Effect of temperature on the volume and surface contributions in the symmetry energy of rare earth nuclei

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Abstract

The effect of temperature on the volume and surface contributions in the nuclear symmetry energy and their ratio in the isotopic chains of rare earth Nd, Sm, Gd, and Dy nuclei with $N = 82-126$ is analyzed in the framework of coherent density fluctuation model (CDFM). The weight function of nuclei, within CDFM, are calculated by using the densities from the temperature-dependent relativistic mean-field (RMF) model. Firstly, we discuss the temperature-dependence of bulk properties of nuclei, within RMF model, such as binding energy, deformation parameter, charge radius and isotopic shift along with comparison with the available experimental data at temperature $T = 0$ MeV. Further, we discuss the thermal evolution of symmetry energy and its volume and surface components. At $T = 0$ MeV, the persistence of a peak in the symmetry energy and components at neutron number $N = 100$ shows the manifestation of deformed shell closure in consonance with an earlier study by one of us [L. Satpathy, S.K. Patra, J. Phys. G 30 (2004) 771; S.K. Ghorui, et al., Phys. Rev. C 85 (2012) 064327]. However, the scenario changes with rise in temperature and the magnitude of peak decreases at higher temperatures. At $T = 3$ MeV, the peak disappears which may be due to shape change in addition to quenching of shell effects since the quadrupole deformation parameter $\beta_2$ decreases with an increase in temperature and nuclei become spherical at $T = 3$ MeV. It indicates that behavior of symmetry energy is closely related to the deformation/shape of the nuclei. We

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have also discussed the values of volume symmetry energy, surface symmetry energy and their ratio which are in consonance with available experimental data. © 2020 Elsevier B.V. All rights reserved.

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1. Introduction

The equation of state (EOS) of asymmetric nuclear matter is one of the crucial issues in the field of nuclear physics. The solutions of a variety of queries encompassing the dynamics of isospin asymmetric heavy ion collisions and supernovae explosions, stability of superheavy elements, structure of neutron stars, stellar nucleosynthesis, frequency and amplitude of gravitational waves from the spiraling binary neutron stars etc. hinge upon the EOS of asymmetric nuclear matter (ANM) [1–11]. The fundamental ingredient of the EOS of ANM is the nuclear symmetry energy, which characterizes the energy needed to convert symmetric nuclear matter into pure neutron matter. It is the primary coefficient in the expansion of binding energy per nucleon in terms of asymmetry parameter \( \delta = (\rho_n - \rho_p)/\rho \) with \( \rho = \rho_n + \rho_p \), given by [12]

\[
E(\rho, \delta) = E(\rho, 0) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4)
\]

where \( E(\rho, 0) \) is the energy of symmetric nuclear matter and \( E_{\text{sym}}(\rho) = 1/2(\partial^2 E(\rho, \delta)/\partial \delta^2)_{\delta=0} \) is the symmetry energy. Several astrophysical observations and development of radioactive ion beam facilities leading to the availability of novel exotic beams with larger span of isospin asymmetry than stable beams, provide major thrust for the explorations of the symmetry energy and its density dependence. The existence of drip-line nuclei depends on the symmetry energy of the exotic nuclei.

Experimentally, the information about the density dependence of symmetry energy at sub-saturation, saturation (\( \rho_0 \)) and above saturation densities (\( \rho \sim 2\rho_0 \)) is extracted through analysis of nucleus-nucleus collisions [3,13,14], nucleon flow [15], pygmy resonances [16], giant dipole resonance [17], and isospin diffusion measurements [18]. The neutron skin thickness measurements in lead nuclei using parity violating electron scattering PREX experiment [19–21] and antiprotonic atoms [22], also facilitate to constrain \( E_{\text{sym}}(\rho) \) due to correlation of slope of symmetry energy with neutron skin thickness. In addition, the astrophysical observations of neutron stars open a window to explore the nuclear matter properties under the extreme isospin asymmetric environment. The investigation of correlations among mass, radius of neutron star and EOS help to put constraints on \( E_{\text{sym}}(\rho) \) [23,24].

The theoretical investigations of symmetry energy in isospin asymmetric nuclear matter were pioneered by Brueckner et al. [25]. The symmetry energy has also been investigated through fitting of ground state masses using different versions of semi-empirical mass formula within liquid drop model [26–28]. Some other formalisms such as Hartree-Fock approach based random phase approximation method [29], Thomas-Fermi (TF) approach using effective nucleon-nucleon Skyrme interaction [30], relativistic nucleon-nucleon interactions [31,32], energy density functional (EDF) of the Skyrme force [33,34], variational approach [35], local density approximation [36–39] have been used to investigate the symmetry energy. Inspite of numerous theoretical efforts, the value of \( E_{\text{sym}}(\rho) \) is quite uncertain at supra-saturation densities and the results using different approaches strongly deviate from each other. The underlying issue is the incomplete un-
derstanding of isospin part of the in-medium nuclear interaction, therefore the value of symmetry energy is model dependent at supra-saturation densities.

