphene in the base fluid. Godsona et al. [21] reviewed the enhancement of heat transfer using different nanofluids and they reported that nanofluid has a great potential for further research in heat transfer enhancement. Kumaresan et al. [22] studied convective heat transfer characteristics of CNT nanofluids in a tubular heat exchanger of various lengths for energy efficient cooling/heating system. Kamali and Binish [23] numerically investigated heat transfer enhancement using carbon nanotube-based non-Newtonian nanofluids. Xu et al. [24] studied the energy dissipation behavior of multiwalled carbon nanotube (MWCNT) based nanofluid and reported the optimum length of the carbon nanotube. Etefagh et al. [25] investigated on the thermal properties of engine oil (SAE 20) based MWCNT nanofluid and reported that 0.1 wt.% improved...
thermal conductivity of 13.2% and flash point 6.7%. Halefendili et al. [26] recently studied the change of viscosity on the influence of temperature and concentration for carbon nanotube water based nanofluids. Kumaresan et al. [27] studied the convective heat transfer characteristic of a secondary refrigerant based CNT nanofluid in a tubular heat exchanger and have reported that the effect of friction factor is very negligible for the 0.15 vol.% of the nanofluid. Harish et al. [28] studied the reason behind the enhancement of thermal conductivity of ethylene glycol based single walled carbon nanotube inclusions and observed that tri-dimensional structure of the CNT is responsible for this. Yousefi et al. [29] experimented on MWCNT–water nanofluid on the efficiency of a flat plate solar collector and reported that an increase of the weight fraction from 0.2 to 0.4% increases the efficiency significantly. Javadi et al. [30] investigated on the performance improvement of solar collector by using different nanofluids and reported that thermal conductivity has a significant effect on improving efficiency of the solar absorption collector. They also reported that there is a lack of study on the transmittance and optical property effect on efficiency analysis and showed economical consideration as a big challenge. The shape of the enclosure depends on the practical case study and a quarter circular shapes have been presented in this paper which is quite similar to a solar collector. Convection heat transfer has been analyzed for this geometry. Normally in the study of natural or forced convection square, rectangular, trapezoidal shape enclosure has been given enormous importance and a lot of numerical simulation and experimental work are available. Introducing nanofluid in this kind of enclosure also has been extensively studied. Triangular shape enclosure gets a little attention on the study of convection heat transfer though it has some very practical case. Circular shape enclosure is quite a new type of enclosure in the field of numerical and experimental heat transfer studies [31–34]. Enclosures of this sort have a mighty chance of being used in the solar thermal collectors, duct designs, heat exchangers and so on. Introduction of nanofluid to such enclosure is an idea that has never been explored before the present work. This novel idea has a very high applicability in solar collector modeling, design, analysis and optimization process.

The effect of solid volume fraction plays a very vital role in heat transfer augmentation. It is expected that an increment in the solid volume fraction of solid particles in the base fluids should enhance the heat transfer rate due to the higher thermal conductivity of the resulting nanofluids [35–37]. Tilt angle also plays a significant role. Effect of tilt angle of the enclosure is analyzed in the paper keeping in mind that the position of the sun changes with time. For both focusing and non-focusing type of collector this angle becomes one of the major parameters which govern the heat transfer rate. Handoyo and Djatmiko Ichsani [38] studied the optimal tilt angle for a solar collector and found that a tilt angle about 40° shows the best result. Jafarkazemi and Saadabadi [39] also studied the tilt angle for solar collector in Abu Dhabi, UAE and reported the optimal tilt angle as 22° which is very much close to the latitude of UAE. The authors also proposed that the tilt angle of the solar collector must be changed twice in a year as the optimum tilt angles vary significantly (−9 to 52°) with the change of the month. Bakirci [40] investigated the tilt angle of the solar collector for a case study of Turkey and reported 65° as an average tilt angle for Turkey. Yadav and Chandel [41] reviewed the tilt angle for maximizing solar irradiation and reported that this tilt angle varies from both time to time and location to location as solar irradiation is different on different parts of the world and it is also different on different months.

