Dynamics for two atoms interacting with intensity-dependent two-mode quantized cavity fields in the ladder configuration

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Exact solutions are obtained for a collective model of two identical two-level atoms interacting with a quantized bimodal field with intensity-dependent coupling terms in a lossless cavity. A unitary transformation method is used to solve the time-dependent problem that also gives the eigensolutions of the interaction Hamiltonian. The atomic population dynamics and the dynamics of the photon statistics of the two cavity modes are studied. We present evidence of cooperative effects.

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I. INTRODUCTION

Two-photon processes are of considerable importance in quantum optics due to the high degree of correlation between the emitted photons. This correlation may lead to the generation of nonclassical states of the electromagnetic field, such as squeezed states [1,2] and states which may violate other classical inequalities [3]. With the successful operation of a single-mode two-photon maser in a high-Q cavity [4], the discussion of the nondegenerate two-photon Jaynes-Cummings models has acquired added importance. In the existing theoretical work much attention has been paid to single-atom two-mode two-photon processes. There have been few publications concerning the many- or two-atom nondegenerate case so far [5,6]. This motivates us to study and analyze the collective dynamics of a system of two identical two-level atoms that resonantly interact with two quantized modes of electromagnetic fields in the ladder configuration.

Cavity QED generally deals with few cavity photons. Hence, atomic emission or atomic absorption effects are expected to change the atom-field interaction strength significantly. Consequently an intensity-dependent coupling constant [7,8] would be appropriate to study the problems related to cavity QED. Taking this into consideration Napoli and Messina [9] generalized the intensity-dependent Jaynes-Cummings model (JCM) to a two-mode cavity field with a two-photon process based upon the dressed-atom states. Singh and Amrita [10] studied the dynamic properties of the intensity-dependent Jaynes-Cummings model for a two-mode cavity field.

In the present paper we study the quantum dynamics of the two-atom two-mode two-photon JCM, generalizing the corresponding Hamiltonian [6–9] by introducing an intensity-dependent coupling term. Although the one-atom case is more convenient to observe the nonclassical properties of the cavity field, the study of two atoms has certain advantages. The nondegenerate two-photon Jaynes-Cummings model is an effective two-level atom with upper and lower states denoted by $a$ and $b$, respectively, interacting with two different modes $\omega_1$ and $\omega_2$ of the field. The Hamiltonian for such a system in the rotating-wave approximation is written as

$$\hat{H} = \hbar \frac{\varrho_0}{2} \hat{\sigma}_3 + \hbar \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \hbar \omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar g (\hat{a}_2^\dagger \hat{a}_1^\dagger \hat{\sigma}_- + \hat{a}_1 \hat{a}_2^\dagger \hat{\sigma}_+).$$

(1)

Here $\varrho_0$ is the atomic transition frequency, $g$ is the atom-field coupling constant, $\hat{\sigma}_3$ is the inversion operator, and
\( \sigma_z \) and \( \sigma_- \) are the Pauli raising and lowering operators, respectively. \( \hat{a}_i^\dagger \) \((\hat{a}_i) \) \((i = 1, 2)\) is the boson creation (annihilation) operator of modes. The number states for the field modes are the direct product of number states for modes 1 and 2, i.e., \( |n_1, n_2\rangle_F = |n_1\rangle_{F_1} \otimes |n_2\rangle_{F_2} \).

The intensity-dependent two-atom two-mode two-photon Jaynes-Cummings model is obtained from Eq. (1) as [7–10,15–18]

\[
\hat{H} = \frac{\hbar \omega_0 \tilde{\sigma}}{2} + \hbar \omega_0 \hat{a}_1^\dagger \hat{a}_1 + \hbar \omega_2 \hat{a}_2^\dagger \hat{a}_2 \\
+ \hbar g (\hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 + \sqrt{2} \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 \sigma_+ + \sqrt{2} \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 \sigma_-).
\]

In two-photon processes the Stark shift caused by the intermediate atomic level plays the role of an intensity-dependent detuning [19–21]. However, if the two fields are tuned in such a way that both have reverse detuning with the intermediate atomic level, the Stark shift will not appear [22,23]. Such a two-photon signal could be achieved by two dye lasers.

