Investigation of heat transfer in porous channels

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Abstract

Purpose – This paper aims to investigate the heat transfer in porous channels.

Design/methodology/approach – Finite element method is used to simulate the heat transfer in porous channels.

Findings – The number and width of channels play a key role in determining the heat transfer of the porous channel. The heat transfer is higher around the channel legs. Smaller base height is better to get higher heat transfer capability.

Originality/value – This study represents the original work to investigate heat transfer in a porous domain having multiple channels.

Keywords Porous channel, Heat transfer, FEM

Paper type Research paper

Nomenclature

$B_w$ = Base height;
$c_p$ = Specific heat of fluid (J/kg°C);
$C_w$ = Channel width;
$g$ = Acceleration due to gravity (m/s²);
$k$ = Thermal conductivity (W/m°C);
$K$ = Permeability of porous medium (m²);
$L$ = Length and Height of domain (m);
$N_c$ = Number of channels;
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\( Nu \) = Local Nusselt number;
\( qr \) = Radiation flux (W/m²);
\( Rd \) = Radiation parameter;
\( Ra \) = Modified Rayleigh number;
\( T, \overline{T} \) = Temperature;
\( u, v \) = Velocity components (m/s);
\( x, y \) = Cartesian co-ordinates; and
\( \pi, \overline{\Pi} \) = Non-dimensional co-ordinates.

Greek Symbols
\( \alpha \) = Thermal diffusivity (m²/s);
\( \beta \) = Coefficient of thermal expansion (1/°C);
\( \rho \) = Density (kg/m³);
\( \mu, \nu \) = Coefficient of Dynamic (kg/m·s) and kinematic viscosity (m²/s), respectively;
\( \sigma \) = Stephan Boltzmann constant (W/m²·K⁴);
\( \beta_r \) = Absorption coefficient (1/m);
\( \Psi \) = Stream function; and
\( \overline{\psi} \) = Non-dimensional stream function.

Subscripts
\( h \) = Hot; and
\( c \) = Cold.

1. Introduction
Because of the tremendous scope of transport phenomena through porous media, the associated research is an evergreen topic for researchers in various fields of contemporary technologies. Its applications include cooling of electronic devices, geothermal engineering, solar energy collectors, thermal insulation, solidification of alloys, environmental engineering, food processing, heat pipes, chemical and nuclear waste disposal, fuel cells, packed bed heat exchangers, catalytic reactors, drying processes, tissue engineering, drug delivery and even in advanced medical applications. More detailed discussion of porous media technology and its applications are covered elaborately in the books authored by Nield and Bejan (1999), Vafai (2000), Pop and Ingham (2001) and Ingham and Pop (1998). The massive applications of porous media demand a more precise understanding of the mechanism of transport phenomenon for application-oriented optimum designs. To enable the choice of explicit operational parameters in porous medium should be understood clearly before venturing to any applications. The research on various aspects of porous medium has received substantial consideration and has been well documented in the past few years. Researchers have paid attention to the variety of geometries including the square cavity (Sheremet et al., 2018; Badruddin et al., 2018; Alsabery et al., 2017; Eswaramurthi et al., 2008; Badruddin et al., 2007; Arpino et al., 2015; Allouë et al., 2010; Nandalur et al., 2019; Saeid and Pop, 2004), porous annulus/cylinder (Jha and Musa, 2018; Xu, 2017; Hasnain and Abbas, 2017; Arpino et al., 2016; Arpino et al., 2013; Badruddin et al., 2015; Ahmed et al., 2014; Badruddin et al., 2007; Badruddin et al., 2019) vertical plate with porous medium (Dash et al., 2011; Chamkha et al., 2011; Olajuwon and Ishola, 2011). The regular square, rectangle or cylindrical geometries with porous medium has been prominent choice for heat transfer investigation. However, the literature suggests that there have been various efforts to study the non-regular porous geometries including the porous duct (Badruddin et al., 2012a; Badruddin et al., 2012b; Nik-Ghazali et al., 2014), trapezoidal enclosure (Baytas and Pop, 2001; Kumar and Kumar, 2004; Varol et al., 2009) and porous medium
confined in non-regular porous domain (Misirlioglu et al., 2005, 2006; Bhardwaj and Dalal, 2013, 2015; Badruddin et al., 2019; Badruddin, 2019). The heat transfer in porous media varies according to the geometry and the boundary conditions being adopted for a particular domain. In general, heat transfer is found to decrease along with the height of square porous cavity due to heated and cooled vertical surfaces (Saeid and Pop, 2004). This trend gets altered if any solid block is present at the bottom of cavity and the location of the block along the horizontal direction further dictates this variation (Sheremet et al., 2018). Heat transfer in annular porous medium is substantially different from that of square or rectangular cavity even though similar boundary conditions as that of square or rectangular cavity might be present. Heat transfer depends on the aspect ratio in case of vertical annulus and it has a peculiar behaviour of maximum heat transfer at a particular aspect ratio (Badruddin et al., 2007). The presence of hollow section at centre of cavity leads to symmetric heat transfer from bottom surface (Badruddin et al., 2012a; Badruddin et al., 2012b). The surface roughness to plays its role in altering the heat transfer from hot surface to porous region. It is noted that the surface roughness in the form of wave makes to heat transfer to fluctuate along the hot surface (Cheng, 2000). Thus, it can be conveniently said that the heat transfer is a function of porous domain and the boundary conditions being applied apart from physical parameters such as Rayleigh number, Radiation parameter, viscous dissipation, and so on. Most of the applications require that the heat transfer should be increased from hot surface to porous medium for various purposes. One of the common practices being followed is to increase the surface area exposed to cold temperatures. The cooling surface area can be easily increased by incorporating multiple channels subjected to cooling temperature in the domain. However, it is noted that the literature lacks in such kind of studies particularly in porous medium. The present article is novel in a way to study the impact of increased cooling surface by incorporating multiple channels in a porous medium. This kind of porous channels can be an effective tool in applications such as fuel cell technology (Crowe, 1973; Prater, 1994) that demands more quality research pertaining to porous media. Hence, any improvement in the transport phenomena in porous channel will further enhance the performance of the fuel cell (Bradean et al., 2002; Yuan et al., 2003; Jen and Yan, 2005).

