An empirical test of modal choice and allocative efficiency: Evidence from US coal transportation

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A B S T R A C T
A generalized shipper transportation cost function is estimated to test whether coal shippers achieve allocative efficiency with respect to market prices when facing limited access to the full range of transportation services. Findings indicate that allocative efficiency with respect to market prices is achieved when shippers have access to all major transportation modes. In contrast, the condition for allocative efficiency is not met with respect to market prices when shippers’ modal choices are limited to trucking and rail services. Findings for the sample of shippers who face limited shipping choices is interpreted as suggesting an over-use of trucks relative to the use of trains due to price distortions of transportation services.

1. Introduction

Pro-competitive policies enacted over three decades ago in the transportation industry were passed in part with the intent of creating a more efficient transportation system. Past research presents compelling evidence of non-trivial productivity gains and cost reductions in rail and trucking transportation services.¹ Shippers have benefited from cost savings associated with enhanced transportation efficiency. For instance, Winston (1998) shows average shipping rates per ton-mile fell from 35% to 75% in trucking and by more than 50% for rail transportation services two decades after the implementation of pro-competitive policies. Coal shippers are a prime example of transportation services users who have benefited from enhanced efficiency and decreasing transportation operating cost. Real average coal transportation rates from 1979 to 1999 fell 29.48% and 33.33%, respectively for rail and trucking services (EIA, 2004).² While extensive research has examined efficiency gains in the business operations of different sectors of the transportation services industry much less attention has focused on whether

¹ Permanent address: Department of Economics, Faculty of Economics and Administration, University of Malaya, 50603 Kuala Lumpur, Malaysia.
² See Winston (1998) for a review of some of the more recent studies on pro-competitive policies and transportation efficiency. He reports average real operating cost reductions from 35% to 75% in the trucking industry due partly to the reduction of empty loads per mile. He also reports a 60% reduction in operating cost for rail carriers due partly to the abandonment of one-third of their track miles since the enactment of pro-competitive policies in the mid-1970s and early 1980s. For the most part, research on post-deregulation rates and productivity growth in the trucking and rail transportation sectors are consistent with Winston’s findings. For instance, Moore (1986) and Ying and Keeler (1991) find a non-trivial reduction of shipping rates and Ying (1990) finds an appreciable post-deregulation increase in trucking transportation productivity growth. Research on rail rates and performance indicates post-deregulation rate declines and productivity growth increases as Barnekov and Kleit (1990) and MacDonald and Cavalluzzo (1992) report rate declines ranging from 16.5% to 24.3%. Bittan and Keeler (2003), Oum et al. (1999), Bereskin (2001), and Wilson (1997) reveal a substantial reduction of rail rates following deregulation.
³ Barge services present another major source of transportation for coal. Their services, however, were never substantially regulated and only provided a 6.23% rate decline for the transportation of coal for the 1979–2001 period of observation.
shippers consider market prices as an appropriate rate to use to attain an allocatively efficient mix of transportation services following the enactment of pro-competitive policies in the transportation services industry. The significance of presenting such an investigation is highlighted by noting that past research reveals that pricing behavior by carriers contributed to a misallocation of transportation services prior to the policy shift promoting inter- and intra-modal competition (Friedlaender, 1969; Martin, 1979).

This study addresses the dearth of research on shipping allocative efficiency during the current competitive business environment in the transportation industry by examining the use of different transportation modes for the delivery of coal in the US. Examining coal shippers’ choice of transportation services contributes to our understanding of the costs associated with providing an energy resource that is critical to the operations of US industries. Evidence of this commodity’s economic significance is depicted by the large proportion of coal production used to generate the electricity that powers machinery and equipment of many US industries. This study estimates the coal industry's transportation cost function to test whether shippers consider market prices as viable indicators of transportation costs when making decision on the efficient combination of rail, trucking, and barge services needed to transport coal. Special consideration is given to the possibility that infrastructure and capacity constraints create a business environment that allows carriers to set prices that do not account for the high opportunity cost associated with relatively poor service quality.

2. Accessibility, capacity constraints, and the choice of transportation modes

Bulk commodity shippers rely heavily on the transportation services of rail, barge, and trucking. Differences in economies of shipment size and economies of distance influence shippers’ choice of transportation mode for hauling bulk commodities such as coal. Rail and barge carriers are able to take advantage of economies of shipment size, in part because of the large hauling capacity of these modes of transportation and the high fixed costs required to purchase large shipping vehicles for these types of operations. In addition, rail and barge carriers are able to lower per unit costs of hauling large loads because they are able to haul larger volumes of freight without proportionately increasing crew sizes. In contrast, trucking carriers have an inherent cost advantage hauling relatively small shipments over short distances due in part to lower fixed costs compared to rail and barge. Economies of shipment size and distance are more difficult to attain for trucking transportation because of the need to employ at least a proportional number of drivers and motor vehicles for 80,000 lb increases in freight. Given these attributes rail and barge carriers are generally better suited for providing service for long-haul shipments, whereas trucking contracts are generally better suited for providing short-haul feeder service.

In addition to the attributes inherent to different transportation modes, carrier pricing behavior also influences shippers' use of rail, barge, and trucking services. For instance, past research argues that prior to the enactment of pro-competitive policies rail prices did not reflect the full cost of transportation when rail carriers were the sole long-distance service provider (Friedlaender, 1969; Harbeson, 1969; Moore, 1975; Peck, 1965; Martin, 1979). These authors report that relatively high rail prices did not discount the cost of low quality service. Research on relative service quality across transportation modes suggests that compared to rail, trucking provides faster and more reliable transit times (Winston, 2005). Rail pricing that did not take into account inter-modal differences in service quality contributed to a less desirable allocation of transportation resources by bulk shippers because shippers based modal choices on the opportunity cost of the service rather than on the market price. Hence, while the attributes of rail service suggest that it is better suited for hauling bulk commodities over long distances, shippers have an incentive to consider trucks if rail prices do not accurately depict the value of the transportation service. Friedlaender reports that the primary source of the less desirable allocation of transportation services was the shifting of bulk traffic away from rail to trucks.