Radiative transitions may be neglected, assuming a long excited lifetime of the atoms. We consider the two-level system and neglect other intermediate states in actual atoms or quantum dots that contribute to the nonresonant transitions that modify or shift the energy levels of the atoms to some extent. The level shift does not alter the transient behavior of the photon statistics.

The many-atom case is constructed just as for the usual Dicke model [14]. We define the collective atomic operators \( \hat{R}_1, \hat{R}_2, \) and \( \hat{R}_3 \) as

\[
\hat{R}_k = (1/2) \sum_{j=1}^N \hat{\sigma}_k^j, \quad \hat{R}_\pm = \sum_{j=1}^N \hat{\sigma}_\pm^j,
\]

where \( N \) is the number of atoms, \( k = 1, 2, 3 \), and \( \hat{\sigma}_k^j \) are the atomic operators for the \( j \)th atom.

The operators \( \hat{R}_1, \hat{R}_2, \) and \( \hat{R}_3 = \hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 \) obey the commutation relations for general angular momentum operators. The Hamiltonian for the \( N \)-atom case is then obtained as

\[
\hat{H} = \hbar \omega_0 \hat{R}_3 + \hbar \omega_0 \hat{a}_1^\dagger \hat{a}_1 + \hbar \omega_2 \hat{a}_2^\dagger \hat{a}_2 \\
+ \hbar g (\hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 \hat{R}_+ + \sqrt{2} \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 \hat{R}_-) \quad \text{(4)}
\]

or

\[
\hat{H} = \hbar \omega_0 \hat{R}_3 + \hbar \omega_0 \hat{a}_1^\dagger \hat{a}_1 + \hbar \omega_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar g (\hat{Q}_+ + \hat{Q}_-) \quad \text{(5)}
\]

with

\[
\hat{Q}_+ = \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 \hat{R}_+; \quad \hat{Q}_- = \sqrt{2} \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_1 \hat{a}_2 \hat{R}_-.
\]

The basis vectors for the atomic system can be represented as \( |\psi\rangle = |m_t\rangle \) where \( m_t \) stands for the number of atoms occupying their lower-energy states. Since the different atom operators commute with each other, the following operations hold:

\[
\hat{R}_3|m_t\rangle = (N/2 - m_t)|m_t\rangle, \\
\hat{R}_+|m_t\rangle = m_t|m_t - 1\rangle, \\
\hat{R}_-|m_t\rangle = (N - m_t)|m_t + 1\rangle.
\]

In what follows, we consider only the special case for two atoms when \( N = 2 \). Hence, the possible two-atom states are

\[
|0\rangle = |a\rangle_{A_1} \otimes |a\rangle_{A_2} = |a, a\rangle_A, \\
|1\rangle = [|a\rangle_{A_1} \otimes |b\rangle_{A_2} + |b\rangle_{A_1} \otimes |a\rangle_{A_2}] = [|a, b\rangle_A + |b, a\rangle_A], \\
|2\rangle = |b\rangle_{A_1} \otimes |b\rangle_{A_2} = |b, b\rangle_A.
\]

where \( A_1 \) and \( A_2 \) refer to atom 1 and atom 2, respectively. It is convenient to introduce the Dicke states \( |j, m\rangle_D \). For the two-atom system we have \( j = 1 \) with \( m = 1, 0, -1 \). These states are related to the two-atom states above according to

\[
|1, 1\rangle_D = |a, a\rangle_A, \\
|1, 0\rangle_D = \frac{1}{\sqrt{2}}[|a, b\rangle_A + |b, a\rangle_A], \\
|1, -1\rangle_D = |b, b\rangle_A.
\]

If we assume that initially both atoms are in the excited state with \( n_1 \) and \( n_2 \) photons with frequencies \( \omega_1 \) and \( \omega_2 \), respectively, then, at \( t = 0 \) we have

\[
|\psi(0)\rangle = |0\rangle|n_1, n_2\rangle_F = |0, n_1, n_2\rangle = |1, 1\rangle_D|n_1, n_2\rangle.
\]

