Nuclear matter properties of finite nuclei using relativistic mean field formalism


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Abstract

In this theoretical study, we establish a correlation between the neutron skin thickness and the nuclear symmetry energy for the even–even isotones for magic neutron N = 20, 40, 82, and 126 within self-consistent relativistic mean-field formalism for non-linear NL3* and density-dependent DD-ME2 parameter sets. The local density approximation is used to formulate the symmetry energy, and its co-efficient, namely, neutron pressure of finite nuclei over the isotonic chains. We find a few moderate signatures of pick and/or depth over the isotonic chains at and/or near the proton magic for symmetry energy and neutron pressure, which is a manifestation of the persistence of shell/sub-shell closure. We determine the symmetry energy for the isotonic chain of expected neutron shell closures N = 172 and also find similar behavior as in the case of known magic neutrons. Furthermore, we show the symmetry energy as a function of neutron-proton asymmetry, which results in similar behavior as persisted in the mass-dependence curve. In addition to these, a comparative study is performed using Coherent-Density-Fluctuation-Model to examine the surface effect over the isotonic chain in terms of nuclear matter quantities at local density. The obtained results are of

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considerable importance since due to shell closure over the isotonic chain, will act as awaiting point in nucleosynthesis of $r$-process and experimental investigations towards the drip-line region of the nuclear chart. © 2020 Elsevier B.V. All rights reserved.

**Keywords:** Relativistic mean field; Local density approximation; Symmetry energy; Neutron pressure; Isospin asymmetry

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1. Introduction

Understanding of the density-dependent nature of the symmetry energy at a large isospin asymmetry and the high density regime has remained a long-standing problem in both nuclear [1–4] and astrophysics [5–7]. Specifically, the nuclear symmetry energy plays a crucial role in understanding the reaction dynamics of the heavy-ion collision [8], the structural properties of the neutron and proton-rich exotic nuclei [9], properties of the neutron star [10–13] and so on. The symmetry energy for an isospin-asymmetry system is defined as the energy cost of converting all the protons into neutrons in a nuclear system. The basic concept of the symmetry energy was first coined by Weizsacker in the semi-empirical mass formula [14] and later the surface symmetry energy was introduced by the Myers and Swiatecki [15]. In recent years lots of work have been devoted to correlate the volume and surface symmetry energy [16].

In infinite nuclear matter system, the equation of state (EoS) can be expanded over symmetry energy around the saturation density as,

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L(\rho_0)}{3} (\frac{\rho - \rho_0}{\rho_0}) + \mathcal{O}[(\frac{\rho - \rho_0}{\rho_0})^2].$$

(1)

Here, the $E_{\text{sym}}(\rho_0)$ is the symmetry energy at the saturation density, which has value in the range $27 \pm 2$ MeV [15,17,3,18,16,19]. The density slope of the symmetry energy at saturation density $L(\rho_0)$, has a value in the wide range such as $20–115$ MeV, depending on various model calculations [20,7,21–23,18,24–30]. The exact value of the $L(\rho_0)$ affects significantly various properties such as the size of the neutron skin in the heavy nuclei like $^{208}$Pb [31,32,22,23,30], relocation of the neutron drip line and emergence of new islands of stability [33,34], the crust-core transition density [35–38] and the gravitational binding energy [39]. Furthermore, the curvature of the symmetry energy has a significant role in determining the stiffness of the EoS and which directly affects the maximum mass of a neutron star. Moreover, the value of the incompressibility also directly connected with the symmetry energy, has a range $230 \pm 20$ MeV form different model calculation and measurements of the excitation of the giant monopole resonances of the heavy nuclei [40–43].