2. Methodology

Consider a porous domain containing channels as shown in Figure 1. The base of the domain is heated isothermally to temperature $T_h$, whereas all interior and top surfaces of the channel are cooled to temperature $T_c$ as depicted in Figure 1. The left and right vertical
surfaces are adiabatic. The governing equations for the problem under investigation can be written as:

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y} &= 0 \\
\frac{\partial \mathbf{u}}{\partial x} - \frac{\partial \mathbf{v}}{\partial y} &= -\frac{g \beta K}{v} \frac{\partial T}{\partial x} \\
\mathbf{u} \frac{\partial T}{\partial x} + \mathbf{v} \frac{\partial T}{\partial y} &= \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial x}
\end{align*}
\]

where \( \mathbf{u} \) and \( \mathbf{v} \) are Darcy's velocities in the \( x \) and \( y \) directions, respectively. The boundary conditions are:

\[
\text{at } y = 0 \quad T = T_h
\]

All the channel walls are maintained at \( T = T_c \) as depicted in Figure 1. All the walls of domain are applied with no-slip condition. Velocity is represented by stream function \( \psi \) as given below:

\[
u = -\frac{\partial \psi}{\partial x}, \quad u = \frac{\partial \psi}{\partial y}
\]

The governing equations can be non-dimensionalised with the help of the following parameters:

\[
\bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}, \quad \frac{\psi}{\alpha}, \quad T = \frac{(T - T_c)}{(T_h - T_c)}, \quad \frac{4\alpha T_c^3}{\beta \gamma}, \quad \frac{Ra}{\nu \alpha} = \frac{g \beta \Delta TKL}{C_23}
\]

Radiation can be given as:

\[
q_r = -\frac{4\alpha}{3\beta} \frac{\partial T^4}{\partial x}
\]

\[
T^4 \approx 4T_c^4 - 3T_c^4
\]

Equations (1)-(3) can be converted in non-dimensional form with the help of above mentioned parameters:

\[
\frac{\partial^2 \psi}{\partial \bar{x}^2} + \frac{\partial^2 \psi}{\partial \bar{y}^2} = -Ra \frac{\partial \bar{T}}{\partial \bar{x}}
\]

\[
\left[ \frac{\partial \psi}{\partial \bar{y}} \frac{\partial \bar{T}}{\partial \bar{x}} - \frac{\partial \psi}{\partial \bar{x}} \frac{\partial \bar{T}}{\partial \bar{y}} \right] = \left( \frac{4Ra}{3} \right) \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}
\]

