Predictive Indirect Matrix Converter Fed Torque Ripple Minimization with Weighting Factor Optimization

Muslem Uddin, Saad Mekhilef
Power Electronics and Renewable Energy Research Laboratory (PEARL), Department of Electrical Engineering, University of Malaya
50603 Kuala Lumpur, Malaysia
muslem_eee04@siswa.um.edu.my

Marco Rivera
Department of Industrial Technologies
Universidad de Talca
Curico, Chile
marcoesteban@gmail.com

Jose Rodriguez
Departamento de Electrónica
Universidad Técnica Federico Santa María (USM)
Chile
jrp@usm.cl

Abstract—Predictive control is a powerful and promising control algorithm in the control of power converter and electrical machine drive’s system. The system performance depends on the selection of weighting factor in the cost function. Therefore, this paper proposed a weighting factor optimization method to reduce the torque ripple of induction motor fed by an indirect matrix converter. Also, predictive torque and flux control with conventional weighting factor is being investigated in this paper and is compared with the proposed optimum weighting factor based predictive control algorithm. The introduced weighting factor optimization method in predictive control algorithm is validated through simulation and shows potential tracking of variables and control with their corresponding references and consequently minimizes the torque ripple compare to the conventional weighting factor based predictive control method.

Keywords—Indirect Matrix Converter, Induction Motor, Predictive Control, Torque Ripple, Weighting Factor.

I. INTRODUCTION

Diverse control targets, variables and constraints can be included in a single cost function at predictive control algorithm and be controlled with the basis of priority control factor is known as weighting factor. For the variables with same nature in the cost function no need to set the weighting factor but when target variables are in different nature (different order of magnitude and different unit) in a single cost function then weighting factor selection become as a great issue for the system stability. Till the date in literature no analytical or numerical methods or control design theories to adjust the weighting factor and currently they are determined with the iterative evaluation method [1]. Though this procedure is extensively used to adjust the weighting factor and potential performances can be attained but this is quietly approximated. Therefore, for the best performance of the system, the optimized weighting factor is needed. In recent past, a weighting factor optimization method is applied for torque ripple reduction fed by three phase voltage source inverter (VSI) in [2] with good performance and predictive two-level inverter fed induction motor control strategy with weighting factor look up table and divide control interval have been investigated in [3]. In [4], a ranking approach based multi objective optimization has been proposed to replace the single cost function at the predictive horizon which allows the predictive control of torque and flux without weighting factors. Therefore, in this investigation, predictive control algorithm has been used to control torque and flux of the induction motor precisely and is specially focused on indirect matrix converter fed weighting factor optimization method to minimize the torque ripple and control the flux of the induction motor. There are different types of AC-AC converter in the power converter field. The cyclo-converter is one of them which transfer the power without any intermediate energy storage devices with a significant amount of harmonic contents at the output frequency due to the commutations and these harmonics cannot be filtered by the load inductance. Also, direct matrix converter (DMC) is another AC-AC converter without DC-link storage device but its control strategy is so complex. On the other hand, indirect matrix converter is an AC-AC converter which has been proposed to remove all the demerits stated above. The most important improvement of this topology is the simplicity and less complex in the control compared to DMC and allowing secure commutation of the system [5] without particular sensing devices as needed for DMC [6]. Furthermore, the indirect matrix converter is with a longer life span and size becomes more compact. Recently, some investigations with the multilevel inverter have been investigated in [7, 8] with digital control method.

Some recent investigations have been carried out with indirect matrix converter, such as: predictive torque and flux control with unity power factor [9], current control [10], imposed sinusoidal source and load current [11], current control with filter resonance mitigation in [12]. Also, predictive control applications with three phase VSI in [13, 14], active front end rectifier control with unity displacement in [15] and for matrix converter (MC) have been investigated in [16, 17]. Furthermore, a comprehensive review on MC have been elaborately presented in [18]. Also, a three-stage 18-level
hybrid inverter circuit and its innovative control method have been presented in [19, 20].