From the abovementioned issues it has been clear that both the use of nanofluid in solar collector and the varying tilt angles are justified. And nowadays different studies are being carried out introducing different nanofluids inside the solar thermal collector. The aim of this paper is to propose a novel model for solar thermal collector to maximize the heat transfer rate to the solar thermal collector. The effect of solid volume fraction of the CNT–water nanofluid and tilt angle has been shown throughout the paper as it has been found from the previous literature that these two parameters can augment the heat accumulation to the thermal energy collector surface. Moreover the heat transfer in a quarter circular shape geometry has been studied which is very rare in the heat transfer analysis though it has some practical application. The results are presented in the streamline, isotherm contour and related graphical analysis has been done. Since the enclosure introduced in the paper can have an inordinate use in making solar collectors, the analysis presented can help to promote the use of renewable energy especially solar energy.
2. Problem formulation and numerical treatment

2.1. Physical modeling

Fig. 1 shows the details of the problem with a specified co-ordinate system. Basically, this solar thermal collector is quarter circular shape. The physical modeling has been done to simulate the real life phenomena of a quarter circular solar thermal collectors. Basically the horizontal line is the solar thermal collector plate, so the boundary condition is modeled as \( T = T_c + (T_h - T_c) \left( \frac{x}{L} \right) \) as variable thermal condition arises on this boundary. The vertical straight part is modeled as adiabatic assuming no heat transfer for this boundary. The circular boundary of the cavity is made of glass or plastic. These boundary temperature is low as heat goes through it and modeled as \( T = T_c \). The sun’s rays go through this glass or plastic cover and it is expected that the heat is absorbed by the collector plate. These boundaries form a quarter circular shape cavities and for the augmentation of heat transfer to the collector plate nano-fluid has been introduced inside the enclosure. Carbon nanotube (CNT)–water nano-fluid has been introduced which is quite new for heat transfer analysis and has a great potential to enhance the heat transfer rate due to the higher thermal conductivity of the carbon nano tube. Gravity effect has been introduced and the convection is the only mechanism of heat removal. Convection is the only mechanism of heat removal.

2.2. Mathematical modeling

A set of governing equation has been formed assuming that the nano-fluid is a Newtonian fluid and the flow is unsteady laminar flow. Incompressible Navier–Stokes equation has been applied for the two dimensional flow. Boussinesq approximation has been applied to assume the constant thermophysical property of the nano-fluid. Conservation of mass, momentum and energy control the system behavior which has been modeled through the mathematical equation. From the above stated assumption of the two dimensional fluid flow field we can write

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho U)}{\partial X} + \frac{\partial (\rho V)}{\partial Y} = \frac{\partial}{\partial X} \left( \rho \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \rho \frac{\partial V}{\partial Y} \right) + S_{\rho}.
\] (1)

Here dependent non-dimensional variables are designated by \( \varphi \), and the corresponding diffusion and source term respectively are defined by \( \Gamma_{\varphi} \) and \( S_{\varphi} \) and they are summarized in Table 1.

The density of nano-fluid which is assumed to be constant can be expressed as

\[
\rho_{nf} = (1-\delta)\rho_{fl} + \delta \rho_{s}.
\] (2)

In the above equation solid volume fraction \( (\delta) \) has significant effect on heat transfer and the thermal diffusivity of nano-fluid which is quite different from the conventional fluid can be expressed as

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho cp)_{nf}},
\] (3)

where heat capacitance of nano-fluid \( (\rho cp)_{nf} \) can be found by

\[
(\rho cp)_{nf} = (1-\delta)(\rho cp)_{fl} + \delta (\rho cp)_{s}.
\] (4)

In addition, the thermal expansion coefficient \( (\rho\beta)_{nf} \) of the nano-fluid is

\[
(\rho\beta)_{nf} = (1-\delta)(\rho\beta)_{fl} + \delta (\rho\beta)_{s}.
\] (5)

Moreover, dynamic viscosity \( (\mu_{nf}) \) of the nano-fluid can be expressed as

\[
\mu_{nf} = \frac{\mu_{fl}}{(1-\delta)\nu_{fl}}.
\] (6)

Effective thermal conductivity of the nano-fluid can be described as

\[
k_{nf} = k_{s} + 2k_{t} - 2\delta(k_{s} - k_{t})
\] (7)

where, \( k_{s} \) is the thermal conductivity of the nanoparticles and \( k_{t} \) is the thermal conductivity of base fluid is the \( k_{t} \).

Scales which have been applied to obtain the non-dimensional governing equation are presented in Eq. (8).