The boundary conditions take the form:
The heat transfer rate at the bottom surface of the domain is given in terms of Nusselt number as:

\[ Nu = \left( 1 + \frac{4Rd}{3} \right) \frac{\partial \bar{T}}{\partial \bar{y}} \bigg|_{\bar{y}=0} = 0 \]  

(12)

### 2.1 Numerical method

Finite element method (FEM) is used to solve the governing equations. A good insight into the FEM can be seen in well-documented literature such as Lewis and Garner (1972), Lewis and Schreiber (1987, 1998), Badruddin et al. (2017), Lewis et al. (1996), Lewis et al. (2004) and Morgan et al. (1984). A triangular element is used to divide the whole domain into smaller segments. The governing equations (9)-(10) are converted into matrix form of equations with the help of Galerkin approach that leads to a global matrix with the dimension $n \times n$, where $n$ stands for the number of nodes. An in-house computer programme is developed using Matlab to solve the matrix form of equations. Appropriate boundary conditions are applied and solution is obtained by solving the simultaneous equations in iterative manner. The methodology is verified by reducing the current domain to a square porous cavity by making the gap between channels as zero. It is worth mentioning that the data pertaining to problem under investigation is not available, thus, it was reduced to square cavity by increasing the width of channels so as to make the gap between two channels as zero. A total of 3,200 elements are used by making sure that they are mesh independent. It is clear from Table I that the methodology followed is accurate enough to predict the heat transfer behaviour.

### 3. Results and discussion

Figures 2 and 3 demonstrate the effect of channel width, $C_w$, on isotherms and streamlines with three and two channel domains, respectively, having other parameters as $B_w = 0.25$.

<table>
<thead>
<tr>
<th>Author</th>
<th>$Ra = 10$</th>
<th>$Ra = 100$</th>
</tr>
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<tbody>
<tr>
<td>Walker and Homsy (1978)</td>
<td></td>
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<tr>
<td>Bejan (1979)</td>
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<td>Beckermann et al. (1986)</td>
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<td>Moya et al. (1987)</td>
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<td>Baytas and Pop (1999)</td>
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<td>3.16</td>
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<tr>
<td>Misirlioglu et al. (2005)</td>
<td>1.119</td>
<td>3.05</td>
</tr>
<tr>
<td>Gross et al. (1986)</td>
<td></td>
<td>3.141</td>
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<tr>
<td>Manole and Lage (1992)</td>
<td></td>
<td>3.118</td>
</tr>
<tr>
<td>Present</td>
<td>1.081</td>
<td>3.201</td>
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**Table I.** Validation of present method for average Nusselt number $Nu$.
Figure 2.
Influence of channel width for three channels on (I) isotherms (II) streamlines.

Notes: (a) $C_w = 0.1$; (b) $C_w = 0.2$; (c) $C_w = 0.3$
The positive and negative sign of dimensionless stream function denotes anti-clockwise circulation and the clockwise circulation, respectively. The shape of streamlines and isotherms are symmetric with respect to the middle line of the geometry. The fluid gets heated at the hot base of the channel by absorbing thermal energy and moves upwards because of density variation between hot and cold surfaces. The fluid perforates much deeper into the upper section of channel legs due to buoyant force created by hot base and upper cold walls of the channels. The streamlines show that the energy level of the fluid has increased to sustain more circulation in channel legs when channel width, $C_w$ is increased from 0.1 through 0.3 for three channel domain and when channel width, $C_w$ is increased from 0.25 through 0.45 for two channel domain. A careful examination of the isotherms of Figures 2 and 3 reveals that the heat transfer rate continuously varies along the base of porous domain vindicated by distortion of isotherms. It is interesting to note that the fluid flows in two distinct directions at central channel for the case of three channels. It is noted that fluid at the central channel flows to relatively lesser height as compared to the other two channels. This happens because of the reason that smaller channel width leads to lesser thermal energy at the central channel as reflected by isotherms [Figure 2(a)]. However, increase in channel width allows better thermal energy distribution in central channel allowing the fluid to occupy a larger channel area [Figure 2(c)]. The fluid is found to circulate in four types of multiple cells in three channels. Similarly, fluid flows in four cells when $C_w$ is relatively small for the case of two channels [Figures 3(a) and (b)]. However, the fluid cells at the base of domain get merged with that of the fluid flowing inside the channel when $C_w$ is increased to 0.45 [Figure 3(c)]. The magnitude of stream function increases with increase in the channel width indicating that the fluid velocity is directly proportional to channel width.