In 1947, Feenberg put on the emphasis for the inclusion of surface contribution in the symmetry energy, in addition to bulk contribution, due to lack of saturation at the nuclear surface [40]. In the following years, the bulk and surface parts of the symmetry energy and their ratio have been extensively probed [41–43]. Danielewicz has shown, in connection to semi-empirical mass formula, that for consistent description of properties of neutron rich nuclei it is indispensable to include surface symmetry term besides the volume symmetry energy [44]. The partition of total asymmetry into bulk and surface components has been discussed with an analogy of two connected capacitors [44–47]. Several efforts have been made to study the volume and surface symmetry energies and their ratio in relation with neutron skin thickness [48–53]. The greater temperature sensitivity of surface part of symmetry energy is reported in comparison to the bulk part of symmetry energy [54]. It has also been remarked that in contrast to infinite nuclear matter, the symmetry energy coefficient of finite nuclei show significant change with variation of temperature [55].

The knowledge of symmetry energy at zero temperature is quite important to investigate the properties of exotic nuclei in ground state as well as that of cold neutron stars in β-equilibrium. On the other hand, the nucleus undergo expansion as it warms up with increasing excitation energy which dictates the temperature dependence of symmetry energy due to change in the density distribution. As the density derivative of symmetry energy define the pressure difference on protons and neutrons [56,57] and thereby plays an imperative role in the formation of neutron skin in nuclei. Moreover, the thermal symmetry energy of hot neutron rich matter is of paramount significance to understand the heavy ion physics, liquid gas phase transition in asymmetric nuclear matter, neutron star properties, supernova explosion process [58] and isotopic dependence of yield in multifragmentation process [59]. The temperature-dependence of symmetry energy in hot nuclei has been studied within thermal Hartree-Fock calculations [60], TF approximation [61], extended TF method [62], Skyrme-Hartree-Fock formalism [63]. The coherent density fluctuation model (CDFM) [64–67] has also been employed to evaluate the surface properties of nuclei, which are obtained by folding the nuclear matter properties with the weight function of nuclei. The distinct advantage of the CDFM approach is that it takes care of the fluctuations due to density distribution via weight function and fluctuations due to momentum distribution via mixed density matrix i.e. Wigner distribution function.

Keeping into view the significance of the symmetry energy, in the present work, we aim to investigate the symmetry energy and its volume and surface components at non-zero temperature within CDFM. It will assist to provide thermal mapping of volume and surface symmetry energies which are poorly studied till now. To study the same, we have chosen the isotopic chains of rare earth Nd, Sm, Gd and Dy nuclei. We have used the temperature-dependent densities from effective field theory motivated relativistic mean field (RMF) model [68–71]. RMF model has been applied successfully to explain the bulk properties of the stable as well as drip-line nuclei throughout the nuclear chart. The RMF densities have been used as an input within the CDFM to investigate the thermal evolution of symmetry energy and its bulk and surface components in the case of rare earth nuclei. The results present the persistence of peak at the low temperature corresponding to neutron number $N = 100$ depicting the deformed shell closure in consonance with an earlier study by one of us [72]. The comparison of results has also been made with the available theoretical results as well as experimental data obtained through measurable quantities such as nuclear masses, neutron skin thickness, isobaric analog states etc.
2. Methodology

2.1. Effective field theory motivated temperature-dependent relativistic mean field model (E-TRMF)

The relativistic mean field (RMF) model is among the most successful and extensively used理论 to study the finite nuclei as well as infinite nuclear matter and neutron stars. It is in fact effective the relativistic version of the Hartree-Fock Bogoliubov theory and the distinct advantage of RMF is that it automatically takes into account the spin-orbit interaction. It is adequate to predict the bulk properties of nuclei in ground and excited states of nuclei throughout the periodic table. Within RMF model, the nucleons are assumed to oscillate independently in a harmonic oscillator motion in the mean field produced via the exchange of mesons and photons. The nucleons interact with each other via the exchange of isoscalar-scalar $\sigma$, isovector-vector $\rho$ and isoscalar-vector $\omega$ mesons. It is realized that the self- and crossed interactions among the mesons have great influence on the EOS, whose effect is substantial on the production of various baryons as well as on the prediction of mass and radius of neutron stars. All the interacting terms have their own importance to determine the properties of finite nuclei as well as the properties of infinite nuclear matter. Thus, the effective field theory motivated relativistic mean field (E-RMF) model is the extension of the RMF model in which all possible interacting mesons are considered. In the present paper, we have included up to fourth order of field expansion which is sufficient to get reasonable results [68–70]. For axially deformed case, the Lagrangian density of nucleons with $\sigma-$, $\omega-$, $\rho-$mesons and photon $A^\mu$ fields within E-RMF is given as [70,73]:

$$\mathcal{L}(r_\perp,z) = \bar{\psi}(r_\perp,z) \left( i\gamma^\mu \partial_\mu - M + g_\omega \sigma - g_\rho \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \tau \rho_\mu - e\gamma^\mu \frac{1 + \tau_3}{2} A_\mu \right) \times \psi(r_\perp,z)$$

$$+ \frac{1}{2} \left( \partial_\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu$$

$$- \frac{1}{4} \tilde{R}^{\mu\nu} \tilde{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^\mu \rho_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - m_\omega^2 \sigma^2 \left( \frac{k_3}{3!} \frac{g_\omega^2 \sigma}{M} + \frac{k_4}{4!} \frac{g_\omega^2 \sigma^2}{M^2} \right)$$