In recent years, several works have been devoted to correlate the effective symmetry energy with the bulk properties of finite nuclei [16,44]. As symmetry energy is an isospin dependent quantity, it is sensitive to the skin thickness of the heavy nuclei [44]. Further skin thickness of heavy nuclei, namely, $^{208}$Pb, a laboratory parameter to constraint the nuclear EoS [44]. The EoS imparts the information about the isovector component of the nuclear interaction and properties of nuclei in analogous to finite nuclear matter at local density. In addition, a few astrophysical observations and the availability of RIBs have further pushed the investigations of symmetry energy on finite nuclei. In this present context, we calculate the symmetry energy of the finite nuclear system over the isotonic chain for neutron magic numbers, such as $N = 20, 40, 82, 126,$ and 172. Here we choose magic and/or shell closures neutron to observe the sensitiveness of
open shell proton in an isotonic chain on symmetry energy and neutron pressure. Here, the local density approximation (LDA) [45–47] is adopted to formulate the symmetry energy of the finite nuclei at local density. The relativistic mean field model for two different parametrizations: NL3* [48,49], the non-linear point coupling, and DD-ME2 [50,51], the density-dependent meson-nucleon interactions are used in the present analysis.

This paper is organized as follows: In Sec. 2, we discuss the relativistic mean field approach along with the meson-nucleon couplings. Sec. 3 is assigned to the discussion of the results of our calculation and of the possible correlations for effective symmetry energy and bulk properties of the finite nucleus. Finally, a summary and brief conclusion are given in Sec. 4.

2. Theoretical formalism

The present section is assigned to discuss theoretical approaches used in this work, which mainly contains two parts. In the first part, we discuss the relativistic mean field (RMF) formalism, which is used to calculate the bulk properties including density distribution of the nucleus. In the second part, we discuss the local density approximation to formulate the symmetry energy and neutron pressure of the finite nuclear system. The RMF model is among the most successful and extensively used theoretical approach to study the finite nuclei as well as infinite nuclear matter and neutron stars [52–67,61,68–71,34]. Within RMF theory, the nucleons are assumed to oscillate independently in a harmonic oscillator motion in the mean field produced via the exchange of mesons and photons. The nucleons interact with each other via the exchange of isoscalar-scalar \( \sigma \), isovector-vector \( \rho \) and isoscalar-vector \( \omega \) mesons. The interaction Lagrangian density of nucleons with \( \sigma -, \omega -, \rho - \)mesons and photon \( A^\mu \) fields within RMF is given as [72,52,66,53,55–65,34],

\[
\mathcal{L} = \bar{\psi} \left[ i \gamma^\mu \partial_\mu - M \right] \psi + \frac{1}{2} \bar{\psi} \Gamma^\mu \partial_\mu \psi - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_\sigma^2 \sigma^3 - \frac{1}{4} g_\omega^2 \sigma^4 - g_\rho \bar{\psi} \psi \sigma \\
\frac{1}{4} \Omega^\mu \nu \Omega_{\mu \nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - g_\omega \bar{\psi} \gamma^\mu \psi \omega_\mu - \frac{1}{4} \bar{B}^\mu \nu \cdot \bar{B}_{\mu \nu} + \frac{1}{2} m_\rho^2 \rho^\mu \cdot \rho_\mu \\
- g_\rho \bar{\psi} \gamma^\mu \tau_3 \psi \cdot \rho_\mu - \frac{1}{4} F^\mu \nu F_{\mu \nu} - e \bar{\psi} \gamma^\mu (1 - \tau_3) \frac{1}{2} \psi A_\mu, \tag{2}
\]

with vector field tensors

\[
F^\mu \nu = \partial_\mu A_\nu - \partial_\nu A_\mu \\
\Omega^\mu \nu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \\
\bar{B}^\mu \nu = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu.
\tag{3}
\]

The \( m_\sigma, m_\omega, \) and \( m_\rho \) and \( g_\sigma, g_\omega \) and \( g_\rho \) are the masses and coupling constants of \( \sigma -, \omega - \) and \( \rho - \)mesons, respectively. These coupling constants represent the interaction strengths of nucleons with the meson fields. The photon field \( (A^\mu) \) gives the electromagnetic interaction due to protons with strength \( \frac{e^2}{4\pi} \). The first-order coupled differential equations for nucleons and four second-order differential equations for the meson fields are obtained from Eq. (2) using Euler-Lagrangian equation [53,61,64,65,34]. The detailed equations can be found in Refs. [53,61,64,65,34,73].