Pro-competitive policies targeting freight transportation in the late 1970s and early 1980s had the potential to change shippers’ usage of transportation services by promoting rate declines for such services. Coal shipping is a prime example of the influence of pro-competitive transportation policies on freight pricing and transportation resource allocation. Shipping rates for coal hauled by rail fell in real 2000 dollars from $14.45 to $10.19 per ton from 1979 to 1999. Rates fell slightly more for truck hauling services as the real price declined from $9.00 to $6.00 per ton for the same period. Much smaller rate

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4 Eighty-eight percent of coal production in 1998 was used to produce electricity as compared to only 42% in 1960s (EIA, 2000; Elmes and Harris, 1996).
5 The utility industry's total transportation costs are examined in this study to account for the shipping rate in addition to service-induced costs.
6 Coal shippers rely much less on the transportation services of collier, pipeline, and conveyor.
7 The purchase of track and locomotives depicts substantial fixed cost for rail carriers. The purchase cost of barges contributes to the significant fixed cost for this form of water transportation.
8 Resor et al. (2004) observe that relatively high terminal and drayage costs contribute to rail's disadvantage relative to trucking service for short hauls. These authors also find that rail has a competitive advantage at distances of greater than 500 miles.
9 The comparatively limited capacity of trucks generally restricts the advantageous utilization of trucks to short-haul deliveries of bulk commodities of typically less than 50 miles (Andersen, 1981). Most states adhere to the 80,000 lb weight limit for trucks. However, tractors with two or more trailers having gross vehicle weight of more than 80,000 lb are limited to certain highways in 21, mostly western states (Association of American Railroads, 2008).
10 These authors observe that rail carriers' practice of setting rates based in part on "value-of-service" amounted to rates that were set at levels which reflect conditions of demand for services rather than for the cost of providing these services.
11 Winston (2005) also reports that following deregulation, rail greatly improved its service, partly through end-to-end mergers. In a separate study he estimates that the annual benefits to shippers due to service quality improvements amount to $5 billion annually in 1990 dollars (Winston et al., 1990).
12 Domestic coal shipping rates are taken from the Energy Information Administration's Coal Transportation Rate Database, 2004.
of non-price cost of rail services is reported by Miklius and Casavant (1975) and Oum (1979) who find that shippers constantly shrink their demand (See Subcommittee on Railroads: Hearing on National Rail Infrastructure Financing Proposals, 2003).

Given the hauling attributes of rail carriers, engaging in such non-competitive pricing contributes to a less desirable structure for rail consists of two dominant coal carriers west of the Mississippi and two different dominant carriers east of the Mississippi. It would seem that barge carriers present a natural alternative to contracting rail carriers servicing non-competitive rail routes. However, bulk commodity shippers’ modal choices are actually further limited by infrastructure constraints in the barge transportation sector. Water transportation services are confined to a network of only 12,000 miles of navigable rivers and lakes making this system the least accessible of the three dominant transportation services for coal (Vachal et al., 2005). The market for shipping by barge is concentrated in the Eastern US, with services provided primarily along the Mississippi, Ohio, Tennessee, Illinois, and Monongahela Rivers. The Ohio and Monongahela are the major shipping corridors used to transport coal as 57% of coal is transported along the Ohio River and coal accounts for 87% of all freight hauled along the Monongahela (Vachal et al., 2005). Even if coal shippers are located near inland waterways they still potentially face accessibility limitations due to seasonal freezing and changes in the depth of these rivers (Medine, 2004).

The lack of accessible long-distance alternatives and the small number of Class-1 rail carriers have the potential to enhance the pricing power of these carriers by creating a business environment that limits intra-modal competition. Monopolistic rail carriers then would have the latitude to engage in the pre-deregulation pricing practice of charging high rates that do not reflect the quality of service. Indeed, during congressional testimony legal representatives of shippers claim that rail customers who lack access to competitive rail transportation often face high rates and inadequate service (Testimony, 2007, 2004). Given the hauling attributes of rail carriers, engaging in such non-competitive pricing contributes to a less desirable allocation of transportation services if the opportunity cost of using rail service differs from the price charged by rail carriers. An allocation of transportation services that is inconsistent with carrier’s hauling attributes arises because cost-minimizing shippers equate marginal productivities for each mode with prices that account for unobserved costs associated with service quality. In contrast, standard allocative efficiency is depicted by equating the marginal productivities with the listed price of transportation services.

Shippers’ modal choices are further influenced by the limited capacity of rail transportation services, given increasing demand for long-haul shipment of bulk commodities. Claims of capacity constraint were presented in testimony to the subcommittee on railroad shippers, as shippers argued that following pro-competitive policies railroad carriers had begun to approach the limits of cargo capacity (Testimony, 2003). Additional concerns over rail capacity is expressed at the 1998 National Agricultural Transportation Summit where the USDA indicates that for the first time in a century there is too little rail capacity available to satisfy the demand of shippers. Capacity constraints in the rail freight transportation sector increases the likelihood that shippers rely more heavily on trucking transportation services, even though trucks are an undesirable choice compared to rail for long-haul shipments of bulk commodities.

Despite shipping challenges posed by infrastructure and capacity constraints coal shippers could still be using allocation of transportation services that is consistent with carrier’s hauling attributes. The almost exclusive use of unit trains to haul coal allows shippers to benefit from economies of shipment size associated with the high capacity volume of these containers (Morrison, 1985). Rail carriers have also expanded the US rail network by increasing their investment in infrastructure. Hence, shippers who are restricted to using rail may not face a significant disadvantage compared to shippers with access to rail and barge if high rail costs are accompanied with superior transportation services. Evidence suggesting inaccurate assessment of non-price cost of rail services is reported by Miklius and Casavant (1975) and Oum (1979) who find that shippers constantly underestimate the quality of rail services.

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13 The Union Pacific and the Burlington Northern-Santa Fe are the dominant carriers west of the Mississippi and CSX and the Norfolk Southern are the dominant carriers east of the Mississippi.
14 Research by the Government Accountability Office (GAO) supports a lack of intra-modal competition in the rail freight industry (GAO, 1994). Shippers' complaints with respect to rail rates and poor service are also reported in the popular press (Davidson, USA Today, 2007) as well as by the Energy Information Administration (EIA, 2004).
15 There is hardly consensus regarding whether rail carriers will invest enough into the maintenance and expansion of the rail system to keep up with growing shippers’ demand (See Subcommittee on Railroads: Hearing on National Rail Infrastructure Financing Proposals, 2003).
16 The Joint Line rail system used to haul coal in the Powder River Basin is an excellent example of the gains from using unit trains and from heavy investment in the capacity expansion of that rail system (EIA, 2007). Even though 95% of coal mined from the Powder River Basin is hauled by freight the shipping rate per ton-mile is less than that charged for coal shipped from the multi-modally served Central Appalachia region for medium–distanced routes (GAO, 2002, p. 13). The EIA reports that railroads serving the Powder River Basin, took advantage of inherent economies of shipment size. Rail rates from the Powder River Basin could be held down, on a cost per ton-mile basis, because the flat terrain and space for loading facilities allow efficiencies throughout the haul. This terrain allows Union Pacific and Burlington Northern-Santa Fe, the two rail carriers in this region, to construct unit trains that are some of the longest on the rail system and comprise some of the highest-capacity bulk railcars in the United States (EIA, 2000, p. 21).
Conflicting research observations on rail rates and opportunity cost associated with service quality indicates the need for empirical analysis that examines whether coal shippers use a desirable mix of transportation services that is consistent with carriers’ hauling attributes.

3. Estimation approach

The same cost estimation approach proposed by Atkinson and Halvorsen (1984) to examine factor input allocation by firms is used in this study to examine whether coal shippers choose an allocatively efficient mix of modal transportation services. As opposed to the neoclassical cost approach, which assumes that firms minimize costs subject to their output constraint, this study uses a generalized cost function that includes shadow input prices as cost determinants. An advantage of using this approach is it allows for testing whether shippers’ modal choice decisions differ from cost minimization choices based on shipping prices charged by carriers (Atkinson and Halvorsen, 1984), and thus makes cost minimization a testable hypothesis rather than an assumption (Eakin and Kniesner, 1988).