This paper is organized in the following manner: Section II is related to the mathematical modeling of the indirect matrix converter topology of the system. Section III present the proposed predictive torque ripple reduction and flux control algorithm with weighting factor optimization method. Section IV states, verification results and discussion of the proposed investigation to reduce the torque ripple corresponding to IV states, verification results and discussion of the proposed algorithm with weighting factor optimization method. Section V presents the proposed predictive torque ripple reduction and flux control converter topology of the system. Section III present the conventional weighting factor based predictive control algorithm and finally, a fruitful conclusion is drawn in section V.

II. INDIRECT MATRIX CONVERTER TOPOLOGY

The Fig. 1 shows the topology of the indirect matrix converter (IMC) which consists of rectifier and inverter part. The IMC has 24 possible switching states that are utilized in the predictive control algorithm to select the best actuation for the converter. The modeling equations of the indirect matrix converter are given in below:

\[
V_{dc} = [S_9 - S_6, S_8 - S_9, S_6 - S_3] V_i
\]

(1)

\[
I_{dc} = \begin{bmatrix}
S_{10} - S_{24} \\
S_{23} - S_{24} \\
S_{20} - S_{24}
\end{bmatrix} I_a
\]

(2)

\[
V_o = \begin{bmatrix}
S_{10} - S_{14} \\
S_{13} - S_{2} \\
S_{20} - S_{22}
\end{bmatrix} V_{dc}
\]

(3)

\[
I_o = \begin{bmatrix}
S_{10} & S_{13} & S_{22}
\end{bmatrix} I_a
\]

(4)

where rectifier switching states = \( S_{10} \) to \( S_{22} \); input voltage \( V_i = [V_{i1}^r, V_{i2}^r, V_{i3}^r] \); rectifier input current \( I_a = [I_a^r, I_a^i, I_a^f] \); output voltage \( V_o = [V_{o1}, V_{o2}, V_{o3}] \); output current \( I_o = [I_o^r, I_o^i, I_o^f] \).

III. PREDICTIVE CONTROL ALGORITHM

Predictive control algorithm uses the finite number of valid switching states of the power converter. The proposed scheme maintains the predictive values closed to their respective references at the end of the sampling instant and maintain positive DC-link voltage between the rectifier and inverter stages which eliminates the extra usage of the large energy storage devices. Hence reduces the size and increase the life span of the converter. This proposed optimum weighting factor based predictive control algorithm and scheme are presented in Fig. 2 and Fig. 3, respectively. Predictive controller satisfies all the aforementioned constraints by using the following five steps:

Steps 1: Supply voltage \( V_{i1}^k \), input voltage \( V_{i2}^k \), stator current \( I_{s1}^k \) and speed \( \omega_{ref}^k \) of the induction motor are measured in the \( k^{th} \) sampling instant.

Step 2: PI controller is used to set nominal torque \( T_{nom} \) from the error signal between the measured and reference speeds of the induction motor where reference speed \( \omega_{ref} \) is known value.

Step 3: Stator reference flux \( \psi_{ref} \) is a given value and a flux estimator has been used to estimate the stator and rotor flux.

Step 4: For each valid switching states of indirect matrix converter, values of torque \( T_{s1}^{k+1} \) and stator flux \( \psi_{s1}^{k+1} \) are predicted in the next sampling period (k+1).

Step 5: All the predictive values are compared with their respective references and determine the cost functions for all possible switching states based on conventional weighting factor and with imposed optimized weighting factor. The switching state corresponds to the minimum cost function is selected in the next sampling time period to actuate the converter.
System Variables for \( S_n \) Sampling Time and Input References

Optimal Switching Actuation, \( S_{opt(k+1)} \)

Torque and Flux Estimation

Weighting Factor Optimization, \( W_{opt} \)

For \( i = 1, 2, 4 \)

Torque and Flux are Predicted

Calculation of Cost Function, \( g_s(k+1) \)

Optimum Switching Selection, \( g_s(k+1)^{opt} \)

Fig. 2. Predictive torque and flux control algorithm.