The RMF model proposed in the Refs. [68,51] allows density dependence of the meson-nucleon coupling, which is parameterized in a phenomenological approach [67,51,68–71]. The coupling of the mesons to the nucleon fields are defined as

\[
g_i(\rho) = g_i(\rho_{sat}) f_i(x) |_{x=\sigma,\omega}, \tag{4}
\]
Table 1
Parameters and infinite nuclear matter properties at saturation density of the non-linear NL3∗ [49] and density-dependent DD-ME2 [51] interaction parameters.

<table>
<thead>
<tr>
<th>Interaction Parameters</th>
<th>Saturation Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NL3∗ [49]</strong></td>
<td><strong>DD-ME2 [51]</strong></td>
</tr>
<tr>
<td>$M=939$ MeV</td>
<td>$M=939$ MeV</td>
</tr>
<tr>
<td>$m_\sigma=502.5742$ MeV</td>
<td>$m_\sigma=550.1238$ MeV</td>
</tr>
<tr>
<td>$m_\omega=782.6000$ MeV</td>
<td>$m_\omega=783.0000$ MeV</td>
</tr>
<tr>
<td>$m_\rho=763.0000$ MeV</td>
<td>$m_\rho=763.0000$ MeV</td>
</tr>
<tr>
<td>$g_\sigma=10.094$</td>
<td>$g_\sigma</td>
</tr>
<tr>
<td>$g_\omega=12.8065$</td>
<td>$g_\omega</td>
</tr>
<tr>
<td>$g_\rho=4.5748$</td>
<td>$g_\rho</td>
</tr>
<tr>
<td>$g_2=-10.8093$ fm$^{-1}$</td>
<td>$a_\sigma=1.3881$</td>
</tr>
<tr>
<td>$g_3=-30.1486$</td>
<td>$b_\sigma=1.0943$</td>
</tr>
<tr>
<td>$c_\sigma=1.7057$</td>
<td>$a_\omega=1.3892$</td>
</tr>
<tr>
<td>$d_\sigma=0.4421$</td>
<td>$b_\omega=0.9240$</td>
</tr>
<tr>
<td>$a_\omega=1.4620$</td>
<td>$d_\omega=0.4775$</td>
</tr>
<tr>
<td>$a_\rho=0.5647$</td>
<td></td>
</tr>
</tbody>
</table>

where,

$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2},$$  \hspace{1cm} (5)

and

$$g_\rho = g_\rho(\rho_{sat})e^{a_\rho(x-1)}.$$  \hspace{1cm} (6)

The details of the functional $x = \rho/\rho_{sat}$ and procedure of parametrization can find in the Refs. [67,51,68–71,34]. We have solved the set of coupled differential equations solved by following the procedure of Vautherin and Brink [53,61,64,65,34,73] with a fourth-order Runge-Kutta algorithm and the meson fields are solved by the Newton-Raphson method [53,61,64,65,34,73]. After getting a convergent solution of the fields, the densities and energy of a nucleus are obtained. The center-of-mass motion energy correction is estimated by the usual harmonic oscillator formula $E_{c.m.} = \frac{3}{4}(41A^{-1/3})$. The total binding energy and other observable are also obtained by using the standard relations, given in Refs. [74]. To describe the nuclear bulk properties of open-shell nuclei, we have considered the constant gap BCS approach with the NL3∗ and the Bogoliubov transformation with the DD-ME2 parameter to take care of pairing correlations into account in the present analysis [75–79,60,62,59,61,80–84,56,85,51,66,34]. Although BCS approach is not reliable in general to deal the pairing correlations in drip-line nuclei, it work reasonably good for the considered nuclei around the β-stable region of the nuclear chart [75–79,60,62,59,61,34]. The reason behind here for considering two kinds of pairing approaches is to examining the pairing effect and their model dependency on the nuclear matter quantities at local density.
2.1. Local density approximation (LDA)