$$+ \frac{1}{4!} \zeta_0 g_\omega^2 (\omega^\mu \omega_\mu)^2 + \Lambda_\omega g_\omega^2 g_\rho^2 (\omega^\mu \omega_\mu) (\rho^\mu \rho_\mu), \quad (2)$$

with

$$V^{\mu\nu} = \partial_\mu \omega^\nu - \partial_\nu \omega_\mu, \quad (3)$$

$$\tilde{R}^{\mu\nu} = \partial_\mu \tilde{\rho}^\nu - \partial_\nu \tilde{\rho}_\mu, \quad (4)$$

$$F^{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu. \quad (5)$$

The $m_\sigma$, $m_\omega$, and $m_\rho$ and $g_\sigma$, $g_\omega$ and $g_\rho$ are the masses and coupling constants of $\sigma-$, $\omega-$ and $\rho-$ mesons, respectively. These coupling constants represent the interaction strengths of nucleons with the mesonic fields. The parameter $\zeta_0$ corresponds to the self-coupling of $\omega$ mesons,
\(\Lambda_\omega\) corresponds to the cross-coupling between \(\omega\) and \(\rho\) mesons and \(\varphi(r_\perp, z)\) is the axially deformed wave function where \(r_\perp = \sqrt{x^2 + y^2}\). The photon field \((A^\mu)\) gives the electromagnetic interaction due to protons with strength \(\frac{e^2}{4\pi}\). The equations of motion for the nucleon and boson fields are obtained by solving the E-RMF Lagrangian density using the variational principle and applying the mean field approximation. Further, the fields are redefined as \(\Phi_1 = g_s \sigma_0, W = g_\omega \omega_0, R = g_\rho \rho_0\) and \(A = e A_0\). The Dirac equation, obtained from the Lagrangian density (Eq. (2)), for nucleons is

\[
-\mathbf{\alpha} \cdot \nabla + \beta [M - \Phi(r_\perp, z)] + W(r_\perp, z) + \frac{1}{2} \tau_3 R(r_\perp, z) + \frac{1 + \tau_3}{2} A(r_\perp, z) \left\{ \varphi_i(r_\perp, z) = \varepsilon_i \varphi_i(r_\perp, z). \right. \tag{6}
\]

The equations of motion of \(\Phi, W, R\) and \(A\) fields are given by

\[
-\Delta \Phi(r_\perp, z) + m^2_\omega \Phi(r_\perp, z) = \frac{g^2_s}{4\pi} \rho_\omega(r_\perp, z) - \frac{m^2_\omega}{M} \Phi^2(r_\perp, z) - \frac{\kappa_3}{2} + \frac{\kappa_4}{3!} \Phi(r_\perp, z) \tag{7}
\]

\[
-\Delta W(r_\perp, z) + m^2_\omega W(r_\perp, z) = \frac{g^2_s}{4\pi} \rho(r_\perp, z) - \frac{1}{3!} g_\omega W^3(r_\perp, z) - 2 \Lambda_\omega g^2_\omega R^2(r_\perp, z) \times W(r_\perp, z), \tag{8}
\]

\[
-\Delta R(r_\perp, z) + m^2_\rho R(r_\perp, z) = \frac{1}{2} g^2_\rho \rho_\omega(r_\perp, z) - 2 \Lambda_\omega g^2_\omega R(r_\perp, z) \times W^2(r_\perp, z), \tag{9}
\]

\[
-\Delta A(r_\perp, z) = e^2 \rho_\omega(r_\perp, z). \tag{10}
\]

The baryon, scalar, isovector and proton densities used in the above equations are defined as

\[
\rho(r_\perp, z) = \sum_i n_i \bar{\psi}_i(r_\perp, z) \psi_i(r_\perp, z) \tag{11}
\]

\[
\rho_\omega(r_\perp, z) = \sum_i n_i \bar{\psi}_i(r_\perp, z) \beta_\omega \psi_i(r_\perp, z) \tag{12}
\]

\[
\rho_3(r_\perp, z) = \sum_i n_i \bar{\psi}_i(r_\perp, z) \tau_3 \psi_i(r_\perp, z) \tag{13}
\]

\[
\rho_\rho(r_\perp, z) = \sum_i n_i \bar{\psi}_i(r_\perp, z) \left( \frac{1 + \tau_3}{2} \right) \psi_i(r_\perp, z). \tag{14}
\]

Here, \(\tau_3\) is the third component of the isospin and the summation \(i\) is taken over for all nucleons. The temperature comes into picture through the occupation number \(n_i\) in the formalism given as \([71]\):

\[
n_i = v_i^2 = \frac{1}{2} \left[ 1 - \frac{\varepsilon_i - \lambda}{\varepsilon_i} [1 - 2 f(\varepsilon_i, T)] \right], \tag{15}
\]
where
\[ f(\tilde{\epsilon}_i, T) = \frac{1}{(1 + \exp[\tilde{\epsilon}_i/T])} \]
is the Fermi Dirac distribution for the quasi particle energy and \( \tilde{\epsilon}_i = (\epsilon_i - \lambda)^2 + \Delta^2 \). The effective mass of nucleon in the presence of mesons is given as \( M^* = M - \Phi(r_{\perp}, z) \) and the vector potential is
\[
V(r_{\perp}, z) = W(r_{\perp}, z) + \frac{1}{2} \tau_3 R(r_{\perp}, z) + \frac{1 + \tau_3}{2} A(r_{\perp}, z).
\]