Figure 4 demonstrates the influence of base height for three channel domain, on isotherms and streamlines at Rayleigh number $Ra = 500$ with constant channel width $C_w = 0.2$. The inference of isotherms indicates that the thermal gradient varies along the width of cavity for smaller values of $B_w$. However, this variation seizes when the size of base height ($B_w$) increases, leading to almost uniform thermal gradient along the base of domain. The implication of increased $B_w$ is further noted in terms of no fluid flow in middle channel, which, in turn, forces temperature to the lowest level in that region. The fluid is found to have four circulation patterns at smaller base height, but it turns into two flow patterns at increased $B_w$. Similar flow pattern and isothermal trend is seen when number of channels are increased to five (Figure 5). It could be argued that the increased base height increases the thermal resistance in vertical direction. On top of that, the larger porous region resulting from increased $B_w$ allows the two fluid cells to merge into one that primarily flows from bottom of domain towards the left or right channel leg. This, in turn, reduces the temperature variations at the interior channels.

Figure 6 shows the influence of number of channels with constant base height ($B_w = 0.25$), channel width ($C_w = 0.1$) and Rayleigh number ($Ra = 500$) on isotherms and streamlines with two, three and five channels. Size of the fluid circulation zone visible in streamlines plot is slightly augmented with increase in number of channels by which enhancement in heat transfer is perceived. The isotherm penetrates into the middle channel as seen in Figures 6(b) and (c). The distribution of isotherms is better in the central channel of cavity as compared to other interior channels. The clubbing of isotherms at middle channel indicates that heat dissipation should be higher in that channel in comparison to other interior channels. Thus, from design perspective, the middle channel is important factor and the length of other interior channels can be reduced, as they do not seem to have higher impact on heat transfer rate. A close look at the mid of the cavity base ($x = 0.5$) reveals that the isotherm moves away from the base as number of channels are increased, thus, decreasing the thermal gradient in that area.
Figure 3.
Influence of channel width for two channels on (I) isotherms (II) streamlines

Notes: (a) $C_w = 0.25$; (b) $C_w = 0.3$; (c) $C_w = 0.45$
Figure 4. Influence of base height for three channels, \( Ra = 500 \) and \( C_w = 0.2 \) on (I) isotherms (II) streamlines

Notes: (a) \( B_w = 0.25 \); (b) \( B_w = 0.5 \); (c) \( B_w = 0.75 \)
Figure 5. Influence of base height for five channels, $Ra = 500$ and $C_w = 0.1$ on (I) isotherms (II) streamlines

Notes: (a) $B_w = 0.25$; (b) $B_w = 0.5$; (c) $B_w = 0.75$
Figure 6.
Influence of number of channels at $B_{w} = 0.25$, $C_{w} = 0.1$ and $Ra = 500$ on (I) isotherms (II) streamlines

Notes: (a) $N_c = 2$; (b) $N_c = 3$; (c) $N_c = 5$
Figure 7.
Influence of radiation parameter on three channels at $B_w = 0.25$, $C_w = 0.2$ and $Ra = 500$ on (I) isotherms (II) streamlines

Notes: (a) $Rd = 0$; (b) $Rd = 0.1$; (c) $Rd = 1$
Figure 8. Influence of Rayleigh number with two channels at $B_w = 0.25$, $C_w = 0.2$ and $Re = 0$ on (I) isotherms (II) streamlines