The energy per nucleon of nuclear matter \( E/A = e(\rho, \alpha) \) (where \( \rho \) is the baryon density) can be expanded by Taylor series expansion method in terms of isospin asymmetry parameter \( \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \),

\[
e(\rho, \alpha) = \frac{E}{\rho_B} - M = e(\rho) + S(\rho)\alpha^2 + \mathcal{O}\alpha^4.
\]

(7)

Here, \( e(\rho), S(\rho) \) and \( M \) are the energy density of symmetric nuclear matter (SNM) \( (\alpha = 0) \), the symmetry energy, and the mass of a nucleon, respectively. The odd powers of \( \alpha \) are forbidden by the isospin symmetry and the terms proportional to \( \alpha^4 \) and higher orders have negligible contribution. The symmetry energy \( S(\rho) \) is defined by,

\[
S(\rho) = \frac{1}{2} \left[ \frac{\partial^2 e(\rho, \alpha)}{\partial \alpha^2} \right]_{\alpha=0}.
\]

(8)

Near the saturation density \( \rho_0 \), the symmetry energy can be expanded through the Taylor series expansion method and is given in Eq. (1), which is important/crucial part to determine the respective quantity at local density. In other words, one can determine the symmetry energy and its co-efficient by using Eq. (1) at local density. More details can find in Ref. [45,46,86,47].

Using the density profile, we can calculate the effective bulk properties of a nucleus by adopting local density approximation (LDA) [45,46,86,47]. The symmetry energy of a nucleus having \( N \) number of neutron and \( Z \) number of proton can be defined as,

\[
S \left( \frac{N - Z}{A} \right)^2 = \frac{1}{A} \int \rho(r) S^{NM}[\rho(r)] \alpha^2(r) d\mathbf{r}.
\]

(9)

In Eq. (9), the quantity \( S^{NM}[\rho(r)] \) is the symmetry energy of the symmetric nuclear matter at local density \( \rho(r) \) and \( \alpha(r) \) is the isospin asymmetry of the corresponding nuclear system. This is a good approximation for the asymmetric nuclear system and works well for the heavy and superheavy nuclear system at finite temperature [45,47]. In other words, one can generate the same expression for finite temperature by using the for the finite respective inputs such as...
density, symmetry energy for finite temperature and it works well for heavy and superheavy nuclei [45,47].

Following the Eq. (9), we can write similar expressions for the neutron pressure and symmetry energy curvature, where the symmetry energy for the nuclear matter is replaced by corresponding nuclear matter quantities at the local density of a nucleus. The detail calculations for the symmetry energy and related quantities within RMF formalism can be found in Refs. [48,49,85,34,87]. The quantity $L_{0}^{NM}$ is the slope parameter of the symmetry energy. It plays an important role in defining the density dependence of the symmetry energy. $L_{0}^{NM}$ has a large uncertainty from various theoretical model analysis of the heavy-ion collision data, nuclear masses, isovector giant dipole resonance, and neutron skin. The slope of the symmetry at saturation density $L_{0}^{NM}$ is defined as,

$$L_{0}^{NM} = 3\rho \left( \frac{\partial S^{NM}}{\partial \rho} \right)_{\rho=\rho_0} = \frac{3p_0^{NM}}{\rho_0}.$$  

(10)

Here, $S^{NM} (\rho) = E_{sym}(\rho)$ of Eq. (1) and $p_0^{NM}$ is the neutron pressure of the nuclear matter at the saturation density. By replacing $S^{NM} (\rho(r))$ by $p^{NM} (\rho(r))$, in Eq. (9), one can obtain the neutron pressure of the finite nuclei. The knowledge of the density dependence of the symmetry energy, which plays a crucial role in the nuclear matter and finite nuclear calculation is still poor. The appreciated range from various calculations is given by $27\pm3$ MeV [88]. In the present context, we calculate the symmetry energy and neutron pressure for the isotonic chain of fixed neutron magic number. We use two parameter sets, non-linear NL3* and density-dependent DD-ME2 of RMF formalism.