Standard neoclassical production theory suggests that coal shippers minimize cost by satisfying the condition depicted by

\[ MP_i/MP_j = P_i/P_j \]

where \( MP_i \) is marginal product of the \( i \)th transportation service and the variable \( P_i \) is the actual price paid by shippers for \( i \)th transportation service. However, this variable may not completely account for the opportunity cost of transportation services to shippers if carriers do not adjust prices to compensate for service quality. A more accurate depiction of shippers’ modal choice behavior is depicted by Eq. (2), which assumes that shippers’ cost-minimizing decisions are based on shadow prices rather than actual prices.\(^{17}\)

\[ MP_i/MP_j = P_i^s/P_j^s \]

The multiplicative form of shadow rates of transportation modes denoted by \( P_i^s = g_iP_i \) is used to depict the relationship between actual and shadow shipping prices (Atkinson and Halvorsen, 1984).\(^{18}\) The variables \( P_i^s \) and \( P_i \) are respectively the shadow and actual shipping rate for the \( i \)th transportation service. The variable \( g_i \) is the factor of proportionality or price efficiency parameter, which measures the shadow price’s divergence from the actual shipping rate. The interpretation of \( g_i \) is generally viewed as a measure of allocative inefficiencies (Oum and Zhang, 1995). According to Eakin and Kniesner (1988) and Balint and Sauer (2006), shippers can be regarded as allocatively efficient with respect to observed market prices only if those prices reflect shippers’ opportunity cost with respect to transportation modes. Conditional allocative efficiency with respect to observed market prices is met if \( g_i = 1 \) and complete allocative efficiency is achieved if \( g_i = 1 \) for all \( i \) transportation services. Thus, if either condition is not met, in the case of coal shippers, the failure to allocate transportation services efficiently with respect to market prices could be attributable to the divergence between the actual rate paid by shippers and the opportunity cost of using a particular transportation mode.

The system of equations depicted by Eqs. (3) and (4) are used to derive the generalized transportation cost function for transportation services

\[ C^s = C^s(P_i^s, Y) \]

\[ M_i^s = P_i^sX_i/C^s \]

where \( C^s \) is coal shippers’ shadow shipping cost, \( M_i^s \) is the shadow shipping cost-share equation for \( i \)th mode, and \( Y \) is total output for coal shippers. These costs are purely variable costs associated with coal transportation. Given that electric utility companies are the shippers examined in this study it is assumed that their costs of transporting coal are separable from electricity generating activities and from other utility activities. The separability assumption allows coal transportation costs to be examined independently from other combination of inputs employed by utilities (Buckley and Westbrook, 1991).

Applying Shephard’s Lemma to the shadow cost function yields the actual demand for the \( i \)th transportation service mode depicted by

\[ \partial C^s/\partial g_i P_i = X_i \]

From Eq. (4), the demand for the \( i \)th transportation service mode can also be expressed as

\[ X_i = C^sM_i^s/g_i P_i \]

Therefore, shippers’ total actual cost function is as follows:

\[ C^a = \sum_i P_i X_i = \sum_i P_i \cdot C^sM_i^s/g_i P_i \]

\[ C^a = C^s \sum_i M_i^s/g_i \]

\(^{17}\) Making such an assumption is consistent with the notion that rail shippers’ modal choices are influenced by the opportunity cost of transportation rather than the prices charged by rail carriers.

\(^{18}\) The expression for shadow prices is interpreted as the first-order Taylor’s expansion of an arbitrary shadow price function, \( g_iP_i \) with properties \( g_i(0) = 0 \) and \( \partial g_i(P_i)/\partial_i > 0 \). An alternative approach used by Eakin and Kniesner (1988) is the additive form of factor of proportionality: \( P_i^s = P_i + g_i \).
or in logarithmic terms,
\[
\ln C^a = \ln C^S + \ln \sum_i M_i^f / g_i
\]

(8)

Eq. (8) indicates that the difference between the shadow cost function and the actual cost function is the shadow cost-share equations of all transportation modes divided by their respective factor of proportionality. Therefore the differences between actual and shadow costs come from distortions in shares of each mode weighted by respective factor of proportionality. The actual cost function equals the shadow function if \( g_i = g_j = 1 \) for inputs \( i \neq j \), which is also the condition for relative price efficiency. Thus the divergence between shadow and actual cost function is the cumulative impact of misallocation of transportation modes, which is captured by the additional term in Eq. (8).

To obtain the expression for the actual cost-share equations, Eqs. (6) and (7) are substituted into the actual cost-share equation to give,
\[
M_i^f = P_i X_i / C^a = \frac{P_i \left( C^S M_i^f / g_i \right)}{C^S \sum_i M_i^f / g_i} = \frac{M_i^f / g_i}{\sum_i M_i^f / g_i}
\]

(9)

Now the actual cost function can be derived by specifying the appropriate functional form for the shadow cost function. Following Atkinson and Halvorsen the translog cost function specified by Eq. (10) is used to approximate the shadow cost function.
\[
\ln C^S = \alpha_0 + \alpha_Y \ln Y + 1/2 \gamma_{yy} (\ln Y)^2 + \sum_i \gamma_{iy} \ln Y \ln (g_i P_i) + \sum_i \alpha_i \ln (g_i P_i) + 1/2 \sum_{ij} \gamma_{ij} \ln (g_i P_i) \ln (g_j P_j)
\]

(10)

The translog shadow cost function still has the standard properties of the translog cost function i.e. homogenous of degree one in input price, but in this case it is homogenous of degree one in effective shipping rates. Hence, a proportionate change in rates of all transportation modes used in transporting coal will change the shipper's total cost proportionately. The necessary and sufficient homogeneity and symmetry conditions to construct a cost function that is linearly homogenous in shipping rates are:
\[
\sum_i \alpha_i = 1 \sum_i \gamma_{iy} = 0
\]
\[
\sum_i \gamma_{ij} = \sum_i \sum_j \gamma_{ij} = 0
\]

The shadow transportation service cost-share equation for modal service \( i \), \( M_i^S \), is then derived by applying logarithmic differentiation on the shadow cost equation with respect to each shadow shipping rate where
\[
M_i^S = \alpha_i + \sum_j \gamma_{ij} \ln (g_j P_j) + \gamma_{iy} \ln Y
\]

(11)

Eqs. (10) and (11) are substituted into Eq. (8) to derive the actual total transportation cost function depicted by Eq. (12).
\[
\ln C^a = \alpha_0 + \alpha_Y \ln Y + 1/2 \gamma_{yy} (\ln Y)^2 + \sum_i \gamma_{iy} \ln Y \ln (g_i P_i) + \sum_i \alpha_i \ln (g_i P_i) + 1/2 \sum_{ij} \gamma_{ij} \ln (g_i P_i) \ln (g_j P_j)
\]
\[
+ \ln \left\{ \sum_i g_i^{-1} \left( \alpha_i + \sum_j \gamma_{ij} \ln (g_j P_j) + \gamma_{iy} \ln Y \right) \right\}
\]

(12)