3-Phase Supply

Input Filter

IMC

V_o

\( V_{ref} \)

\( V_{ref} \)

PI

Optimum Switching Selection

Flux Estimation

Torque and Flux Prediction

Optimization of Weighting Factor

Fig. 3. Control scheme with weighting factor optimization.

A. Torque and Flux Prediction

The predictive flux, current and torque equations are given in equations (5)-(7) respectively.

\[
\psi_{s}^{i+1} = \psi_{s}^{i} + V_{a}^{i+1}T_s - R_s I_{s}^{i+1}T_s
\]

\[
I_{s}^{i+1} = \psi_{s}^{i} (V_{a}^{i+1} + (\tau, k_s - jk, \omega^s)) \frac{T_s}{\sigma L_s} + I_{s}^{i} (1 - \frac{T_s}{\sigma L_s})
\]

Here, \( r_s = R_s + R_i k_s \), \( \tau_s = L_s \), \( k_s = \frac{L_m}{L_s} \), \( k_r = \frac{L_m}{L_r} \) and \( \sigma = 1 - k_s k_r \).

\[
T_r^{i+1} = \frac{3}{2} p (\psi_{s}^{i+1} \times I_{s}^{i+1})
\]

where \( R_s \) and \( R_r \) are the stator and rotor resistance respectively.

B. Weighting Factor Optimization Method

Torque ripple of the induction motor can be derived in the following.

\[
T_r^2 = \frac{1}{T_s} \int (D_r + m_t)^2 dt
\]

where, \( D_r = T_r - T_{nom} \), \( T_r \) = Torque ripple, \( T_s \) = Sampling time, \( T_r \) = Torque of induction motor, \( T_{nom} \) = Nominal torque and \( m_t \) = Ascending torque slope and is given in below:

\[
m_t = \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r} + K(V_{ad} - \psi_{ad} + \psi_{r} + \psi_{id} + \psi_{d} - \psi_{r})
\]

where \( V_{ad} \) = Stator voltage \( \alpha \)-axis component; \( V_{ad} \) = Stator voltage \( \beta \)-axis component and \( \sigma = 1 - k_s k_r = 1 - \frac{L_m}{L_s L_r} \).

\[
k = \frac{3}{2} p
\]

The simplified torque ripples can be represented as follows:

\[
T_r^2 = \frac{1}{T_s} \int (D_r^2 + m_t^2 t^2 + 2 m_t D_r) dt
\]

The first derivative of torque ripple to the weighting factor has to be set to zero in order to find the weighting factor that minimizes the ripple of torque. The relation is as follows:

\[
T_r^2 = T_r^2(m_t) = m_t = m_t(V_r) \Rightarrow V_r = V_{opt}(W_{opt})
\]

Therefore, \( T_r^2 = T_r^2(W_{opt}) \)

To find the optimum weighting factor in the cost function the derivative of the torque ripple must be zero. Therefore,

\[
\frac{dT_r^2}{dW_{opt}} = 0
\]

By solving the equation (12), optimum weighting factor becomes,

\[
G = \frac{3D_r}{2KT_r} + \omega^s (\psi_{ad} - \psi_{r} - \psi_{id} - \psi_{d} - \psi_{r}) + \frac{1}{K} \left( \frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r} \right) T_r
\]

\[
W_{opt} = \beta (\psi_{ad}) - \beta (\psi_{r})
\]

Equation (14) represents the optimum weighting factor.