The symmetry energy coefficient of a finite nuclear system is a bulk property and it depends strongly on the isospin asymmetry of that system. In approximation, the symmetry energy of a finite nucleus can be expressed in term of volume and surface symmetry energy coefficients, which mainly depends on the mass number [45]. It is worth mentioning that LDA is one of the simplest procedures to generate the symmetry energy coefficient for the finite nuclear system. Further, the surface term plays a significant role in determining the symmetry energy of the light mass nuclei due to the large surface effect and it becomes negligible for the heavy and super heavy nuclei. The surface symmetry energy coefficient is proportional to $A^{-1/3}$, where $A$ is the mass number [45], while the volume symmetry energy is almost independent of the shape of the nucleus [45,89]. For a comparative study, we also adopted Coherent Density Fluctuation Model [90–92,34,93] to determine the surface effect for a few cases, which will be discussed in the preceding section. Further, in Ref. [94], the authors show that the deformation of a nuclear system does not change the symmetry energy coefficient significantly, for example, the relative change in symmetry energy is about 0.4 MeV in case of large deformation ($\beta_2 \sim 0.6$). The effect of the deformation on the symmetry energy decreases with the mass number [94]. Our calculations deal with isotonic chains of the neutron magic $N = 20, 40, 82, 126, and 172$, where the ground state deformations is of magnitude $0.0 \leq \beta_2 \leq 0.2$. Hence the effects of deformation on the symmetry energy are very small and can be neglected. For the sake of computational simplicity, we take the spherical density distribution of the nucleus in the present analysis.

3. Calculations and results

In the present work, we explore the nuclear symmetry energy and its coefficient over the isotonic chains of neutron magic, $N = 20, 40, 82, 126$, and 172 (predicted). We perform the self-consistent calculations, within spherically symmetric relativistic mean field model [49,51] for
widely used non-linear NL3* [49] and density-dependent DD-ME2 [51] parameter sets. These parameter sets are successful in predicting the properties of both the finite nuclei and infinite nuclear matter at high isospin asymmetry. From the self-consistent RMF calculations, we obtain the bulk properties such as binding energy, root-mean-square radius (charge, proton, and neutron), single particle, pairing energy and nuclear matter density distributions, which are not listed in the manuscript. Instead of concentrating on these quantities, we use the mean-field densities to estimate the nuclear matter quantities such as symmetry energy and neutron pressure within LDA [45] using Eq. (9). As one can notice in Eq. (9), the symmetry energy of nuclear matter is the key quantity in determining the respective values in finite nuclei. In Fig. 1, we show the symmetry
energy as a function of density for the symmetric nuclear matter for both NL3* and DD-ME2 force parameter sets. It is clear from the figure that the symmetry energy at saturation density for the NL3* has larger value rather than that of DD-ME2 set. Nature of the changes in the symmetry energy with density for the parameter NL3* is different from the DD-ME2 parameter set. In lower density region for DD-ME2, the symmetry energy is stiffer than that of the NL3* but the this nature gets reverse in the higher density region. Further, the symmetry energy for the NL3* parameter set monotonically increases where as for the DD-ME2 shows a strong density dependence due to the density dependent nucleon-meson coupling constants. At high density, the
DD-ME2 gets softer as compared to NL3* parameter set, which also reflects in the finite nuclei, will be discussed in the preceding sections.