Eq. (13) denotes the modal share equation for actual costs
\[
M_i^A = \left[ \alpha_i + \sum_j \gamma_{ij} \ln (g_j P_j) + \gamma_{iy} \ln Y \right] / g_i
\]
\[
\sum_i \left[ \alpha_i + \sum_j \gamma_{ij} \ln (g_j P_j) + \gamma_{iy} \ln Y \right] / g_i
\]

(13)

The parameters for factor of proportionality are normalized so that one of the factors serves as the numeraire and all other factors are measured relative to the benchmark factor of proportionality which is normalized to be equal to one. Thus, shippers use an efficient mix of transportation services with respect to observed market prices if the factors of proportionality are not statistically significantly different from one. Given that the benchmark factor of proportionality equals one, the actual cost function would then closely resemble the shadow cost function for allocative efficiency.
4. Data

Data reporting coal transportation information from electric utility contracts are taken from the 1979–2001 Coal Transportation Rate Data Base (CTRDB) to estimate the generalized transportation cost function for coal shippers.\(^{19}\) Files from this data source report information on coal transportation contracts for 153 utility shippers’ rates, tons shipped, distance shipped, originating and destination state, and shipping service for seven different transportation modes.\(^{20}\) The analysis for this study is limited to utilities reporting transportation cost and shipping information for coal hauled by rail, truck, or barge. These three transportation modes are chosen as the selection criteria because of the extremely small sample of utility contracts providing rate information on collier, pipeline, conveyer, and multi-mode services.\(^{21}\) An additional criterion imposes shipment along originating–destination (O–D) state pairs as the measurement observation level criterion rather than using utility level information, because for each observation at the utility level the CTRDB reports shipping rates for only one transportation mode. Hence, there is a complete absence of rate variation across observations at the utility level.\(^{22}\) Originating state–destination state pair information are constructed as mean weighted values using total tons of coal shipped as weights. These weighted data allow examining rate variations across modes because each observation includes rate information for different transportation modes used to ship freight along the same originating–destination route. Examination at the originating–destination pair observation level presents the additional benefit of clearly defining the market for transportation services as freight hauled along a given originating–destination pair. In contrast, utility level information combines shipments of coal on different routes that might not provide the same modal choices and thus may provide bias results on shippers’ allocation of transportation services. Indeed, Buckley and Westbrook (1991) reveal the significance of accurately defining the service market when examining shippers’ modal choice as they find that estimation results on modal choices using data aggregated at the utility level depict transportation modes as substitute services when they were actually operating in separate markets.

Two sample populations are constructed for the analysis. They include information that separately report shipments when there is accessibility to all three major transportation modes and shipments when there is accessibility to only rail and trucking transportation services. Analysis using these two sample populations provides the opportunity to test whether accessibility to transportation services influences shippers’ ability to satisfy the condition of allocative efficiency with respect to market prices. The sample population consists of 71 observations for eight originating state–destination state pairs when the selection criterion is restricted to routes using all three major transportation modes. The sample increases to 84 observations for twenty originating state–destination state pairs when shippers only have access to trucking and rail transit services. These relatively small sample sizes suggest some caution when interpreting cost results.

\(^{19}\) Information used to construct the Coal Transportation Rate Database is collected by the Federal Energy Regulatory Commission (FERC) as mandated by the Public Utility Regulatory Policy Act of 1978. This act requires responses from all jurisdictional utilities that operate at least one steam-electric generating station of 50 mW or greater capacity, or have an ownership interest in a jointly-owned steam-electric station of 50 mW or greater capacity, and that may claim changes to regulated power rates under FERC’s Fuel Use Adjustment Clause.

\(^{20}\) The seven transportation modes are rail, trucking, barge, collier, pipeline, conveyer, and multi-mode services.

\(^{21}\) Even if rate information were available these transportation modes provide a small share of coal transportation.

\(^{22}\) Cost estimations using utility level weighted means are not pursued in this study, in part because cost function estimation results do not satisfy the concavity condition for that set of information.

### Table 1

Descriptive statistics on coal transportation.

<table>
<thead>
<tr>
<th></th>
<th>1—Rail–truck–barge (n = 71)</th>
<th>2—Rail–truck (n = 84)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shipping rates ($ per ton-mile)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail rate</td>
<td>0.0387 (0.0184)</td>
<td>0.0567 (0.10680)</td>
</tr>
<tr>
<td>Truck rate</td>
<td>0.2791 (0.6486)</td>
<td>0.1041 (0.1179)</td>
</tr>
<tr>
<td>Barge rate</td>
<td>0.0230 (0.0367)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Shipping volume (annual tonnage)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>3876867.15 (3413938.4)</td>
<td>2388201 (2684188)</td>
</tr>
<tr>
<td>Truck</td>
<td>764968.44 (608558.57)</td>
<td>813959.5 (913962.2)</td>
</tr>
<tr>
<td>Barge</td>
<td>1612056.97 (1142077.33)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Shipping distance (miles)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail distance</td>
<td>161.47 (80.4775)</td>
<td>166.3533 (109.8347)</td>
</tr>
<tr>
<td>Truck distance</td>
<td>61.30 (88.3119)</td>
<td>79.6309 (76.4531)</td>
</tr>
<tr>
<td>Barge distance</td>
<td>181.15 (122.3173)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Modal share of shipping costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>0.6706 (0.6706)</td>
<td>0.6873 (0.25085)</td>
</tr>
<tr>
<td>Truck</td>
<td>0.1324 (0.1164)</td>
<td>0.3127 (0.25085)</td>
</tr>
<tr>
<td>Barge</td>
<td>0.1970 (0.2002)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Modal share of shipping volume</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>0.5442 (0.2170)</td>
<td>0.6520 (0.26099)</td>
</tr>
<tr>
<td>Trucking</td>
<td>0.1341 (0.0757)</td>
<td>0.3479 (0.26099)</td>
</tr>
<tr>
<td>Barge</td>
<td>0.3217 (0.2248)</td>
<td>–</td>
</tr>
<tr>
<td>Average distance</td>
<td>148.30 (78.760)</td>
<td>136.4543 (98.6239)</td>
</tr>
</tbody>
</table>
Descriptive statistics on coal transportation are presented in Table 1. Column-1 presents information for the sample of originating–destination state pairs with accessibility to the three major transportation modes. These findings suggest that for this sample population barge shipping services provide the least expensive transportation mode at 2.30 cents per ton-mile. Shipping coal by rail, however, is not appreciably more expensive as rail rates are 3.87 cents per ton-mile. In contrast, the per ton-mile rate charged by trucking carriers is more than ten times the rate charged by barges and seven times the rate charged by rail carriers. The relatively high rates charged by trucking carriers possibly influence demand for their services as the share of coal shipped by this mode of transportation is appreciably lower than the shares transported by rail and barge. For instance, for this study’s sample population rail and barge carriers ship 54.42% and 32.17% of the coal, respectively compared to only 13.41% for trucking carriers. The findings reported in column-1 also show that rail and barge carriers are used for much longer hauls compared to the hauling distance for truck carriers. Using rail and barge for longer distances allows shippers to take advantage of economies of distance associated with these transportation modes.