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{\alpha_{fl} t}{L^2}, \quad U = \frac{uL}{\alpha_{fl}}, \quad V = \frac{vL}{\alpha_{fl}}, \quad P = \frac{(P + \rho \beta g L^2)}{\rho_{fl} \alpha_{fl}^2},
\] (8)

\[
\theta = \frac{\delta \rho \beta f L^3 (T_h - T_c)}{\alpha_{fl} \nu_{fl}}, \quad Pr = v_{fl}/\alpha_{fl}.
\]

In the above equation \( \theta \) is the non-dimensional temperature. Rayleigh number and Prandtl number are designated by Ra and Pr.

Initial and boundary conditions in the dimensionless form for the present problems can be defined by \( \tau = 0 \)

Entire domain : \( U = V = 0, \quad \theta = 0 \)

\[ \tau > 0 \]

on the horizontal wall : \( U = V = 0, \quad \theta = X(1-X) \)

on the round walls : \( U = V = 0, \quad \theta = 0 \)

Table 1

<table>
<thead>
<tr>
<th>Equations</th>
<th>( \varphi )</th>
<th>( \Gamma_{\varphi} )</th>
<th>( S_{\varphi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>U-momentum</td>
<td>( \frac{\mu_{nf}}{\mu_{fl} \alpha_{fl}} )</td>
<td>( -\frac{\partial P}{\partial X} + (\frac{\partial \varphi_{nf}}{\partial X} - (\rho \beta)_{nf}) )</td>
<td>Ra Pr ( \theta )</td>
</tr>
<tr>
<td>V-momentum</td>
<td>( \frac{\rho_{nf}}{\rho_{fl}} )</td>
<td>( -\frac{\partial P}{\partial Y} + (\frac{\partial \varphi_{nf}}{\partial Y} - (\rho \beta)_{nf}) )</td>
<td>Ra Pr ( \theta )</td>
</tr>
<tr>
<td>Energy</td>
<td>( \theta )</td>
<td>( \alpha_{nf} / \alpha_{fl} )</td>
<td>0</td>
</tr>
</tbody>
</table>
on the vertical wall $U = V = 0$. \( \frac{\partial \theta}{\partial x} = 0. \) (9d)

Average Nusselt number has been evaluated for the bottom horizontal heated surface and calculated from the following expression

$$\text{Nu}_{av} = -\frac{k_{nf}}{k_f} \int_0^1 \frac{\partial \theta}{\partial x} dX. \quad (10)$$

Stream function $\psi$ is a mathematical trick which has been introduced to present the fluid motion. Stream function has been defined from velocity components $U$ and $V$. The relationship between the stream function and velocity component for a two dimensional flow has been given by

$$U = \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x}. \quad (11)$$

2.3. Nanofluid

For the last few decades using nano fluid for the augmentation of heat transfer rate is very common. A great deal of the study has been carried out to investigate about different combinations of metal, metal oxide, and carbon made material with different base fluids. Among the available nano fluid very few of them are being commercially used nowadays. Most common commercial nano fluids are $\text{Al}_2\text{O}_3$–water, $\text{TiO}_2$–water, $\text{Cu}$–water, $\text{CuO}$–water and so on. This type of nano fluid has been chosen due to its low cost and availability [42]. Besides this nano fluid there are some emerging nano fluids which are showing extraordinary characteristic in the heat transfer phenomena. Carbon nanotube (CNT)–water and graphene based nano fluid are such kinds of nano fluid. $\text{Ag}$ is a metal with high thermal conductivity (reported to be about 406 Wm$^{-1}$K$^{-1}$) and consequently used as nano material in a thermal fluid frequently. On the other hand carbon nanotubes (single walled) have thermal conductivity in the range of 3500 Wm$^{-1}$K$^{-1}$. This value is about 9 times greater than that of silver. Graphene has more unique properties and characteristic which is used for the enhancement of heat transfer. But one of the constraints for this emerging nano fluid is that the material cost is still so high. And for this reason though they are showing extensively good thermal characteristic we cannot use it as a commercial nano fluid.

2.4. Thermophysical property of nano fluid

The heat transfer rate largely depends on the fluid we are using for the heat transfer. Due to low performance of the conventional fluid nano fluid has been introduced and nano fluids are giving satisfactory performance due to its thermophysical property. For this paper carbon nanotube (CNT)–water nano fluid has been used and its thermophysical property has been presented in Table 2. These thermophysical properties have been taken from the established literature [43–44].