On the other hand, if we assume that initially both the modes are in coherent states, at \( t = 0 \) we have

\[
|\psi(0)\rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} C_{n_1}(\alpha_1)C_{n_2}(\alpha_2)|0, n_1, n_2\rangle \\
= |1, 1\rangle_D \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} C_{n_1}(\alpha_1)C_{n_2}(\alpha_2)|n_1, n_2\rangle
\]

where

\[
|C_{n_1}(\alpha_1)|^2 = P_{n_1}(n_1) = |\langle n_1|\alpha_1\rangle|^2 = \exp(-\tilde{n}_1) \frac{\tilde{\alpha}_1^{n_1}}{n_1!} \quad \text{(i = 1, 2)}
\]

and \( \tilde{n}_1 \) is the initial average number of photons in the \( i \)th mode. \( P_{n_1}(n_1) \) and \( P_{n_2}(n_2) \) represent the coherent field probability distribution functions for photon numbers in Poisson statistics.

The general state vector for the system at any time \( t \) can be expressed as the linear combination of the basis eigenkets of the interacting system designated as

\[
|\psi(t)\rangle = \sum_{n_1, n_2=0}^{\infty} \left[ A_{n_1, n_2}(t)|0, n_1, n_2\rangle_F \\
+ A_{n_1, n_2}(t)|a, a\rangle_A|n_1, 1\rangle_F \\
+ A_{n_1, n_2}(t)|b, a\rangle_A|n_1 + 1, n_2 + 1\rangle_F \\
+ A_{n_1, n_2}(t)|b, b\rangle_A|n_1 + 1, n_2 + 2\rangle_F \right],
\]

\[
|\psi(t)\rangle = \sum_{n_1, n_2=0}^{\infty} \left[ A_{n_1, n_2}(t)|0, n_1, n_2\rangle_F \\
+ A_{n_1, n_2}(t)|1, n_1 + 1, n_2 + 1\rangle_F \\
+ A_{n_1, n_2}(t)|2, n_1 + 1, n_2 + 2\rangle_F \right],
\]

\[023810-2\]
or in terms of the Dicke states of Eq. (8)

\[
|\psi(t)| = \sum_{n_1, n_2=0}^{\infty} \left[ A_t^{n_1, n_2}(t) |1, 1 \rangle_d |n_1, n_2 \rangle_F \\
+ \sqrt{2} A_t^{n_1, n_2}(t) |1, 0 \rangle_d |n_1 + 1, n_2 + 1 \rangle_F \\
+ A_t^{n_1, n_2}(t) |1, -1 \rangle_d |n_1 + 2, n_2 + 2 \rangle_F \right],
\]

(11)

where the relabeling of the A coefficients from Eqs. (10) and (11) is obvious. The coefficients of \(|0, n_1, n_2\rangle, |1, n_1 + 1, n_2 + 1\rangle, |2, n_1 + 2, n_2 + 2\rangle\) in the state vector are the probability amplitudes \(A_t^{n_1, n_2}(t)\), \(A_t^{n_1, n_2}(t)\), and \(A_t^{n_1, n_2}(t)\), respectively.

The state vector |\psi(t)\rangle develops from the state vector |\psi(0)\rangle at \(t = 0\) according to

\[
|\psi(t)| = \hat{T}(t)|\psi(0)\rangle,
\]

(12)

where the unitary operator \(\hat{T}(t)\) satisfies

\[
\frac{i}{\hbar} \frac{d\hat{T}(t)}{dt} = \hat{H} \hat{T}(t).
\]

(13)

Since \(\hat{H}\) given by Eq. (4) is time independent in the Heisenberg representation under the condition \(\omega_1 + \omega_2 \approx \omega_0\), an integration of Eq. (13) with \(\hat{H}\) given by Eq. (4) gives the transformation operator representing the two-atom two-mode ladder model with an intensity-dependent coupling as

\[
\hat{T} = \exp[-i t (\omega_0 \hat{R}_3 + \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 + g(\hat{Q}_+ + \hat{Q}_-))].
\]

(14)

Considering the initial state of the system as \(|0, n_1, n_2\rangle\) we get from Eq. (12)

\[
|\psi(t)| = \hat{T}(t)|0, n_1, n_2\rangle.
\]

(15)

The right-hand side of Eq. (15) can be obtained by expanding \(\hat{T}(t)\) given by Eq. (14) and then operating each term of the expansion on \(|0, n_1, n_2\rangle\). This gives the expression for the general state vector as obtained in Refs. [10,13].