We have solved the set of coupled differential Eqs. (6)-(10) self consistently by expanding the Boson and Fermion fields in an axially deformed harmonic oscillator basis with \( \beta_0 \) as the initial deformation. After getting a convergent solution of the fields, the densities and energy of a nucleus are obtained. The total energy of a nucleus at finite temperature \( T \) is given by the expression [71,74–76],
\[
E(T) = E_{\text{part}} + E_{\text{mes}} + E_C + E_{\text{pair}} + E_{\text{c.m.}} - AM,
\]
where \( E_{\text{part}} \) is the sum of the single-particle energies of the nucleons and \( E_{\text{mes}} \), \( E_C \) are the contributions of the mesons and Coulomb fields. \( E_{\text{pair}} \) is the pairing energy obtained from the BCS formalism [77] given as
\[
E_{\text{pair}} = -\Delta \sum_{i>0} u_i v_i = -\frac{\Delta^2}{G},
\]
where \( u_i^2 \) and \( v_i^2 \) are the probabilities of unoccupied and occupied states, respectively, and \( \Delta \) is the pairing gap. The details formalism of BCS can be found in Ref. [71], and \( E_{\text{c.m.}} = -\frac{3}{4} \times 41 A^{-1/3} \) MeV, is the centre of mass energy correction obtained from the non-relativistic approximation [78]. The total binding energy \( E(T) \) is obtained by subtracting the rest mass energy \((AM)\) of nucleus, where \( A \) is the mass number and \( M \) is the nucleonic mass. The quadrupole deformation parameter \( \beta_2 \) is calculated using the resulting proton and neutron quadrupole moments,
\[
Q = Q_n + Q_p = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} AR^2 \beta_2.
\]
The mean square radii of neutrons and protons is defined as
\[
< R_{n,p}^2 > = \frac{1}{A} \int \rho(r_{\perp}, z)r^2 d\tau,
\]
where \( \rho(r_{\perp}, z) \) is the deformed density, \( A \) is the mass number and root mean square (r.m.s.) radii for protons and neutrons are given by \( R_{n,p} = < R_{n,p}^2 > ^{1/2} \). The neutron-skin thickness is calculated as the difference between neutron and proton radii: \( \Delta R = < R_n^2 > ^{1/2} - < R_p^2 > ^{1/2} \).
After obtaining the axially deformed densities from E-RMF, we have changed it into spherical equivalent density by the following steps: Initially we have the axially deformed density \( \rho(r_{\perp}, z) \) which is converted to one dimensional as \( \tilde{\rho}(r_{\perp}) \) where \( r_{\perp} = \sqrt{x^2 + y^2} \) by performing the \( z \)-integration over the whole space as done in [79,80]
\[
\rho(r_{\perp}) = \int_{-\infty}^{\infty} \tilde{\rho}(r_{\perp}, z) dz.
\]
Then $z$-integrated density $\tilde{\rho}(r_\perp)$ is fitted to a two point Gaussian function expressed as $\rho(r) = \sum_{i=1}^{2} c_i \exp[-a_i r^2]$, where the co-efficient $c_i$ and range $a_i$ are given with initial values for respective nuclei. The spherical equivalent densities are normalized to mass number ($A$) of the nucleus. Then we have used the spherical equivalent densities for further calculations of symmetry energy and its bulk and surface components within coherent density fluctuation model, discussed briefly in following sub-section.

2.2. The coherent density fluctuation model (CDFM)

The expression of nuclear energy within the droplet model, an extension of the Bethe-Weizsäcker liquid drop model, incorporating the volume and surface asymmetry is written as \cite{81,82}:

$$E(A, Z) = -B A + E_S A^{2/3} + S_V A \frac{(1 - 2Z/A)^2}{1 + S_S A^{-1/3}/S_V} + E_C \frac{Z^2}{A^{1/3}} + E_{di} \frac{Z^2}{A} + E_{ex} \frac{Z^{4/3}}{A^{1/3}} + a A^{-1/2}. \tag{22}$$

In the above eq. $B \sim 16$ MeV presents the binding energy per particle of symmetric matter at saturation. $E_S$, $E_C$, $E_{di}$, and $E_{ex}$ represent the coefficients corresponding to the surface energy of symmetric matter, the Coulomb energy of a uniformly charged sphere, the diffuseness correction, and the exchange correction to the Coulomb energy, respectively, and the last term provides the pairing corrections. $S_V$ is the volume symmetry energy parameter and $S_S$ is the modified surface symmetry energy parameter in the liquid drop model (see Ref. [81]). In the present work, to study the temperature-dependence, the symmetry energy (3rd term on the R.H.S. of Eq. (22)) is rewritten in the form $S(T)(N - Z)^2/A$ where

$$S(T) = \frac{S_V(T)}{1 + \frac{S_S(T)}{S_V(T)} A^{-1/3}} = \frac{S_V(T)}{1 + \frac{A^2}{\kappa(T)}}. \tag{23}$$

From the above Eq., the following Eqs. can be written:

$$S_V(T) = S(T) \left(1 + \frac{1}{\kappa(T) A^{1/3}}\right) \tag{24}$$

and

$$S_S(T) = \frac{S(T)}{\kappa(T)} \left(1 + \frac{1}{\kappa(T) A^{1/3}}\right). \tag{25}$$