In Fig. 2, we show the variation of the symmetry energy for the isotonic series of neutron magics, N = 20, 40, 82, 126, and 172 (predicted) as a function of neutron skin thickness. Mainly, the idea here is to fix the neutron number to these particular value of magic numbers and change the proton number. Moving through an isotonic chain, we observe the effects of Z-dependence of the symmetry energy and its coefficient. Here, only N = 172 (predicted magic) has some ambiguity, but many of the models have conformed this to be the next neutron magic number [95–98]. It is worth mentioning that the skin thickness (∆R = Rn − Rp) are calculated within RMF model for NL3* and DD-ME2 parameter sets. We observe that at a fix proton number (isotope), the ∆R for NL3* is always greater than DD-ME2 parameter set, which is the effect of strong repulsive isovector component of NL3* Lagrangian density. We notice the symmetry energy for a particular isotope has a large value by ∼ 2 MeV for NL3* parameter set than that of DD-ME2 force parameter. For example, for particular isotones (N = 40, Z = 32), the symmetry energy for the DD-ME2 is around 17 MeV, while for NL3* is around 20 MeV. The reason behind the difference of the symmetry energy for two force parameter sets for a particular isotones is due to the saturation value of the symmetry energy, which is slightly depends on the parameters. From the Table 1 and also from Fig. 1, one can notice that NL3* has nuclear matter symmetry energy at saturation ∼ 38.7 MeV, while the value for the DD-ME2 is 32.3 MeV. Even the saturation value of symmetry energy for nuclear matter for these two parameter sets differ by ∼ 5 MeV, whereas the difference reduce in finite nuclei due to the structure effect and also the average density of finite nuclei always lay below the nuclear matter saturation density ρ ∼ 0.16 fm−3.

To clarify the Z-dependency of the symmetry energy over the isotonic chain, in Fig. 3, we show the variation of the symmetry energy with proton number. From Fig. 3, we notice that the symmetry energy monotonically increases with the atomic number for all the isotonic chains. As we notice in Fig. 2, here we can see the transparent picture of symmetry energy for a particular N and Z values. Quantitatively, the symmetry energy value for NL3* is more than that of DD-ME2 parameter set and the difference over isotonic chain almost remains constant throughout the series for a particular neutron number. More careful inspection shows that the relative difference in ∆R for NL3* and DD-ME2 increase with mass number. In other words, the relative difference in ∆R for NL3* and DD-ME2 is minimum in the isotonic series of the N = 20, while it is maximum for the isotonic series of the N = 172. Further, one can observe a very small pick/change in each isotonic chain for magic proton over the chain, which is due to the double shell closures isotones. The signature is very small, because here we consider the isotonic chain of neutron magic, which is already playing a great role in the magicity of the drip-line nuclei. Hence, the effect of shell closures by a proton magic is dominated by the neutron magic. This feature will be more cleared in case of isotopic chain contain both open-shell nuclei and attain neutron magic over the chain [34]. In addition, the present calculation performed within spherical symmetry, by inclusion of more shape degrees of freedom may have slight effect to enhance the signature, which will be studied in near future.

Exploration of the density dependence of nuclear symmetry energy and its isospin dependence are the key issues in contemporary nuclear physics. In addition, the structure analysis of shell/sub-shell closures in drip-line region is also crucial to determine the isospin dependence of effective nuclear symmetry over the isotonic chain, specially for neutron magic. In Fig. 4, we show the variation of the symmetry energy with isospin asymmetric in an isotonic chain of N = 20, 40, 82, 126, and 172 (predicted) for non-linear NL3* and density-dependent DD-ME2 parameter sets using Eq. (9). From the figure, it is clear that the symmetry energy linearly varies
with the iso-spin asymmetry \( \frac{N-Z}{N+Z} \) for all isotonic chains. At a particular value of \( \frac{N-Z}{N+Z} \), the higher value of neutron \( N \) gives the higher symmetry energy and vice-versa. In other words, the \( \frac{N-Z}{N+Z} \) value increases with mass number i.e., the heavier isotones have larger value of symmetry energy as compare to the lighter one. From Eq. (9), one can notice that the symmetry energy depends on various factors such as number of proton, nucleonic density distribution, and relative symmetry energy of the finite nuclei. For example, even the system having same isospin asymmetry, we get different effective symmetry energy for both force parameter sets as it depends on the local density distribution of the nucleus. The skin thickness for a particular isotope is drastically differ for both the force parameter sets (see Fig. 2). Hence, it is crucial and also not easy to trace the exact (\% of) dependency of these factors on the symmetry energy of finite nuclei. Whereas, the present study ensures that the symmetry energy mostly depends on the isospin asymmetry of the system as compare to other factors.