2.5. Numerical procedures

The entire enclosure of the solar thermal collector has been discretized into several elements and governing equations mentioned above have been applied to these discretized elements for the numerical analysis. The elements are of triangular shape as triangular mesh has been used. Galerkin Weighted Residual scheme of Finite Element Analysis is used to analyze the problem numerically. The entire domain was discretized by forming six nodded triangular mesh elements. A set of algebraic equation has been found by applying the specified boundary condition of the problem and they are solved by Newton Raphson iteration technique. The iteration has been carried out until the problem converges. For this process limit of tolerance has been set to $10^{-5}$. So that $|\gamma^{m+1} - \gamma^{m}| \leq 10^{-5}$ where $m$ is the number of iteration and $\gamma$ is the general dependent variable.

2.6. Grid test

Governing equations are applied to the triangular element and it has been found that numerical results vary with the variation of the element number. To check the accuracy of the numerical procedure a grid test has been performed and its results have been presented in Fig. 2. The grid independence test has been performed for the non-dimensional time $\tau$ with solid volume fraction $\delta = 0.04$ and Rayleigh number $Ra = 10^5$. For checking the accuracy of the numerical solution several element numbers have been checked. From the figure it is evident that Nusselt number which is the basis of our comparison does not change significantly for element numbers 3111, 4051 and 5243. To reduce the computational time and effort element number 3111 has been set as the standard grid size and other numerical simulation has been carried out taking this grid as independent.

2.7. Code validation

Code validation test has been performed to compare the numerical accuracy of the present numerical solution with the establish literature. The present work has been validated with Tirari and Das [32]. Code validation has been done in the light of average Nusselt number in the heated surface. The deviation of the present work with established literature has been presented in Table 3. From the table we can see that the present numerical procedure is accurate and there are very small deviations from the previous study.

3. Results and discussion

As mentioned earlier the problem has been modeled using the partial differential equations that govern the physics and the set of equations has been solved using finite element method. In this paper, the problem is analyzed for varying solid volume fractions, Rayleigh numbers and inclination angles and the result is expressed in terms of streamline, isotherm contours for time parameter $\tau = 0.1$ and 1.

<table>
<thead>
<tr>
<th>Fluid (water)</th>
<th>$c_p$</th>
<th>$\rho$</th>
<th>$k$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid (CNT)</td>
<td>4179</td>
<td>997.1</td>
<td>0.613</td>
<td>$2.1 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>650</td>
<td>1350</td>
<td>3500</td>
<td>$4.2 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Fig. 2. Grid independency study with $\delta = 0.04$, $\varphi = 30^\circ$ and $Ra = 10^5$. 

Table 2
Thermophysical properties of water and nanoparticles [43–44].
the end, a comparison of cases has been made in terms of average Nusselt number versus $\tau$ for different solid volume fractions and inclination angles while varying the Rayleigh number. A brief albeit sufficient explanation follows each result.

The effect of solid volume fraction on streamlines for the selected values of Ra (10$^5$ to 10$^8$) with $\phi = 30^\circ$ and $\tau = 0.1$ has been portrayed in Fig. 3. As can be seen from the figure, for no CNT particles in the fluid, increasing the value of Ra increases the strength of the vortex inside the enclosure. For the boundary conditions of the problem there forms a counterclockwise rotating vortex inside the cavity for low to moderate value of Ra. The fluid inside the cavity takes heat from the bottom wall and due to the buoyant effect the fluid moves up. On the other hand, relatively colder fluid adjacent to the curved wall comes down along the path traced by the wall. As a result a vortex is formed inside the cavity. For Ra = 10$^8$ the convection inside the enclosure becomes very strong and strong eddies are formed along with the primary vortex. If the solid volume fraction $\delta$ is increased there is not much of a change in the pattern of streamline for cases where Ra is 10$^5$ to 10$^7$ except for the fact that there is a very weak and small eddy at one corner for Ra = 10$^5$. The vortex inside the cavity becomes slightly stronger. The highest value for stream function is achieved at $\delta = 0.08$.