The calculations of the symmetry energy and its volume and surface components are performed within CDFM \cite{83}. In the CDFM, the one-body density matrix of a nucleus can be written as a coherent superposition of infinite number of one-body density matrices $\rho_x(r, r')$ for spherical pieces of the nuclear matter called Fluctons \cite{66,84,85} with, $\rho_x(r) = \rho_0(x) \Theta(x - |r|)$, and $\rho_0(x) = \frac{3 A}{4 \pi x^3}$ is the Flucton density. The generator coordinate $x$ is the spherical radius of the nucleus contained in an uniformly distributed spherical Fermi gas. In finite nuclear system, the one body density matrix can be given as \cite{66,84,85},

$$\rho(r, r') = \int_0^{\infty} dx |F(x)|^2 \rho_x(r, r'). \tag{26}$$
where, $|F(x)|^2$ is the weight function. The term $\rho_x(r, r')$ is the coherent superposition of the one body density matrix. For a detailed analytical derivation, one can follow the Refs. [84]. The T-dependent symmetry energy $S(T)$ is calculated by

$$S(T) = \int_0^\infty dx |F(x, T)|^2 S[\rho(x, T)],$$  \hspace{1cm} \text{(27)}$$

where the weight function $|F(x, T)|^2$ depends on the density distribution,

$$|F(x, T)|^2 = -\left(\frac{1}{\rho_0(x)} \frac{d\rho(r, T)}{dr}\right)_{r=x}.$$  \hspace{1cm} \text{(28)}$$

Following Refs. [44,45] an approximate expression for the ratio $\kappa(T)$ can be written within the CDFM,

$$\kappa(T) = \frac{3}{R\rho_0} \int_0^\infty dx |F(x, T)|^2 x\rho_0(x) \left(\frac{S(\rho_0)}{S(\rho(x, T))} - 1\right),$$  \hspace{1cm} \text{(29)}$$

where $\rho_0$ is the equilibrium nuclear matter density and $S(\rho_0)$ is the symmetry energy corresponding to equilibrium nuclear matter density $\rho_0$ and $T = 0$ MeV. Employing the density dependence of symmetry energy [44]:

$$S[\rho(x, T)] = S^V(T) \left(\frac{\rho(x, T)}{\rho_0}\right)^\gamma.$$  \hspace{1cm} \text{(30)}$$

There exist various estimations for the value of the parameter $\gamma$ [45,46]. The density dependence parametrization of the symmetry energy in the form of a power (Eq. (30)) is widely used in several theoretical calculations as well as experimental analysis of heavy ion collisions [86–88]. We also intend to use more realistic parametrization contemplating the different density dependence of the kinetic and potential parts of the symmetry energy as given in Ref. [89] and comparison of results with that using Eq. (30) is of interest, which will be presented in next paper [90]. Using the above Eq. (30) and $S(\rho_0) = S^V$, the Eqs. (27) and (29) can be re-written as follows:

$$S(T) = S(\rho_0) \int_0^\infty dx |F(x, T)|^2 \left(\frac{\rho(x, T)}{\rho_0}\right)^\gamma,$$  \hspace{1cm} \text{(31)}$$

and

$$\kappa(T) = \frac{3}{R\rho_0} \int_0^\infty dx |F(x, T)|^2 x\rho_0(x) \times \left(\left(\frac{\rho_0}{\rho(x, T)}\right)^\gamma - 1\right).$$  \hspace{1cm} \text{(32)}$$

3. Calculations and discussion

In this section, we present the results and related discussions of the symmetry energy and its volume and surface components for isotopic chains of Nd, Sm, Gd, and Dy rare earth nuclei using the relativistic mean field densities, with IOPB-I and NL3 parameter sets, within CDFM. The IOPB-I [70] is recently developed parameter which works well for finite nuclei as well as
for nuclear matter. The widely used standard NL3 parameter set has been used for the comparison purpose. Many nuclei in the isotopic chains of rare earth nuclei are identified as deformed and shape transitions are expected. Therefore, E-TRMF equations have been solved for axially deformed configuration to calculate the temperature-dependent densities. The self consistent calculations have been done using Boson major shell number $N_B = 20$ and Fermion major shell number $N_F = 12$, at which the solutions converge.