In Fig. 5, we correlate the neutron skin thickness (\( \Delta R \)) of finite nuclei with the symmetry energy and its co-efficient, namely, the neutron pressure over the isotonic chains of neutron magics \( N = 20, 40, 82, 126, \) and \( 172 \) (predicted) for non-linear NL3* and density-dependent DD-ME2 force parameter sets. By the definition in the Eq. (10), the neutron pressure is related with the slope of the symmetry energy and which is a quantity having significant importance in astrophysics and radioactive ion beam (RIB) experiments. The value of the slope parameter has large uncertainty, hence the study of neutron pressure, which is differ by a factor of 3 may help us to squeeze the uncertainty over the isotonic chains of neutron magics \( N = 20, 40, 82, 126, \) and \( 172 \) (predicted) for non-linear NL3* and density-dependent DD-ME2 force parameter sets. In Fig. (1) of the Ref. [34], the authors have shown the correlation of neutron pressure with skin thickness in an isotopic chain. The graph shows that over an isotopic chain the neutron pressure increases with decrease in the skin thickness. At a certain value of the \( N \) and \( Z \), it shows a strong signature (kink), which manifest with the shell/sub-shell closures. Here, in our case we find the similar trend of decrements of neutron pressure with the increase of the skin thickness but no strong signature at any particular \( Z \) value as appear in case of symmetry energy. Further, a careful inspection shows that like the symmetry energy curve, the relative difference in neutron pressure for NL3* and DD-ME2 parameter sets increases with neutron number. In other words,
Fig. 5. (Color online) The neutron pressure $p_0$ for the isotonic chain of $N = 20, 40, 82, 126,$ and 172 (predicted) as a function of the neutron skin-thickness as calculated using RMF model for non-linear NL3* (black line) and density-dependent DD-ME2 (red line) interaction parameter sets.

The heavier isotones have large neutron pressure as compare to the lighter masses. In Fig. 6, we clarify the $Z$-dependency of the neutron pressure for all the isotonic chains. From the figure, it is clearly observed that the neutron pressure goes on increasing with atomic number for a fixed neutron number for both the parameter sets. For $N = 20$ isotones, the neutron pressure shows some monotonic change for both the parameter sets NL3* and DD-ME2, with a small the difference therein (see the Fig. 3 and 5). Similar trend is followed by $N = 40$ isotonic chain. But in case of the higher mass isotonic chain of $N = 82, 126,$ and 172, the difference between the neutron pressure of NL3* and DD-ME2 parameter sets remain constant.
Fig. 6. (Color online) The neutron pressure $p_0$ for $N = 20, 40, 82, 126,$ and $172$ (predicted) isotonic series as a function of the proton number as calculated using RMF approach for non-linear NL3* (black line) and density-dependent DD-ME2 (red line) interaction parameter sets.

From the above analysis, we find that there is a minimal signature for shell effect noticed over all the isotonic chain for proton magic. As our study include the isotonic chain for neutron magic (i.e., $N = 20, 28, 40, 82, 126,$ and $172$), the signature for magicity and/or closed-shell proton is insignificant. Further, the surface effect on finite nuclei, where the LDM has a narrow window to include the surface effect to predict the nuclear matter properties at local density [24]. In this context, we have done the same calculation by using the Coherent Density Fluctuation Model (CDFM), where the surface part is taken through improved approximation. More details of CDFM and a few recent works by us and others can be found in Refs. [90–92,34,93]. Both semi-classical, non-relativistic Skyrme-Hartree-Fock and relativistic mean-field models
The neutron pressure $p_0$ for $N = 20, 40, 82, 126,$ and 172 (predicted) isotonic series as a function of the proton number as calculated using RMF approach for non-linear NL3* (black line) and density-dependent DD-ME2 (red line) interaction parameter sets.