Fig. 4 illustrates similar effects as were discussed in Fig. 3, only for $\tau = 1$. So this result is a continuation of the previous analysis in the time domain. For no CNT particles in the base fluid the strength of the primary vortex is slightly higher than it was in the case of $\tau = 0.1$ and Ra = 10$^5$. Besides there is an eddy at the corner rotating in the opposite direction relative to the primary vortex. As the value Ra is increased the strength of the primary vortex becomes higher and the eddy vortex also becomes stronger. For Ra = 10$^7$ the position of the smaller vortex is shifted towards the upward direction from the corner along the curved wall. For Ra = 10$^8$ convection is the strongest and there are two large vortices inside the cavity with $\psi_{min} = 122.92$ and $\psi_{max} = 168.78$. There is also a weaker vortex at the top corner of the cavity. These vortices prove the existence of strong convection at high Rayleigh number value. The general pattern of streamline distribution does not change much as the value of $\delta$ is increased. Although the strength of the vortices is somewhat maximum for $\delta = 0.08$ indicating the most suitable percentage of nanoparticle in the fluid for high heat transfer rate.

In Fig. 5 the effect of solid volume fraction on isotherms has been shown for selected values of Ra with $\phi = 30^\circ$ and $\tau = 0.1$. These results are indicators of how the heat is diffused in the fluid inside the cavity and give us some idea as to the mode of heat transfer. For $\delta = 0$ and Ra = 10$^5$ it has been seen that isotherm contours are almost parallel to each other and are clustering near the heated wall. This indicates domination of conduction mode of heat transfer at those conditions. As the value of Ra is increased the isotherms become more congested near the heated wall and the contours seem to elongate in an irregular manner along the cold wall. Actually this deformed or irregular pattern of the isotherms indicates the path of heat flow and also from the distorted isotherm contour it can be concluded that convection heat transfer mode is the dominant mode of heat transfer in those regions. The cluster of isotherms which are nearly parallel to each other proves

<table>
<thead>
<tr>
<th>Ra</th>
<th>$\text{Nu}_{av}$</th>
<th>Tirari and Das [32]</th>
<th>Present study</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>1.087</td>
<td>1.094</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>$10^4$</td>
<td>2.195</td>
<td>2.2013</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$10^5$</td>
<td>4.450</td>
<td>4.4512</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$10^6$</td>
<td>8.803</td>
<td>8.8125</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>

Table 3
Comparison of $\text{Nu}_{av}$ with those of Tirari and Das [32].
Fig. 4. Effect of solid volume fraction on streamlines for the selected values of Ra with $\phi = 30^\circ$ and $\tau = 1$.

Fig. 5. Effect of solid volume fraction on isotherms for the selected values of Ra with $\phi = 30^\circ$ and $\tau = 0.1$. 
Fig. 6. Effect of solid volume fraction on isotherms for the selected values of \( Ra \) with \( \phi = 30^\circ \) and \( \tau = 1 \).

Fig. 7. Effect of inclination angle on streamlines for the selected values of \( Ra \) with \( \delta = 0.05 \) and \( \tau = 0.1 \).
that conduction heat transfer is the primary mode of heat transfer adjacent to the hotter wall. For higher value of solid volume fraction the isotherm pattern doesn’t evolve much. But the density of isotherms for high value of Ra increases along the colder wall, which denotes better convection at high value of Ra and δ.

Fig. 6 shows the continuation of the effect that was shown in Fig. 5 in time domain (at τ = 1). The first distinguishing feature of the present result is that, the isotherms are more dispersed in nature which means that the heat is diffused into the fluid inside the cavity over time. There is no significant effect of adding CNT particles to the fluid is noticeable as the isotherm contour distribution pattern is the same for every value of δ. For Ra = 10^4 the isotherms are parallel to each other near the horizontal and the curved wall. So it can be concluded that conduction heat transfer is dominating in those regions. In the middle of the enclosure the isotherms are more distorted denoting the presence of convection. As the value of Ra is increased the isotherm contours become more clustered near the horizontal and the curved walls. In these regions these are parallel to each other while in the middle the isotherms are distorted. Although the pattern of change is the same, for δ = 0.08 and Ra = 10^5 isotherms seem to indicate a good combination of conduction and convection heat transfer.