3.1. Temperature-dependent bulk properties of nuclei

Here, we present the change in the bulk properties such as binding energy per nucleon (B.E./A), deformation parameter ($\beta_2$) and skin thickness ($\Delta R$) with respect to rise in temperature ($T$), with IOPB-I parameter set. Fig. 1(a) presents the change in B.E./A with temperature. At a particular temperature, as the mass number of nucleus increases, the B.E./A and hence the stability of nuclei decreases. With an increase in $T$, the decrease in B.E./A is pronounced for more neutron-rich nuclei while drifting to higher $T = 2$ and 3 MeV. It is seen that at $T = 3$ MeV, the decrease in B.E./A with mass is quite sharp. Also, the comparison of theoretical B.E./A calculations within RMF with available experimental data [91] at $T = 0$ MeV is made, which shows good agreement. Next, we present the temperature-dependence of deformation (shape) of nuclei since deformation is known to play a key role in describing the properties like isotopic shift and nuclear size. The calculated deformations are shown in Fig. 1(b), which depicts the evolution of quadrupole deformation parameter $\beta_2$ with mass of rare earth nuclei in the isotopes chains at different temperatures. $^{142}$Nd, $^{144}$Sm, $^{146}$Gd and $^{148}$Dy nuclei are nearly spherical in shape. At $T = 0$ MeV, the prolate deformation increases dramatically with mass number up to neutron number $N = 100$ corresponding to $^{160}$Nd, $^{162}$Sm, $^{164}$Gd and $^{166}$Dy nuclei with maximum deformation $\beta_2 = 0.35$ in the corresponding isotopic chains. Afterwards, with increase in neutron number, the prolate deformation decreases and $^{186}$Nd, $^{188}$Sm, $^{190}$Gd and $^{192}$Dy nuclei with $N = 126$ acquire the spherical shape. The comparison with available FRDM calculations of quadrupole deformation parameter $\beta_2$ by Möller et al. [27] is also made, which shows slight over estimation of calculated $\beta$-values for nuclei with $N = 90-110$. Further, it is noted that temperature has indispensable impact on the nucleus shape. With increasing temperature, the value of
deformation parameter $\beta$ decreases in comparison to at $T = 0$ MeV. At $T = 2$ MeV, there is noteworthy change in $\beta$-value since with increasing thermal energy, the nucleus undergo vibration and expansion. Further with rise in temperature at $T = 3$ MeV, the nuclei become spherical in shape. Essentially, this change in quadrupole deformation $\beta$ with temperature will affect the density distribution, which is the key quantity in the determination of symmetry energy. We found almost similar values of quadrupole deformation parameter $\beta_2$ with the NL3 parameterization (not shown here).

Fig. 2(a) shows the root mean square (r.m.s.) radii of neutrons $R_n$ (solid lines) and protons $R_p$ (dotted lines) as a function of mass number $A$ in the isotopic chain of Nd, Sm, Gd, and Dy nuclei, with IOPB-I parameter set. It is seen that increase in r.m.s. radii of protons is less compared to that of neutrons at all temperatures. It is obvious since the number of protons remains the same within a particular isotopic chain. Moreover, the results at $T = 0$ and 1 MeV are close to each other but with rising temperature, there is a steep increase in the $R_n$ with increasing mass number. Fig. 2(b) gives the difference between r.m.s. radius of neutron and proton distribution i.e. $\Delta R = R_n - R_p$, which gives the measure of emergence of neutron skin thickness in Nd, Sm, Gd and Dy isotopic chain. For a particular mass $A$, the neutron skin thickness increases while moving to higher temperature and this increase in $\Delta R$ with temperature is more for more neutron-rich nuclei. For a particular temperature, there is a monotonic increase in $\Delta R$ with increasing mass or neutron number in the isotopic chains of considered nuclei. The positive value of $\Delta R$ shows that radial distribution of neutron is more extended in comparison to proton. It increases considerably with rising temperature and its value is larger at $T = 2$ and 3 MeV than at lower temperatures. In Ref. [92], the investigation of temperature-dependence of neutron skin formation mechanism in Sn isotopes reveals an enhancement in neutron skin with temperature which is principally attributed to the effect of temperature on the occupation probabilities of single particle states near the Fermi energy.

Further, we note that the charge radius and deformation are closely related and the charge radii have been used to find isotopic shifts $\delta < r^2_{ch} >= < r^2_{ch} > - < r^2_{ch} >$(ref) for all isotopic chains. The $< r^2_{ch} >$(ref) denotes the r.m.s. charge radius of the reference nucleus for a particular isotopic chain. The nuclei $^{142}$Nd, $^{144}$Sm, $^{146}$Gd, and $^{148}$Dy corresponding to $N = 82$ have been used as reference nuclei. Fig. 3 presents the isotopic shifts in the isotopic chains of all considered rare earth nuclei. It is noted that for a particular temperature, the isotopic shift increases as a function
Fig. 3. Variation of isotopic shift of $\delta < r^2_{ch} >$ with mass number $A$ at different temperatures with IOPB-I parameter set.

Fig. 4. The variation of spherical equivalent density $\rho$ (upper panel) and weight function $|F(x, T)|^2$ (lower panel) of $^{144}$Sm and $^{162}$Sm nuclei at $T = 0, 3$ MeV for IOPB-I parameter set.

of mass number or neutron number. While with increment in temperature, the magnitude of isotopic shift reduces which indicates the isotopic shift is affected by the $\beta_2$ of nuclei since $\beta_2$ changes with change in temperature.