Column-2 presents information for the sample of originating–destination state pairs with accessibility to only rail and trucking services. Findings mirror the pattern presented in column-1 as results in column-2 reveal that compared to trucking services rail transportation remains cheaper, and rail carriers haul greater volume over longer distances. What is notable is the markedly lower mean shipping rate charged by trucking carriers for the sample population in column-2 compared to the results in column-1. This smaller trucking rate contributes to a substantially smaller mean rail–truckling rate presented in column-1. The relatively high rates charged by trucking carriers possibly influence demand for their services as the share of coal shipped by this mode of transportation is appreciably lower than the shares transported by rail and barge. For instance, for this study’s sample population rail and barge carriers ship 54.42% and 32.17% of the coal, respectively compared to only 13.41% for trucking carriers. The findings reported in column-1 also show that rail and barge carriers are used for much longer hauls compared to the hauling distance for truck carriers. Using rail and barge for longer distances allows shippers to take advantage of economies of distance associated with these transportation modes.

Even though descriptive statistics suggest that coal shippers use different transportation modes in a manner that lowers costs, a more direct test is needed to examine whether these shippers choose a cost-savings mix of transportation services that satisfy the condition of allocative efficiency with respect to observed market prices. Eq. (14), which is a modified version of the generalized cost function derived in the previous section is estimated to make such a test.23

\[
\text{C} = \alpha_0 + \alpha_1 \ln(TM) + \alpha_2 \text{time} + \alpha_3 \ln(Dist) + \beta_1 \ln(rail') + \beta_2 \ln(truck') + \beta_3 \ln(barge') + \theta_1 1/2 \ln(TM)^2 + \theta_2 1/2 \text{time}^2 + \theta_3 1/2 \ln(Dist)^2 + \gamma_1 1/2 \ln(rail')^2 + \gamma_2 1/2 \ln(truck')^2 + \gamma_3 1/2 \ln(barge')^2 + \theta_12 \ln(rail') \cdot \ln(time) + \theta_13 \ln(TM) \cdot \ln(Dist) + \theta_22 \ln(time) \cdot \ln(Dist) + \gamma_12 \ln(rail') \cdot \ln(truck') + \gamma_13 \ln(rail') \cdot \ln(barge') + \gamma_23 \ln(truck') \cdot \ln(barge') + \varphi_{11} \ln(rail') \cdot \ln(TM) + \varphi_{21} \ln(truck') \cdot \ln(TM) + \varphi_{31} \ln(barge') \cdot \ln(TM) + \varphi_{12} \ln(rail') \cdot \ln(time) + \varphi_{22} \ln(truck') \cdot \ln(time) + \varphi_{32} \ln(barge') \cdot \ln(time) + \varphi_{13} \ln(rail') \cdot \ln(Dist) + \varphi_{23} \ln(truck') \cdot \ln(Dist) + \varphi_{33} \ln(barge') \cdot \ln(Dist) + \ln \sum_{i=1}^3 MS_i + e_i
\]

where, the variable \(C\) is the total cost of transporting coal, which is derived by taking the sum of all modal transportation costs (Buckley and Westbrook, 1991). Input prices are denoted as \(rail', truck',\) and \(barge'.\) The variable \(TM\) denotes output measured in ton-miles and the variable \(Dist\) measures the average hauling distance. The variable \(time\) is a time-trend for the 1979–1999 sample population.24 Last \(\sum MS_i\) is the summation of the shadow cost-share for the \(i\)th transportation service divided by that service’s factor of proportionality.

All explanatory variables excluding the time-trend are divided by their sample mean. Making this adjustment allows for ease of interpretation as the estimated coefficients on the first-order terms of the cost equation not only depict cost elasticity but for the transportation shipping rate parameter these estimates also depict each mode’s share of total transportation costs when calculated at the mean (Bitzan, 1999). The own second-order terms in the cost equation depict the rate of change of total transportation costs associated with a change in the respective explanatory variable. These terms are also used to derive the partial elasticities of substitution for the different modes of transportation services. These elasticities provide valuable information for examining modal choices by coal shippers. Knowing whether modes are substitutes or complements helps in understanding whether shippers are more likely to view different modes of transportation as a choice of competing shipping services or as a set of services used together to deliver coal.25 The Allen–Uzawa (1962) elasticity of substitution is given by

\[
\sigma_{ij} = \left(\frac{\gamma_{ij}}{\alpha_j} \right) + 1 \quad \text{for} \quad i \neq j
\]

Elasticity of input demand with respect to prices is as follows:

\[
e_{ij} = \left(\frac{\gamma_{ij}}{\alpha_i} \right) + \alpha_i
\]

\[
e_{ai} = \left(\frac{\gamma_{ai}}{\alpha_i} \right) + \alpha_i - 1
\]

---

23 Cost-share equations for \(n - 1\) modes of transportation are also estimated. The system of total cost and cost-share equations is estimated using a seemingly unrelated regression (SURE) method.

24 CTRDB information for the 2000–2001 report too few observations on transportation rates and thus are excluded from the sample population.

25 Detailed algebraic derivation of the elasticities can be found in Appendix B.
where the subscripts \( i \) and \( j \) denotes the \( i \)th mode of transportation. The symbols \( x \) and \( y \) are, respectively, the parameter estimates for the first- and second-order terms of the generalized cost function.

Using the standard empirical approach to estimate the cost parameters might introduce bias and inconsistency if output \((TM)\) is determined endogenously. For instance, Berndt et al. (1991) argued that it is reasonable to ask whether output and prices should be treated as exogenous due to the rate-setting freedom carriers experience in the current pro-competitive business environment.\(^{26}\) Output endogeneity occurs because the cost of transporting coal influences shippers demand for coal. Theory suggests that variables affecting the demand for coal best serve as instruments for output since these output determinants are endogenously determined by profit-maximizing behavior of the electric utilities and do not influence the cost of transporting coal (Berndt et al., 1991).\(^{27}\) The list of instruments for the study’s output equation includes coal prices, natural gas prices, electricity sales, and per capita income in the destination states.\(^{28}\) A test of these variables’ validity as instruments supports their use for the two sample populations.\(^{29}\) A Durbin–Wu–Hausman test for the relevancy of output endogeneity, however, generates \( f \)-scores of 0.01 and 0.54 for the three-modal and two-modal sample populations, respectively. This lack of

\(^{26}\) Harris and Winston (1983), however, noted that due to the route-specific nature of the data, output could be considered as exogenous since output is predetermined at the destination point and the choice of transportation is made by utilities at the originating point. We use the endogeneity test to determine whether or not the output is endogenous.

\(^{27}\) Kim (1985) suggests an additional equation of revenue to be estimated simultaneously with the cost system of equations for multiple output firms to take care of endogeneity. Our model is a single output one so we use two-stage-least-squares estimation approach.

\(^{28}\) Data for coal prices, natural gas prices, and electricity sales are taken from EIA’s website. Data on income measures and population are taken from the Bureau of Economic Analysis (BEA)’s website. Prices, income measures, and transportation rates are deflated using GDP deflator with 1996 as the base year. The prices are measured in dollars per ton and the electricity sales are measured in trillion BTUs.