The effect of inclination angle on streamlines for the selected values of Ra (10^5 to 10^8) with δ = 0.05 and τ = 0.1 is illustrated in Fig. 7. From the figure it can be seen that, for no nanoparticle in the fluid and Ra = 10^5 there are two vortices inside the enclosure rotating in the opposite direction with low strength. Since in this case the inclination angle is zero, the fluid inside the enclosure gets proper chance to induce the thermal buoyant flow due to the boundary condition. As a result low strength currents are seen originating from the heated bottom wall and diffuse in the enclosure along the line traced by adiabatic and cold walls. As the value of Ra is increased the strength of convective current is increased as expected. For Ra = 10^7 there is a large clockwise rotating vortex in the middle and two eddies at the right and top corners. Here \( \Psi_{\text{max}} = 98.09 \) and \( \Psi_{\text{min}} = -14.14 \). For Ra = 10^8 there is drastic change in convection pattern with high strength flow. In this case \( \Psi_{\text{max}} = 309.84 \). Along with this very strong vortex there are other local convective cells with high strength that are to be seen. For \( \phi = 30^\circ \) at low Ra values, there is a dominating vortex inside the enclosure which has moderate to high strength. Alongside this central vortex a small eddy can be seen at the right corner. As value of Ra is increased, the central vortex becomes stronger. For Ra = 10^7 this vortex takes up the entire region with its eye stretching over the entire enclosure. At Ra = 10^8 the convection is strongest with no particular pattern and is in fact unstable. For higher inclination angle, the only difference in the distribution of streamlines is that the strength of eddy in this case is very low and the eddy has no particular effect on the streamlines inside the enclosure. Another interesting observation is that, the strength of the central vortex seems to be lowering with increasing inclination angle except for the case of Ra = 10^5. Actually for low value Ra, there is not much of an impact of Rayleigh number on the convective heat transfer inside the enclosure. The strength increases due to the geometric effect rather than fluid properties and hence the increment is not large.

Fig. 8 shows the effects discussed in Fig. 7 at a later instant, to be precise, at the end of the process. For zero inclination angle and relatively lower Ra value, the two vortices formed at \( \tau = 0.1 \) seem to have been stabilized. As a consequence, the strength of the vortices is lowered. For Ra = 10^5 there are three vortices inside the enclosure with \( \Psi_{\text{max}} = 48 \) and \( \Psi_{\text{min}} = -64.13 \). These cells are so distributed that it seems, under this condition that the convective current is more or less distributed equally over the entire enclosure and as a result the heat is transferred inside the cavity in a regular manner. For Ra = 10^8, the convective currents are very strong and distributed in almost equal proportion over the entire cavity. For greater inclination angle there is a pattern in the formation of streamline inside the enclosure. Here for low Ra value,
\( \phi = 0^\circ \) 
\( \phi = 60^\circ \) 
\( \phi = 45^\circ \) 
\( \phi = 30^\circ \) 
\( \phi = 0^\circ \)

\( R_a = 10^6 \)  
\( R_a = 10^5 \)  
\( R_a = 10^7 \)  
\( R_a = 10^8 \)

Fig. 9. Effect of inclination angle on isotherms for the selected values of \( R_a \) with \( \delta = 0.05 \) and \( \tau = 0.1 \).

\( R_a = 10^7 \)  
\( R_a = 10^6 \)  
\( R_a = 10^5 \)  
\( R_a = 10^8 \)

Fig. 10. Effect of inclination angle on isotherms for the selected values of \( R_a \) with \( \delta = 0.05 \) and \( \tau = 1 \).
there is a large vortex inside the enclosure with an eddy at one corner. As the value of Ra is increased the larger cell gradually spread itself to the entire cavity superseding the small eddy. For high value of Ra at first this large cell is stretched and has a very high strength. For inclination angle of 30° the large vortex breaks up into smaller cell. But for higher value of inclination angle it retains its shape and at $\phi = 60°$ it assumes a dumbbell shape with two eyes. Just like the previous case, with increasing inclination angle the strength of vortex seems to be retarding.

The effect of inclination angle on isotherms for the selected values of Ra with $\delta = 0.05$ and $\tau = 0.1$ has been depicted in Fig. 9. For zero inclination angles, at low Ra values the isotherms are parallel to the bottom wall which is the source of heat. This means that under that condition heat is transferred in conduction mode. As the value of Ra rises, the isotherms assume a mushroom like shape (at Ra = $10^6$) and near the bottom wall these contours are close to each other. For higher value of Ra, the isotherms seem to extend along the colder wall while at the bottom wall these contours are closely packed. These distribution patterns suggest that conduction is the dominant mode of heat transfer near the bottom wall. For higher value of inclination angle the pattern of formation remains quite the same except for the fact that, there is no mushroom like distribution for Ra = $10^6$. This happens due to the geometric constraint in the cavity. For high value of inclination angle, the convective stream is bound to follow a specific path. So in a nutshell, for low value of Ra, conduction is stronger and convection becomes stronger with increasing value of Ra.