3.2. Temperature-dependent densities and weight functions of nuclei

The temperature-dependent densities of nuclei are calculated within the E-TRMF with IOPB-I and NL3 parameter sets. The evolution of spherical equivalent density with temperature is
shown for $^{144}\text{Sm}$ and $^{162}\text{Sm}$, comparatively, in Fig. 4 (upper panel) corresponding to IOPB-I parameter set. It shows that with rise in temperature, the density decreases in the central region of nucleus. The nuclear surface shows a little more diffuseness with an increase in $T$, which occurs due to weakening of shell effects with rising temperature. Further, we give details of weight function $|F(x,T)|^2$ calculations in Fig. 4 (lower panel) since it is closely related with density and hence the structural peculiarities of a particular nucleus. It is noted that weight function (Eq. (28)) shows the bell shape form with maxima at $r$ where the density reduces significantly compared to its central value. Therefore, in this region the values of $S(\rho)$ contribute significantly in the evaluation of $S(T)$ and $\kappa(T)$ (refer Eqs. (31) and (32)). In other words, it is observed that substantial values of weight function including the peak value lies in the domain which corresponds to the surface region of the density and hence these quantities are termed as surface properties. At $T = 0$ MeV, the magnitude of central density of $^{162}\text{Sm}$ is higher than $^{144}\text{Sm}$ but it has very small influence on the weight function calculations. It is evident from Fig. 4 (lower panel), which shows that value of weight function is quite small up to $x \sim 3$ fm. It is to be noted that the density and weight function are important factors in the calculation of symmetry energy defined in Eq. (31). Moreover, as the temperature increases, the peak in the weight function curve shifts down and moves little towards right. It happens due to change in density with increasing temperature since the density is the fundamental ingredient in the evaluation of weight function. At next step, the symmetry energy at local coordinate $x$ is folded with weight function (Eqs. (31), (24), (25)) to calculate the corresponding effective surface properties of the nucleus such as symmetry energy and its volume and surface components, which are discussed in the next section.

3.3. Temperature-dependent symmetry energy and its volume and surface contributions and their ratio

The temperature-dependent symmetry energy and its volume and surface contributions and their ratio $\kappa$ is calculated from Eqs. (31) and (32) and the temperature-dependent weight function $|F(x,T)|^2$ by using the Eq. (28) within the CDFM. The investigation of temperature-dependence of the mentioned quantities depicts a certain sensitivity of the results to the $\gamma$ parameter value (shown below). Therefore, to choose its value we impose the condition that the value of studied quantities at $T = 0$ MeV should be comparable with available experimental data [44–46]. The effect of temperature on the variation of symmetry energy ($S$) and its volume $S_V$ and surface $S_S$ components and their ratio $\kappa$ as a function of neutron number $N$ is shown in Figs. 5–7 for the isotopic chains of Nd, Sm, Gd, and Dy nuclei. The results are shown for two values of the parameter $\gamma = 0.3$ and 0.4 and the reason for this choice is already mentioned above. It is noted that at $T = 0$ MeV with $\gamma = 0.4$ (Fig. 7), the value of $\kappa$ is 2.25-2.6 and is in agreement with $\kappa$ values extracted from nuclear characteristics such as from skin and masses $2.0 \leq \kappa \leq 2.8$ [44] and from skins and isobaric analog states $2.6 \leq \kappa \leq 3.0$ [45]. For $T = 0$ MeV with $\gamma = 0.3$ (Figs. 5-6), the maximum value of $\kappa$ is nearly 1.56 which is in consonance with $1.6 \leq \kappa \leq 2.0$ in Ref. [46]. It is noted from Figs. 5-6 with $\gamma = 0.3$ that the CDFM results corresponding to (i) IOPB-I parameter set give the value of symmetry energy as $22 \leq S \leq 24.5$ MeV, volume symmetry energy as $24.5 \leq S_V \leq 27.5$ MeV and that of surface symmetry energy $16 \leq S_S \leq 20$ MeV and (ii) NL3 parameter set give the value of symmetry energy as $24.5 \leq S \leq 27.5$ MeV, volume symmetry energy as $27.5 \leq S_V \leq 31$ MeV and that of surface symmetry energy $17.5 \leq S_S \leq 22.2$ MeV.
Fig. 5. Effect of temperature on the variation of symmetry energy $S$, volume $S_V$ and surface symmetry $S_S$ energies and their ratio $\kappa$ as a function of neutron number $N$ for isotopic chain of (a) Nd and (b) Sm nuclei with IOPB-I (left panel) and NL3 parameters (right panel) with $\gamma = 0.3$.

Fig. 6. Same as Fig. 5 but for isotopic chain of (a) Gd and (b) Dy nuclei.

The available results of $S_V$ from the different analyses are: $27 \leq S_V \leq 31$ MeV [44]; $30 \leq S_V \leq 32.5$ MeV [45]; $31 \leq S_V \leq 33$ MeV [46]. Our calculations present the values of $S_V$, which
are close to that of Ref. [44]. Moreover, it is seen that value of ratio ($\kappa$) of volume and surface symmetry energies corresponding to $\gamma = 0.3$ lies in the range 1.37-1.56. We note that this value is somewhat smaller in comparison to results of Danielewicz et al. extracted from the available wide range of data on the B.E. and fitting of other nuclear properties like excitation energies of isobaric analog states and neutron skin size [45]. The value of $\kappa$ with $\gamma = 0.4$ is 2.25-2.6. The combination of neutron skin thickness and nuclear masses fitting put constraints on the range of volume symmetry energy ($S_V$) as 27-31 MeV and that of volume to surface symmetry energy ratio ($\kappa$) 2.0-2.8 [44]. The values of ratio $\kappa$ obtained from different nuclear characteristics as shown in Table 2 of Ref. [46] are: $1.6 \leq \kappa \leq 2.0$ [46], $2.0 \leq \kappa \leq 2.8$ from skin and masses [44] and $2.6 \leq \kappa \leq 3.0$ from skins and isobaric analog states [45]. The calculated $\kappa$ values, in the present work with $\gamma = 0.4$ lie close to the range given by Ref. [44,45].