\(^{29}\) The \( f \)-scores derived from using the weak identification test for the 3 and 2 transportation modal samples are, respectively, distributed as \( F(4,46) = 4.29 \) and \( F(4,45) = 6.51 \). These scores exceed the critical value for these samples. Results from using the Sargan identification test also suggests that the instrumental variable specification used in this study statistically satisfies the over-identifiable condition as the chi-squared values are 2.7619 and 0.882 for the 3 and 2 transportation modal samples.
The finding on the parameter estimate measuring the volume of coal shipped (TM) suggests that total transportation costs decreases by 0.41% for each percentage increase in distance hauled. Last, the estimated cost functions are also concave for nearly all observations. The conditions for concavity for the translog specification are presented in Appendix A. The complete concavity results are available from the authors on request.

Findings for the estimated coefficients on the control variables for the sample population of shippers reporting access to only rail and truck services are reported in column-2. The findings for transportation services rates’ first-order terms indicate that rail accounts for a 0.7804 share of total transportation costs. The estimated coefficients total on the other first-order terms is only statistically significant for volume of coal shipped. The finding for this parameter estimate suggests that transportation costs increase by 0.9359% for each percentage increase in output. The estimated coefficient on the miles of freight hauled variable (Dist) suggests that total transportation costs decreases by 0.41% for each percentage increase in distance hauled. The negative value of this estimated coefficient suggests that total transportation costs change over time at a decreasing rate. The rate of decline results in total transportation cost actually declining after 1989.31 The estimated coefficients on the time-squared second-order term. The negative value of this estimated coefficient suggests that total transportation costs increase by 0.078% annually over the 21 observation years. The time-trend finding is better understood when considering the estimated coefficients on the time-trend parameter and its estimated coefficient provides the value of the parameter time when total transportation costs start to decline. That value is 10, and since the initial observation year for the sample population is1979, the predicted year for total transportation cost reduction is 1989.

### Table 2

<table>
<thead>
<tr>
<th>Demand elasticities</th>
<th>1–truck–barge (n = 71)</th>
<th>2–truck (n = 84)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Own-price</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truck</td>
<td>–0.3998 (0.1123)</td>
<td>–0.2482 (0.1255)</td>
</tr>
<tr>
<td>Barge</td>
<td>–1.1697 (0.6122)</td>
<td>–0.4672 (0.4698)</td>
</tr>
<tr>
<td>Cross-price</td>
<td>–0.9423 (0.0732)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Rail–truck</strong></td>
<td>0.1645 (0.0681)</td>
<td>0.2482 (0.1255)</td>
</tr>
<tr>
<td><strong>Truck–rail</strong></td>
<td>0.9679 (0.6173)</td>
<td>0.4672 (0.4698)</td>
</tr>
<tr>
<td><strong>Rail–barge</strong></td>
<td>0.2354 (0.0486)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Barge–rail</strong></td>
<td>0.8131 (0.1276)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Truck–barge</strong></td>
<td>0.2019 (0.0411)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Barge–truck</strong></td>
<td>0.1292 (0.0617)</td>
<td>–</td>
</tr>
<tr>
<td><strong>Allen–Uzawa partial elasticity of substitutions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail–truck</td>
<td>1.0051 (0.0034)</td>
<td>0.7153 (0.5065)</td>
</tr>
<tr>
<td>Rail–barge</td>
<td>1.0079 (0.0032)</td>
<td>–</td>
</tr>
<tr>
<td>Truck–barge</td>
<td>1.0009 (0.0015)</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 3

Mean own and cross-price elasticities of modal transportation demand, and their mean elasticities of substitutions.

<table>
<thead>
<tr>
<th><strong>Demand elasticities</strong></th>
<th>1–rail–truck–barge (n = 71)</th>
<th>2–rail–truck (n = 84)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Own-price</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail</td>
<td>–0.3998 (0.1123)</td>
<td>–0.2482 (0.1255)</td>
</tr>
<tr>
<td>Truck</td>
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</tr>
<tr>
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<tr>
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<td>–</td>
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</tr>
<tr>
<td>Truck–barge</td>
<td>1.0009 (0.0015)</td>
<td>–</td>
</tr>
</tbody>
</table>

5. Findings

Table 2 reports coal shipping cost results when separately estimating the actual cost function for the study’s two sample populations. Overall, the model explains almost perfectly the variation in the cost of transporting coal as the value of $R^2$ is 0.93 and 0.97, respectively, for the sample that reports transportation access to all three major modes and the sample that reports access to only rail and truck services. Despite the relatively small sample population size, a large percentage of the parameter estimates is statistically significant. Generally the properties needed to satisfy the condition for a cost function are met. The estimated cost functions increase with increasing rates for modal transportation services and monotonicity is satisfied for modal transportation rates. Last, the estimated cost functions are also concave for nearly all observations. Before examining what the cost estimation results suggest about coal shippers’ use of transportation services an analysis of the estimated coefficients on the control variables excluding the factors of proportionality is presented. Column-1 presents information for the sample of originating–destination state pairs with accessibility to the three major transportation modes. The findings on the estimated coefficients transportation services rates’ first-order terms presented in column-1 indicate that rail accounts for a 0.668 share of total transportation cost. Rail’s cost share is appreciably higher than the shares for barge and trucking, as these transportation services account for 0.220 and 0.111 of coal shippers’ total transportation costs, respectively when shippers have access to all three major transportation services. The estimated coefficients on the other first-order terms are all statistically significant. The finding on the parameter estimate measuring the volume of coal shipped (TM) suggests that total transportation costs increases by 1.292% for each percentage increase in output. The estimated coefficient on the miles of freight hauled variable (Dist) suggests that total transportation costs decreases by 0.41% for each percentage increase in distance hauled. Last, the results on the time-trend parameter suggest that total transportation costs increase by 0.078% annually over the 21 observation years. The time-trend finding is better understood when considering the estimated coefficients on the time-squared second-order term. The negative value of this estimated coefficient suggests that total transportation costs change over time at a decreasing rate. The rate of decline results in total transportation cost actually declining after 1989.31 The estimated coefficients on the remaining second-order terms are only statistically significant for the interaction of modal transportation rates and the volume of coal shipped (TM). The findings for these estimated coefficients suggest a statistically significant increase in rail’s cost share with increasing amounts of coal shipped, which matches trucking and barge’s cost share reduction.
second-order terms are statistically significant for the interaction of transportation rates and volume of coal shipped. This result suggests that rail’s cost share increases with increasing amounts of coal shipped.