Notes: (a) $Ra = 10$; (b) $Ra = 100$; (c) $Ra = 1,000$
The effect of physical parameters such as $Rd$ and $Ra$ are demonstrated in Figures 7-8. The constant parameters taken in Figure 7 are $B_w = 0.25$, $C_w = 0.2$, $N_c = 3$ and $Ra = 500$. The fluid circulation area reduces though the maximum value of steam function is same ($\theta_f = 7$) with increase in radiation parameter from 0 to 1. This shows that the fluid moves at a faster rate that helps in dissipating the thermal energy at increased rate. It is known that the radiation parameter increases the velocity inside the boundary. The length of channels especially the middle channels can be reduced for the cases where radiation is present because of the fact that the fluid flow confines to smaller area and temperature at upper side of channels is lowest. The influence of increased Rayleigh number on isotherms and stream lines with two channel domain is shown in Figure 8. The case considered has constant base height ($B_w = 0.25$) and channel width ($C_w = 0.2$).
The radiation parameter, $Rd$ is assumed zero. The isotherms and stream lines are consecutively plotted for $Ra = 10$, $Ra = 100$ and $Ra = 1,000$. As it is known, the increase in Rayleigh number consequences an increase in fluid velocity. The fluid carries enough energy to sustain the circulation in channel legs when Rayleigh number is increased from 10 to 1,000. The higher velocity caused by increased Rayleigh number splits the fluid into four circulation pattern [Figure 8(c)] as compared to two patterns at low Rayleigh number. The higher fluid velocity enhances the convective heat transfer as obvious from twisted isotherms of Figure 8(c).

Figures 9 and 10 shows the variation of local Nusselt number along the length of the domain for the various channel widths in three and two channel domains, respectively, at
$B_w = 0.25, Ra = 500, Rd = 0$. It is noted that in three channel domain, the Nusselt number oscillates in a wavy form for all the channel width studied, with minimum $Nu$ at left surface, middle and right surface of domain. The maximum Nusselt number is observed at mid of two channels. Smaller the channel width $C_w$, lower the Nusselt number at $x = 0.5$. This is because the isotherms tend to move away from hot surface at smaller values of $C_w$ (Figure 2), creating lower thermal gradient in that region. The Nusselt number is also found to be of wavy form for the case of two channels. However, unlike three channel case, the two channels produced almost same minimum $Nu$ for all studied values of $C_w$. The two channel case had an exception of wavy behaviour where $C_w = 0.45$ (Figure 3(c)) as compared to four patterns of all other values of $C_w$ (Figures 3(a) and (b)).
Figure 11 shows the effect of $B_w$ for three channels at $C_w = 0.2$, $Ra = 500$ and $Rd = 0$. It is noted that the Nusselt number is higher for the case of lower base height ($B_w$) and it decreases with an increase in base height. Nusselt number for $B_w = 0.25$ exhibits some form of wavy fluctuations due to multiple fluid circulation patterns [Figure 4(a)] but $Nu$ for $B_w = 0.5$ and $0.75$ remains almost constant along the width of domain. This is because of the reason that the increased base height leads to increase in thermal resistance, which reduces the heat transfer from bottom surface to interior of porous medium. However, this can be counterbalanced by increasing the number channels ($Nc = 5$) and reducing the width of channels ($C_w = 0.1$) as shown in Figure 12. It is found that increasing number of channels at $B_w = 0.25$ and $C_w = 0.1$ produces almost similar heat transfer rate (Figure 13).

The effect of radiation parameter and Rayleigh number are illustrated in Figures 14-15. Figure 14 is obtained at $Ra = 500$, $B_w = 0.25$, $C_w = 0.2$ and $Nc = 3$, whereas Figure 15 corresponds $Rd = 0$, $B_w = 0.25$, $C_w = 0.2$ and $Nc = 3$. The Nusselt number increase with in radiation parameter, which is in line with other studies being reported with respect to radiation (Badruddin et al., 2007; Raptis, 1998). It must be noted that the thermal gradient decreases along the hot surface when radiation parameter is increased (Figure 7). However, the overall heat transfer increases because of combined effect of conduction and radiation as reflected by a factor $4Rd/3$ in the Nusselt number equation. Effect of Rayleigh number is depicted in Figure 15. Expectedly, increased Rayleigh number increases the Nusselt number because of higher fluid movement. The heat transfer is found to be almost constant at lower Rayleigh number.

4. Conclusion
The present study is carried out to investigate the heat transfer analysis in a porous channel with bottom of porous domain being heated isothermally to temperature $T_h$, and its inner channel walls are exposed to cool temperature $T_c$. FEM is used to solve the governing equations. Influence of various parameters on isotherms and streamlines along with its effect on Nusselt number is studied. The various parameters include channel width, base height, number of channels, radiation parameter and Rayleigh number. The obtained results show that the increase of the base height reduces natural
convection from the bottom to the top. Smaller base height has a higher heat transfer capability. It is also observed that the Nusselt number is generally higher around channel legs. Similar to the other natural convection heat transfer problems, the increase of the Rayleigh number enhances the strength of the heat and flow, consequently Nusselt number increases.

References


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