Fig. 10 shows the similar effect as Fig. 9 for a later instant ($\tau = 1$). As expected the isotherms are more diffused inside the enclosure in this case. But there is no fundamental change that can be noted. The pattern suggests that, for higher inclination angle the diffusion of heat is quicker. And other findings are just like the findings in Fig. 9.

Average Nusselt number at the heated surface versus dimensionless time for different values of $\delta$ at (a) Ra = $10^5$, (b) Ra = $10^6$, (c) Ra = $10^7$ and (d) Ra = $10^8$ is presented in Fig. 11. From the analysis of this plot the following findings can be listed:

1. For higher value of Ra, maximum value of average Nusselt number is high.
2. For Ra = $10^5$ and Ra = $10^6$ there is a general decreasing pattern in average Nusselt number value whereas for higher value of Ra the number of average Nusselt number tends to become more or less constant after a specific amount of time.
3. For $\delta = 0.12$, average value of Nusselt number is the highest. But $\delta = 0.12$ and $\delta = 0.08$ give comparable values.
4. For no nanoparticle in the fluid the average Nusselt number is the lowest.

Average Nusselt number at the heated surface versus dimensionless time for different values of $\phi$ at (a) Ra = $10^5$, (b) Ra = $10^6$, (c) Ra = $10^7$ and (d) Ra = $10^8$ is presented in Fig. 12. This figure presents some interesting results.

1. For the case of $\phi = 0$ the average value of Nusselt number is lowest for Ra = $10^6$. But for Ra = $10^6$ the value of Nusselt number is initially the highest for $\phi = 0$ and then gradually it comes down. For Ra = $10^7$, the value of average value of Nusselt number changes sinusoidally for $\phi = 0$. For Ra = $10^8$ and $\phi = 0$, the Nusselt number value is the higher than the other cases. So it indicates that, as the value of Ra is increased the heat transfer characteristics become better for $\phi = 0$.
2. The higher the value of Ra, the better heat transfer.
3. Keeping $\phi = 0$ aside, among the rest of the cases, $\phi = 60$ seems to produce the highest value of average Nusselt number while, $\phi = 30$ seems to produce the lowest.
4. The average value of Nusselt number seems to come down with time for Ra = $10^5$ and Ra = $10^6$. For higher value of Ra, the average Nusselt number becomes somewhat constant after a specific amount of time.

Fig. 11. Average Nusselt number at the heated surface versus dimensionless time for different values of $\delta$ at (a) Ra = $10^5$, (b) Ra = $10^6$, (c) Ra = $10^7$ and (d) Ra = $10^8$. 

Fig. 12. Average Nusselt number at the heated surface versus dimensionless time for different values of $\phi$ at (a) Ra = $10^5$, (b) Ra = $10^6$, (c) Ra = $10^7$ and (d) Ra = $10^8$. 

4. Conclusions

A detail analysis of the problem has been presented in the paper with solution scheme, equations and a thorough analysis of the results. As it has been attempted to introduce a relatively new sort of problem in this paper, for the convenience of the readers across the scientific world, every result is discussed in a detailed manner. To sum up the outcome of the research the following points can be drawn to attention:

- In order to ensure a very good convective heat transfer without using nanoparticle the value of Rayleigh number must be kept high. This is true and is independent of inclination angle.
- For solid volume fraction value of \( \delta = 0.08 \) the convection current seems to be very strong.
- A combination of solid volume fraction value of \( \delta = 0.12 \) and inclination angle of \( \psi = 60^\circ \) should provide the strongest convective current.
- Effect of inclination angle is more severe than the effect of proportion of nanoparticle.
- If anyone is interested to control the convective current pattern, it is recommended to keep \( Ra = 10^7, \delta = 0.08–0.12 \) and inclination angle of \( 30^\circ \).
- For low value of \( Ra \), geometric constraints become the key factor to control the convection inside the cavity.

The use of different materials as the solid nanoparticle in a base fluid in order to enhance the heat transfer rate is increasing across the world. Different materials have been tried for this purpose and the search for perfect nanoparticle is still going on. This paper sheds light on the pros and cons of using CNT as the solid nanoparticle in water as the base fluid. The geometry introduced in this paper is relatively new but has a wide prospect for use especially in the solar collector. The aim of this paper is to draw the attention of the scientific community across the world to this particular type of geometry and related problems. It is the authors' hope, that the problem will be further investigated by the experimentalists to help the search for a newer and better nanofluid for heat transfer.

References