The remaining set of transportation modal-share interaction terms is used to compute the demand elasticities and partial elasticities of substitution for rail, barge, and trucking transportation services. A summary of these elasticities is presented in Table 3. The contents of column-1 report elasticity findings for originating–destination pairs that allow shipping by rail, truck, or barge. These findings suggest that coal shippers are relatively less sensitive to changes in rail-shipping rates compared to rate changes for trucking and barge as the own-demand elasticity is lowest for rail service. Cross-demand elasticity findings suggesting that rail transportation service is a substitute for barge and truck transportation services provide further evidence of rail’s ability to lower coal shippers’ demand for other transportation services by lowering rail rates. In contrast, the cross elasticity findings for barge and trucking services do not indicate that their prices substantially influence cross demand between these two services. The findings on the elasticities of substitution indicate that all three modes of transportation are substitutes for each other.33 The contents in column-2 of Table 3 report elasticity findings for originating–destination pairs that only report shippers’ access to rail and truck transportation services. These findings closely resemble the findings reported in column-1. For instance, the demand elasticity for rail is measurably less elastic than the demand elasticity for trucking. Cross-demand elasticity findings suggest that changes in rail prices have an appreciably larger influence on the demand for trucking services compared to the influence of trucking prices on the demand for rail services. The finding on the elasticity of substitution for rail and trucking services is nearly one.

The final entries in Table 2 report the estimated factors of proportionality. These parameters are standardized by using rail–shipping rate as the benchmark comparison factor input price. Hence, the factor of proportionality parameter $g_1$ equals one. The contents of column-1 indicate that the factor of proportionality for barge $g_2$ and trucking $g_3$ are higher than the benchmark factor of proportionality with values equaling 1.659 and 1.028, respectively. The log-likelihood test does not reject the hypothesis that the factors of proportionality are equal to one at the 1% significance level. The individual $t$-test also failed to reject the hypothesis that the factor of proportionality equals one, suggesting that the three modes of transportation services examined in this study are allocated in an efficient manner with respect to observed market prices when shippers have access to each mode.34 The contents of column-2 indicate that the factor of proportionality for trucking $g_3$ is lower than the benchmark factor of proportionality with a value equaling 0.3117 when shippers have access to only rail and trucking transportation services. Inference testing rejects the hypothesis that this factor of proportionality is statistically significantly equal to one at the 1% significance level as the $t$-test for $g_3 = 1$ is $-6.828$. The low value for trucking’s factor of proportionality suggests price distortion is associated with an over-use of trucking services relative to rail when modal choices are limited to these two transportation services.

6. Conclusion

Past research shows that prior to pro-competitive policies enacted in the late 1970s and early 1980s bulk shippers frequently misallocated transportation resources in part because rail rates did not accurately account for the full cost of transportation, especially on routes that lacked competition (Friedlaender, 1969; Martin, 1979). This study contributes to the analysis on shippers’ use of transportation services by examining modal choices of coal shippers following the enactment of pro-competitive transportation policies. A generalized transportation cost function is estimated to examine whether coal shippers use an allocatively efficient mix of transportation services. Separate sample populations that include coal transportation information from shippers who have access to all three major transportation modes and information from shippers who have access to only rail and trucking transportation services are used to estimate cost functions. Comparing transportation findings for these two samples is significant because it allows for testing whether limited modal choices influence shippers’ ability to use an efficient mix of transportation services with respect to observed market prices. Examining transportation costs following deregulation contributes to our understanding whether enhanced competition created an environment that promotes a use of freight hauling services that is consistent with the hauling attributes of different modes of transportation.

Findings on transportation modes’ elasticities of substitution suggest that rail, trucking, and barge services present competing choices for coal shippers. Hence, it is appropriate to assume that choosing among different modes of transportation is a relevant decision for coal shippers. Findings on factors of proportionality for modal shipping rates show that when access to the three major modes of coal transportation are available, coal shippers use an efficient mix of these services with respect to observed market prices. Findings on factor proportionality for modal shipping rates show that when shippers’ access to transportation services is limited to rail and truck delivery their modal choices do not satisfy the condition of allocative efficiency with respect to observed market prices. Rather, coal shippers use an undesirable high level of trucking services relative to rail services, given rails attributes for hauling bulk freight. These results are interpreted as suggesting that transportation prices do not accurately account for the full cost of transporting coal when modal choices are limited. The undesirable mix arises because cost-minimizing shippers make decisions based on the opportunity cost of transportation not on the price charged by transportation carriers. These findings underscore the benefits derived from using the general-

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32 It is not likely that trucking service is a close substitute for long-haul services provided by rail, since over 500 short-haul classes 2 and 3 rail carriers compete for service with trucking carriers (Winston, 2005).
33 The elasticity of substitution for all modal transportation combinations is close to 1, which corresponds to the Cobb-Douglas case for input substitutability.
34 The log-likelihood ratio for $g_2 = 1$ and $g_3 = 1$ is $\chi^2 = 1.67$, and the respective $t$-scores for $g_2$ and $g_3$ are 0.1016 and 1.273.
ized transportation cost function to estimate the divergence of actual transportation prices charged to shippers and the shadow prices associated with shippers modal choice decisions.

Acknowledgement

The authors thank participants at the Transportation and Public Utilities Group (TPUG) session at the 2009 ASSA meetings in San Francisco, California for suggestions. We are also grateful for the comments and suggestions provided by John Bitzan and Richard Perlman, as well as the insightful comments provided by the referees.

Appendix A

A.1. Concavity test

The concavity of the cost function requires that it satisfies first and the second-order conditions. The first-order condition is the derivative of the cost with respect to the input prices must be more than zero, which is also the non-decreasing condition. The second-order condition requires that the principal minors of the Hessian matrix be alternative in sign. The elements of the Hessian matrix are given by the second derivative of the cost function with respect to the input prices.

\[
H = \begin{bmatrix}
\frac{\partial^2 C}{\partial P_1^2} & \frac{\partial^2 C}{\partial P_1 \partial P_2} & \frac{\partial^2 C}{\partial P_1 \partial P_3} \\
\frac{\partial^2 C}{\partial P_2 \partial P_1} & \frac{\partial^2 C}{\partial P_2^2} & \frac{\partial^2 C}{\partial P_2 \partial P_3} \\
\frac{\partial^2 C}{\partial P_3 \partial P_1} & \frac{\partial^2 C}{\partial P_3 \partial P_2} & \frac{\partial^2 C}{\partial P_3^2}
\end{bmatrix}
\]

The principal minors for the Hessian matrix are as follow:

\[|H_1| = \frac{\partial^2 C}{\partial P_1^2} < 0\]

\[|H_2| = \begin{vmatrix}
\frac{\partial^2 C}{\partial P_1^2} & \frac{\partial^2 C}{\partial P_1 \partial P_2} & \frac{\partial^2 C}{\partial P_1 \partial P_3} \\
\frac{\partial^2 C}{\partial P_2 \partial P_1} & \frac{\partial^2 C}{\partial P_2^2} & \frac{\partial^2 C}{\partial P_2 \partial P_3} \\
\frac{\partial^2 C}{\partial P_3 \partial P_1} & \frac{\partial^2 C}{\partial P_3 \partial P_2} & \frac{\partial^2 C}{\partial P_3^2}
\end{vmatrix} = \left\{ \frac{\partial^2 C}{\partial P_1^2} \left( \frac{\partial^2 C}{\partial P_2^2} \right)^2 - \left( \frac{\partial^2 C}{\partial P_2 \partial P_1} \right)^2 \right\} > 0\]

\[|H_3| = \begin{vmatrix}
\frac{\partial^2 C}{\partial P_1^2} & \frac{\partial^2 C}{\partial P_1 \partial P_2} & \frac{\partial^2 C}{\partial P_1 \partial P_3} \\
\frac{\partial^2 C}{\partial P_2 \partial P_1} & \frac{\partial^2 C}{\partial P_2^2} & \frac{\partial^2 C}{\partial P_2 \partial P_3} \\
\frac{\partial^2 C}{\partial P_3 \partial P_1} & \frac{\partial^2 C}{\partial P_3 \partial P_2} & \frac{\partial^2 C}{\partial P_3^2}
\end{vmatrix} - 2 \left\{ \frac{\partial^2 C}{\partial P_1 \partial P_2} \cdot \frac{\partial^2 C}{\partial P_2 \partial P_3} \right\} < 0\]