Qualitatively, at $T = 0$ MeV the symmetry energy and its volume and surface contributions exhibit the rise and fall trend with a peak at neutron number $N = 100$. This peak demonstrates that more energy is required to convert neutrons into protons for $N = 100$ nuclei and it finds origin in the peculiar density distribution of corresponding nuclei since the weight factor (Eq. (28)), used in the integrand of symmetry energy calculations (Eq. (31)), depends upon the density of the nuclei. As mentioned earlier and also can be seen from Eq. (31), the weight function and/or density are crucial to determine the symmetry energy of a nucleus. The small change in the weight function results in a significant difference in the symmetry energy when it integrates over $\chi$ [see Eqs. (28) and (31)] in an isotopic chain. From Fig. 4 we note that magnitude of weight function for $^{162}$Sm is higher than for $^{144}$Sm. This difference in density and weight function results in the peak at $^{162}$Sm ($N = 100$) nucleus over the isotopic chain. In other words, the area covered by weight function for $N = 100$ nuclei for considered nuclei are larger in magnitude over the isotopic chains of rare earth nuclei leading to peak at $N = 100$. This peak presents a signature of deformed shell/sub-shell closure at $N = 100$, discussed in detail in Ref. [93]. This fact is fur-

Fig. 7. Same as Fig. 6 for Dy isotopic chain with $\gamma = 0.4$. 
ther supported by the ground state neutron single particle spectra of $^{160}\text{Nd}$, $^{162}\text{Sm}$, $^{164}\text{Gd}$, and $^{166}\text{Dy}$ rare earth nuclei shown in Fig. 8 for NL3 parameter set. A considerable energy gap at neutron number $N = 100$ is noted, which is an indicator of shell closure. It is noted that although the proton number is varying but the shell gap at $N = 100$ continue to persist. This result is in consonance with the earlier studies by one of us [72,94]. Later on, Patel et al. have experimentally confirmed the existence of this deformed shell closure at $N = 100$ in the $^{162}\text{Sm}$ and $^{164}\text{Gd}$ isotones [95].

However, the scenario changes at higher temperatures of $T = 1$ and 2 MeV, where the magnitude of peak decreases and finally at $T = 3$ MeV the peak in the symmetry energy and its components disappears. A plateau is seen in the studied quantities at $T = 3$ MeV. A kink/dip is seen at deformed magic neutron number $N = 100$ in the variation of ratio ($\kappa$) of volume and surface symmetry energies as a function of neutron number at lower temperatures and this dip also vanishes at $T = 3$ MeV. It is observed that value of bulk and surface symmetry energies decrease with an increase in temperature whereas their ratio $\kappa$ shows the reverse trend. It is also noted that the surface symmetry energy presents more sensitivity towards the temperature change compared to the volume symmetry energy. Quite interesting, we note that the evolution of symmetry energy and its volume and surface components is closely related to the evolution of quadrupole deformation parameter $\beta_2$ at different temperatures (see Fig. 1(b)). With rise in temperature, there is a decrease in $\beta_2$ value and at $T = 3$ MeV, the shape of nuclei changes to spherical one. Therefore, the disappearance of the peak at $T = 3$ MeV in symmetry energy and its components may be due to shape change in addition to shell quenching at higher $T$.

4. Summary

In the present work, the temperature-dependence of symmetry energy, its volume and surface contributions and their ratio ($\kappa$) in the isotopic chains of rare earth Nd, Sm, Gd, and Dy nuclei has been analyzed within the formalism of CDFM. For the calculation of weight functions, the temperature-dependent densities from E-TRMF, with NL3 and recently developed IOPB-I parameterizations, have been used as an input. At first, we have explored the effect of temperature
The rise in temperature leads to decrease in total B.E. and stability of nuclei and this decrease is larger for more neutron-rich nuclei. The deformation of nuclei is found to change significantly with an increase in temperature and hot nuclei become spherical in shape at $T = 3$ MeV. Further, it is noted that neutron radii changes considerably compared to proton radii while drifting to higher temperatures and hence the magnitude of neutron skin thickness increases with an increase in temperature. The value of central density reduces while the tail portion of density is extended with rise in temperature and this change leads to the shifting of peak towards the right in the weight function curve.

The temperature-dependent weight functions are further used to probe the thermal evolution of symmetry energy and its volume and surface contributions and their ratio. The calculated values of $S_V$ and ratio $\kappa$ are in agreement with the results of Refs. [44] and [46], respectively. At lower temperatures, the variation of symmetry energy and its volume and surface contributions as a function of neutron number show rise and fall trend with a peak at $N = 100$. This signifies the presence of deformed shell/sub-shell closure at $N = 100$. This fact is supported by considerable energy gap in the neutron single particle spectra of rare earth nuclei. While drifting to higher temperature the curve shifts downwards and the magnitude of peak decreases and this peak vanishes at $T = 3$ MeV due to quenching of shell effects along with shape change of nuclei since careful examination of temperature evolution of quadrupole deformation and symmetry energy of nuclei unveils that these quantities are closely related. In addition, it is seen that surface symmetry energy turns out to be more sensitive to temperature compared to the volume symmetry energy.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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