C. Cost Function Calculation

The cost functions used in conventional and optimized weighting factor based predictive control are given in (15) and (16) respectively.

\[
g^{i+1} = X_1 \left| T_r^{i+1} - T_{nom} \right|^2 + X_2 \left| \psi_{s}^{i+1} - \psi_{ro} \right|^2
\]

\[
g^{i+1} = \frac{1}{2} \left( \left| T_r^{i+1} - T_{nom} \right|^2 + W_{opt} \left| \psi_{ad}^{i+1} - \psi_{ad} \right|^2 \right)
\]

where, \( X_1 \), \( X_2 \) are the conventional weighting factors and \( W_{opt} \) is the optimized weighting factor.
IV. RESULTS AND DISCUSSION

The predictive control of induction motor fed by an indirect matrix converter is verified in MATLAB Simulink environment to justify the performance of the proposed optimization of weighting factor. The parameters used in verifications are given in Table I and the simulations has been carried out with sampling time, $T_s = 20 \mu s$. In this investigation, two cases are analyzed. First, the validation of the predictive control algorithm is performed with conventional weighting factor, while in the second case, an optimization method is adopted to select the optimized weighting factor for the predictive control algorithm. In both cases, the induction motor starts at 0.01s without any load torque, varying the reference speed from 0 to 35 rad/s and the torque is limited to 7 N-m. A load torque of 3 N-m is applied at time of 0.4s and a reverse torque at 0.5s is applied to change the speed in the reverse direction from 35 rad/s to -35 rad/s. In this investigation, stator reference flux has been assumed as 1.1 Wb in all the verifications. The speed controller generates torque references at transients which is different from zero and be appreciated as a good tracker of speed given in Figs. 4(a) and 5(a), of torque indicated in Figs. 4(b) and 5(b), of stator flux depicted in Figs. 4(c) and 5(c) in both the conventional and proposed optimum weighting factor based predictive control schemes respectively. Also the stator flux $\alpha$-$\beta$ representations are plotted in Figs. 4(d) and 5(d) for the both cases respectively and follows the reference magnitude of around 1.1 Wb accurately.

Fig. 4. Verification results (conventional weighting factor): (a) Motor speed, $(\omega^k)$ [rad/s] and reference speed, $(\omega^r_{ref})$ [rad/s]; (b) Motor torque, $(T_e)$ [N-m] and Nominal torque, $(T_{nom})$ [N-m]; (c) Stator flux, $(\psi_s)$ [Wb] and reference flux, $(\psi^r_{ref})$ [Wb]; (d) $\alpha$-$\beta$ presentation for the stator flux, $(\psi_s)$ [Wb].
Fig. 5. Verification results (optimized weighting factor): (a) Motor speed, \( \omega^f \) [rad/s] and reference speed, \( \omega_{ref} \) [rad/s]; (b) Motor torque, \( T_e \) [N-m] and Nominal torque, \( T_{nom} \) [N-m]; (c) Stator flux, \( \psi_s \) [Wb] and reference flux, \( \psi_{ref} \) [Wb]; (d) \( \alpha-\beta \) presentation for the stator flux, \( \psi_s \) [Wb].

A. Torque Ripple Reduction in Forward Speed Region

The Fig. 6(a) shows the maximum value of the torque ripples is 5 N-m and the minimum is 1.85 N-m in a certain time range of verification. Therefore the difference between the maximum and minimum peak value of the torque ripples is 3.15 Nm for the predictive method based on conventional weighting factor. On the other side, for the proposed weighting factor optimization method, the maximum and minimum peaks are in between 4 N-m and 2 N-m respectively in the same time interval in Fig. 6(b) and the difference of the values is only 2 N-m. Therefore, the proposed weighting factor optimization method has reduced the torque ripples (3.15 - 2.0) N-m or, 1.15 N-m corresponding to conventional predictive control algorithm in the forward speed region.

![Fig. 6. Torque ripples (N-m) (forward speed): (a) Using conventional weighting factor; (b) With imposed weighting factor optimization.](image)

B. Torque Ripple Reduction in Reverse Speed Region

The Fig. 7(a) implies that, the maximum value of the torque ripples is 5.5 N-m and the minimum is 1.1 N-m in a certain range of verification. Therefore the difference between the maximum and minimum value of the torque ripples is 4.4 N-m for the method of conventional weighting factor. On the other hand, for the proposed weighting factor optimization method, the maximum and the minimum values of the torque ripples are in between 3.6 N-m and 2.2 N-m respectively in the same time interval and the difference of the values is only 1.4 N-m. Hence, the proposed weighting factor optimization method has reduced the torque ripples by (4.4 – 1.4) N-m, or 3.0 N-m in the reverse speed corresponding to conventional weighting factor based predictive control.