In our case, each element in the Hessian matrix cannot be directly obtained by twice differentiation of the cost function since we are using a translog cost function. For a translog cost function,

\[\ln C = C(ln P_i, ln Y), i = 1, 2, \ldots, n\]

The off diagonal elements are obtained by taking the second cross partial derivative with respect to \(P_i\) and \(P_j\)

\[
\frac{\partial}{\partial \ln P_i} \left( \frac{\partial \ln C(P_i, Y)}{\partial \ln P_j} \right) = \frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j}
\]

Since

\[
\frac{\partial \ln C}{\partial \ln P_j} = \frac{\partial C}{\partial P_j} \frac{P_j}{C}
\]

\[
\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = \frac{\partial \left( \frac{C}{\partial P_j} \frac{P_j}{C} \right)}{\partial \ln P_i}
\]

\[
= P_j \left[ \frac{\partial \left( \frac{C}{\partial P_j} \frac{P_j}{C} \right)}{\partial P_i} \right]
\]
We can express:
\[
\frac{P_j C}{C} = P_j C^{-1}
\]
\[
\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = P_i \left\{ \left( \frac{\partial^2 C}{\partial P_i \partial P_j} \right) \right\} + \left[ \left( -P_i C^{-2} \right) \left( \frac{\partial C}{\partial P_i} \right) \left( \frac{\partial C}{\partial P_j} \right) \right] \]
\[
\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = P_i \left\{ \left( \frac{\partial^2 C}{\partial P_i \partial P_j} \right) \right\} - P_i \frac{\partial C}{\partial P_i} \frac{\partial C}{\partial P_j}
\]
(A.5)
(A.6)

Using Shephard’s Lemma
\[
\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = P_i \left\{ \left( \frac{\partial^2 C}{\partial P_i \partial P_j} \right) \right\} - P_i \frac{\partial C}{\partial P_i} \frac{\partial C}{\partial P_j}
\]
(A.7)

Since for a translog cost function \( \frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = \gamma_{ij} \), therefore
\[
\gamma_{ij} = \frac{P_i P_j}{C} \frac{\partial^2 C}{\partial P_i \partial P_j} - \frac{P_i P_j}{C^2} X_i X_j
\]
\[
= \frac{P_i P_j}{C} \frac{\partial^2 C}{\partial P_i \partial P_j} - \alpha_i \alpha_j
\]
(A.8)
(A.9)

Solve for \( \frac{\partial^2 C}{\partial P_i \partial P_j} \)
\[
\frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{C}{P_i P_j} (\gamma_{ij} + \alpha_i \alpha_j)
\]
(A.10)

Using a similar procedure, we can show that the diagonal elements are
\[
\frac{\partial^2 C}{\partial P_i \partial P_i} = \frac{C}{P_i^2} (\gamma_{ii} + \alpha_i^2 - \alpha_i)
\]
(A.11)

Appendix B

B.1. Allen–Uzawa partial elasticity of substitution

Uzawa (1962) shows that
\[
\sigma_{ij} = \frac{C \cdot \partial X_i / \partial P_j}{X_i X_j}
\]
(B.1)

Due to Shephard’s Lemma,
\[
X_i = \frac{\partial C}{\partial P_i} \quad \text{for} \quad i = 1, 2, \ldots, n
\]
(B.2)

Therefore,
\[
\frac{\partial X_i}{\partial P_j} = \frac{\partial^2 C}{\partial P_i \partial P_j} \quad \text{for} \quad i, j = 1, 2, \ldots, n
\]
(B.3)

Hence, (B.1) can be expressed as
\[
\sigma_{ij} = \frac{C \cdot \frac{\partial^2 C}{\partial P_i \partial P_j}}{X_i X_j} = \frac{\partial^2 C}{X_i X_j \partial P_i \partial P_j}
\]
\[
= \sum_{i=1}^{n} \frac{P_i X_i}{X_i X_j} \frac{\partial^2 C}{\partial P_i \partial P_j}
\]
(B.4)
(B.5)

Substituting (A.10) into (B.4) will give us
\[
\sigma_{ij} = \frac{C}{X_i X_j} \frac{C}{P_i P_j} (\gamma_{ij} + \alpha_i \alpha_j) = \frac{C^2}{P_i P_i P_j P_j} (\gamma_{ij} + \alpha_i \alpha_j)
\]
(B.6)

The share of mode \( i \) in total cost is defined as \( \alpha_i = \frac{P_i X_i}{C} \). Substitute this into (B.6)
\[
\sigma_{ij} = \frac{1}{\alpha_{ij}} \left( \gamma_{ij} + \alpha_i \alpha_j \right) = \frac{\gamma_{ij}}{\alpha_{ij}} + 1 \quad \text{for} \quad i \neq j
\] (B.7)

The same procedure applied to the case of \( i = j \) will give us

\[
\sigma_{ii} = \frac{\gamma_{ii}}{\alpha_{i}^2} + 1 - \frac{1}{\alpha_i}
\] (B.8)

**Own-price and cross-price elasticities of demand**

Given the information from the Allen–Uzawa partial elasticities of substitution, we can derive the own-price and cross-price elasticity of demand for mode of transportation \( i \).

Cross-price elasticity of demand is defined as

\[
e_{ij} = \frac{\partial X_i}{\partial P_j} \frac{P_j}{X_i}
\] (B.9)

From (B.2) and (B.3) we know that the following is true

\[
e_{ij} = \frac{\partial^2 C}{\partial P_i \partial P_j} \frac{P_j}{X_i}
\] (B.10)

From (B.4)

\[
\frac{\partial^2 C}{\partial P_i \partial P_j} = \sigma_{ij} \frac{X_i X_j}{C}
\] (B.11)

Substitute into (B.10)

\[
e_{ij} = \sigma_{ij} \frac{X_i X_j}{C} \frac{P_j}{X_i} = \sigma_{ij} \cdot \alpha_j
\] (B.12)

Or alternatively, substituting (B.7) gives us

\[
e_{ij} = \frac{\gamma_{ij}}{\alpha_i} + \alpha_j
\] (B.13)

The own-price elasticity is derived using the same approach. The own-price elasticity is given by

\[
e_{ii} = \sigma_{ii} \frac{X_i}{C} \frac{P_i}{X_i} = \sigma_{ii} \cdot \alpha_i
\] (B.14)

**References**