have been used in these studies to obtain the nuclear matter properties for finite nuclei. Further the symmetry energy, and neutron pressure used as the effective observable to determine the closed-shell/sub-shell magic nucleons (proton and/or neutron) over the isotopic chain. To examine the surface effect in terms of nuclear matter quantities at local density, we calculate the symmetry energy and neutron pressure for the isotonic chain of $N = 126$ (neutron magic) as a representative case. It is worth mentioning that the trend will be altered for all the considered isotonic chains. As a comparative analysis, the calculated results for the symmetry energy and neutron pressure within LDM and CDFM for $N = 126$ isotonic chain are shown in Fig. 7. From the figure, it is clear that both the procedures (LDM and CDFM) are following a similar pattern/trend with a difference in magnitude for symmetry energy and neutron pressure, which is a well-known effect already examined [24,92,34,93]. Very careful inspection over the isotonic chain shows that the minute signature at proton magic within LDM is slightly enhanced in the case of CDFM. Comparing the magnitude of the pick appears at proton magic in the present work for an isotonic chain is lower in magnitude as compared to the predicted pick for neutron magic over an isotopic chain [91,92,34,93]. Hence, one can conclude that the magnitude of the pick and/or the signature corresponding to proton magic is get suppressed over the isotonic chain of neutron magic. In other words, the signature at proton magic over an isotopic chain get, which is not clear in the case of LDM. Further, CDFM is marginally superior to LDM in predicting the shell/sub-shell closer over the isotopic and isotonic chain. A more systematic analysis in this regard is highly welcome.
4. Summary and conclusions

In the present study, we investigate the possible relationships between the neutron skin thickness of neutron-rich nuclei and symmetry energy and its co-coefficients. A microscopic approach based on a spherical symmetric relativistic mean-field with the non-linear NL3∗ and density-dependent DD-ME2 interaction parameters is used. Effective nuclear matter properties such as the symmetry energy $S_0$, and the neutron pressure $p_0$ are determined for finite nuclei at local density using the local density approximation. The calculation follows two steps, in the first step, we obtain the ground state nuclear bulk properties such as the binding energies, nuclear density distributions and root-mean-square radius by using the self-consistent RMF with the non-linear NL3* and density-dependent DD-ME2 parameter sets. We consider the even–even isotopic chain of neutron magic $N = 20, 40, 82, 126, \text{and} 172$ (expected) for the present analysis. In the second step, we calculated the effective nuclear matter characteristics such as symmetry energy $S_0$, and neutron pressure $p_0$ for the finite nuclei using the density distribution within LDA. For all of the isotonic chains, we found that there exists a moderate correlation between the neutron skin thickness and symmetry energy. We also determine the $Z$-dependency of the symmetry energy and its coefficient called neutron pressure ($p_0$) over the isotonic chains. We find a very weak signature (small pick) for $S_0$ and $p_0$ for magic proton (double shell closures) due to the dominant of the neutron magic effect over the isotonic chain. To examine the surface effect, we perform a comparative study by using CDFM, which reproduces the same trend with a reasonably good picture of the shell/sub-shell closure over the isotonic chain. Hence, one can conclude that CDFM is a comparatively better approach to predict the shell structure in an isotopic and/or isotonic chain over LDM. A more detailed quantitative analysis is opened to determine the exact dependence of various factors on the symmetry energy of finite nuclei.

CRediT authorship contribution statement

M. Bhuyan, S. K. Patra, N. Yusuf, and H. A. Kassim: Calculations, Results analysis, and Supervision
S. K. Biswal, M. K. Abu El Sheikh, and N. Biswal: Calculations and Writing

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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