![Fig. 7. Torque ripples (N-m) (reverse speed): (a) Using conventional weighting factor; (b) With imposed weighting factor optimization.](image)
ψ = ψ = 1

\[ \text{Fig. 8. Supply voltage, } V_{s}^{*} [V] \text{ vs input current, } I_{s}^{*} [A]: (a) Without reactive power minimization; (b) With reactive power minimization.} \]

In the indirect matrix converter, input side current is sinusoidal which is shown in Fig. 8. The Fig. 8 shows the relation between the supply voltage and input current to the converter.

From the result it is clear that in Fig. 8(a), a chaotic behavior is observed in input current of indirect matrix converter regardless of reactive power compensation in the cost function and this reactive power can be minimized by adding a reactive power minimization term as the investigation of [9] and the result is shown in Fig. 8(b).

Consequently, at the input side current becomes more sinusoidal as well as unity power factor is maintained properly.

### TABLE I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{s} )</td>
<td>Sampling time</td>
<td>20 µs</td>
</tr>
<tr>
<td>( V_{s} )</td>
<td>Supply phase voltage (RMS)</td>
<td>500 V</td>
</tr>
<tr>
<td>( f_{s} )</td>
<td>Supply frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>( R_{f} )</td>
<td>Input filter resistance</td>
<td>0.5 Ω</td>
</tr>
<tr>
<td>( L_{f} )</td>
<td>Input filter inductance</td>
<td>400 µH</td>
</tr>
<tr>
<td>( C_{f} )</td>
<td>Input filter capacitance</td>
<td>21 µF</td>
</tr>
<tr>
<td>( \omega_{ref} )</td>
<td>Reference speed</td>
<td>35 rad/s</td>
</tr>
<tr>
<td>( T_{rms} )</td>
<td>Nominal torque</td>
<td>7 N-m</td>
</tr>
<tr>
<td>( R_{s} )</td>
<td>Stator resistance</td>
<td>1.35 Ω</td>
</tr>
<tr>
<td>( L_{s} )</td>
<td>Stator inductance</td>
<td>0.2861 mH</td>
</tr>
<tr>
<td>( R_{r} )</td>
<td>Rotor resistance</td>
<td>7.2037 Ω</td>
</tr>
<tr>
<td>( L_{r} )</td>
<td>Rotor inductance</td>
<td>0.2861 mH</td>
</tr>
<tr>
<td>( L_{m} )</td>
<td>Mutual inductance</td>
<td>0.2822 mH</td>
</tr>
<tr>
<td>( p )</td>
<td>Number of poles</td>
<td>2</td>
</tr>
<tr>
<td>( X_{r} )</td>
<td>Weighting factor</td>
<td>63</td>
</tr>
<tr>
<td>( X_{r} )</td>
<td>Weighting factor</td>
<td>13500</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
K &= \frac{3}{2} p, \quad \sigma = 1 - k_{j} k_{s} = 1 - \frac{I_{s}^{*}}{L_{r}} \\
A. Torque Slope Calculation
\end{align*} \]

The torque for the induction motor can also be determined by the following relation,

\[ T_{s} = T = K(\psi_{s}^{*}\psi_{r} - \psi_{s}\psi_{r}^{*}) \]  

First derivative of (17) implies the slope of the torque as follows,

\[ \frac{dT}{dt} = K(\frac{d}{dt}\psi_{s}^{*}\psi_{r} - \frac{d}{dt}\psi_{s}\psi_{r}^{*}) + \frac{d}{dt}(\psi_{s}^{*}\psi_{r} - \psi_{s}\psi_{r}^{*}) = \frac{d}{dt}\psi_{s}\psi_{r} - \frac{d}{dt}\psi_{s}^{*}\psi_{r} \]  

Considering the induction motor dynamic model from equation (18),

\[ \frac{dT}{dt} = K((V_{s} + W_{s})\psi_{s}^{*}\psi_{r} + W_{s}^{*}\psi_{s} - (V_{r} + W_{r})\psi_{r}^{*}\psi_{s} + W_{r}^{*}\psi_{r}) \]  

where,

\[ W_{s} = \begin{bmatrix} Q_{1} & Q_{2} & Q_{3} & Q_{4} \\ Q_{5} & Q_{6} & Q_{7} & Q_{8} \\ Q_{9} & Q_{10} & Q_{11} & Q_{12} \\ Q_{13} & Q_{14} & Q_{15} & Q_{16} \end{bmatrix} \psi_{s}^{*} \]  

\[ W_{r} = \begin{bmatrix} Q_{1} & Q_{2} & Q_{3} & Q_{4} \\ Q_{5} & Q_{6} & Q_{7} & Q_{8} \\ Q_{9} & Q_{10} & Q_{11} & Q_{12} \\ Q_{13} & Q_{14} & Q_{15} & Q_{16} \end{bmatrix} \psi_{r} \]

\[ \frac{dq}{dt} = -\frac{R}{L} \left( \frac{d\psi_{s}}{dt} \right), \quad \frac{dr}{dt} = -\frac{R}{L} \left( \frac{d\psi_{r}}{dt} \right) \]

\[ B. Relationship Between Weighting Factor and Stator Voltage \]

Applying the Taylor expansion around the nominal values in equation (17) to express the model predictive variables in a linear manner as follows:

\[ T_{s} = T_{rms} + K(\psi_{s}^{*}\Delta\psi_{s}^{*} - \psi_{s}^{*}\Delta\psi_{s} + \psi_{s}\Delta\psi_{r} - \psi_{s}\Delta\psi_{r}^{*} - \psi_{s}^{*}\Delta\psi_{r} - \psi_{s}^{*}\Delta\psi_{r}^{*}) \]

\[ \left| \frac{d\psi_{s}}{dt} \right|^{2} = \left( \frac{d\psi_{r}}{dt} \right)^{2} + 2\psi_{s}^{*}\Delta\psi_{s}^{*} + 2\psi_{s}\Delta\psi_{r} \]

\[ \left| \frac{d\psi_{r}}{dt} \right|^{2} = \left( \frac{d\psi_{r}}{dt} \right)^{2} + 2\psi_{s}^{*}\Delta\psi_{s}^{*} + 2\psi_{s}\Delta\psi_{r} \]

\[ \left| \frac{d\psi_{s}}{dt} \right|^{2} + \left| \frac{d\psi_{r}}{dt} \right|^{2} = \left( \frac{d\psi_{s}^{*}}{dt} \right)^{2} + \left( \frac{d\psi_{r}^{*}}{dt} \right)^{2} + 2\psi_{s}^{*}\Delta\psi_{s}^{*} + 2\psi_{s}\Delta\psi_{r} \]

\[ \left| \frac{d\psi_{s}}{dt} \right|^{2} + \left| \frac{d\psi_{r}}{dt} \right|^{2} = \left( \frac{d\psi_{s}^{*}}{dt} \right)^{2} + \left( \frac{d\psi_{r}^{*}}{dt} \right)^{2} + 2\psi_{s}^{*}\Delta\psi_{s}^{*} + 2\psi_{s}\Delta\psi_{r} \]

\[ \left| \frac{d\psi_{s}}{dt} \right|^{2} + \left| \frac{d\psi_{r}}{dt} \right|^{2} = \left( \frac{d\psi_{s}^{*}}{dt} \right)^{2} + \left( \frac{d\psi_{r}^{*}}{dt} \right)^{2} + 2\psi_{s}^{*}\Delta\psi_{s}^{*} + 2\psi_{s}\Delta\psi_{r} \]
Torque and flux displacements are related to stator and rotor flux as follows:

\[ Y(t_o) = G.X(t_o) \]  

(24a)

Hence, (16) becomes,

\[ Y = \begin{bmatrix} \Delta T_r \\ \Delta \psi_r \end{bmatrix} = G \begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} \Delta \psi_{sd} \\ \Delta \psi_{sq} \end{bmatrix} \]

(24b)

On the other case, in stationary reference frame induction motor discrete model can be described as below:

\[ X(t_o) = R.X(t_o) + S.U(t_o) \]

(25a)

The parameters regarding the matrix \( R \) have been introduced before in (19b). For discrete nature of the system considering in the next sampling periods the before mentioned equations gives:

\[ Y(t_{o+1}) = G.X(t_{o+1}) = G.R.X(t_o) + G.S.U(t_o) \]

(26)

The appropriate input vector satisfies the following set of equations

\[ \frac{d}{dV_{eo}} g_e = 0 \]

(27a)

\[ \frac{d}{dV_{eo}} g_o = 0 \]

(27b)

From equations (16), (26) and (27) lead to the following voltage displacement

\[ \Delta V_{eo} = \alpha_1 + \frac{\beta_1}{W_{opt}} \]

(28a)

\[ \Delta V_{eo} = \alpha_2 + \frac{\beta_2}{W_{opt}} \]

(28b)

where,

\[ \alpha_1 = \frac{-f_1(e_{12}^2e_{14} - e_{12}e_{14}e_{23})}{(e_{11}e_{22} + e_{12}e_{21})^2}, \quad \alpha_2 = \frac{-f_2(e_{12}^2e_{14} - e_{12}e_{14}e_{23})}{(e_{11}e_{22} + e_{12}e_{21})^2} \]

(29a)

\[ f_1 = G.R, \quad f_2 = G.R \]

(29b)

\[ \begin{bmatrix} \Delta \psi_{sd} \\ \Delta \psi_{sq} \end{bmatrix} = G \begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} \Delta \psi_{sd} \\ \Delta \psi_{sq} \end{bmatrix} \]

(29c)

These equations are expressed the stator voltage and weighting factor relationship.

C. Optimization of Weighting Factor

Let, derivative of the torque ripples with respect to weighting factor equals to zero.

\[ \frac{d}{dW_{opt}} T_r^2 = \frac{d}{dW_{opt}} (m_1 T_r^2 + m_2 D_r T_r + D_r^2) = 0 \]

(30)

Therefore,

\[ \frac{d}{dW_{opt}} m_1 = 0 \]

(31)

And,

\[ m_1 = -\frac{3D_r}{2T_r} \]

(32a)

The derivatives of the stator voltage to the weighting factor are obtained as:

\[ \frac{d}{dW_{opt}} V_{eo} = \frac{d}{dW_{opt}} (V_{eo}^2 + \Delta V_{eo}) = \frac{d}{dW_{opt}} (\Delta V_{eo}) = -\frac{\beta_1}{W_{opt}^2} \]

(32a)

\[ \frac{d}{dW_{opt}} V_{eo} = \frac{d}{dW_{opt}} (V_{eo}^2 + \Delta V_{eo}) = \frac{d}{dW_{opt}} (\Delta V_{eo}) = -\frac{\beta_2}{W_{opt}^2} \]

(32b)

As a result, equation (31) is not a suitable equation to calculate the optimized weighting factor parameter because of each cancel from both sides of the equation (33). Therefore, equations (32a) and (32b) are the best criterion for weighting factor optimization. Therefore,

\[ \frac{3D_r}{2T_r} = K(W_{eo} V_{eo} - V_{eo}^2) - \alpha_1 (\Delta V_{eo}) \]

(35)

where, \( G = V_e w_{eo} - V_{eo} w_{eo} \)

(36)

The equation (36) is the criterion for weighting factor optimization. Combining the equations (28) and (36) optimized weighting factor can be determined.

ACKNOWLEDGMENT

The authors wish to thank the financial support from the University of Malaya through HIR-MOHE project U.M.C/HIR/MOHE/ENG/17.

REFERENCES