At two-photon resonance the operators \(\hat{Q}_+\) and \(\hat{Q}_-\) become time independent in the Heisenberg picture and we use the following relations for the operators at two-photon resonance:

\[
\hat{Q}_- |0, n_1, n_2\rangle = 2(n_1 + 1)(n_2 + 1)|1, n_1 + 1, n_2 + 1\rangle,
\hat{Q}_- |1, n_1 + 1, n_2 + 1\rangle = (n_1 + 2)n_2 + 2|2, n_1 + 1, n_2 + 2\rangle,
\hat{Q}_+ |1, n_1 + 1, n_2 + 1\rangle = (n_1 + 1)|0, n_1, n_2\rangle,
\hat{Q}_+ |2, n_1 + 2, n_2 + 2\rangle = 2(n_1 + 2)\hat{a}_2^\dagger \hat{a}_2|1, n_1 + 1, n_2 + 1\rangle,
\hat{Q}_+ \hat{Q}_- |0, n_1, n_2\rangle = 2(n_1 + 1)^2|0, n_1, n_2\rangle,
\hat{Q}_+ |2, n_1 + 2, n_2 + 2\rangle = 0, \text{ etc.}
\]

(16)

The transition probability \(|A_t^{n_1, n_2}(t)|^2\) for the transition of the atom from the higher state to the lower state with the emission of two photons of frequency \(\omega_1\) and two photons of frequency \(\omega_2\) is obtained from the coefficients of \(|2, n_1 + 2, n_2 + 2\rangle\) in the expression for the general state vector as

\[
|A_t^{n_1, n_2}(t)|^2 = \frac{4g^4(n_1 + 1)^2(n_2 + 1)^2}{\Omega_{n_1, n_2}^4}\left[1 - \cos \Omega_{n_1, n_2} t\right],
\]

(17)

where

\[
\Omega_{n_1, n_2} = \sqrt{\Omega_R^2 + \delta_{12}^2}
\]

(18)

with

\[
\Omega_R = \sqrt{2g^2((n_1 + 1)^2(n_2 + 1)^2 + (n_1 + 2)(n_2 + 2)^2)}
\]

(19)

and \(\delta_{12} = (\omega_1 + \omega_2) - \omega_0\) is the two-photon detuning.

Collecting the coefficients of \(|1, n_1 + 1, n_2 + 1\rangle\) in the expression for the general state vector and making algebraic simplifications, we obtain the transition probability \(|A_0^{n_1, n_2}(t)|^2\) for transition of the atom from the higher to the lower state, emitting a photon of frequency \(\omega_1\) and another photon of frequency \(\omega_2\), as

\[
|A_0^{n_1, n_2}(t)|^2 = \frac{2g^2(n_1 + 1)^2(n_2 + 1)^2}{\Omega_R^2 + \left(\frac{\delta_{12}}{2}\right)^2}\sin^2\left[\sqrt{\Omega_R^2 + \left(\frac{\delta_{12}}{2}\right)^2} t\right].
\]

(20)

In the case of two-photon resonance \(\delta_{12} = 0\) so that

\[
\Omega_{n_1, n_2} = \Omega_R = g\sqrt{2[(n_1 + 1)^2(n_2 + 1)^2 + (n_1 + 2)(n_2 + 2)^2]}
\]

and Eq. (20) becomes

\[
|A_0^{n_1, n_2}(t)|^2 = \frac{2g^2(n_1 + 1)^2(n_2 + 1)^2}{\Omega_R^2}\sin^2\Omega_R t.
\]

(21)

Similarly, from the coefficients of \(|0, n_1, n_2\rangle\) in the expression for the general state vector the transition probability \(|A_t^{n_1, n_2}(t)|^2\) is obtained as

\[
|A_t^{n_1, n_2}(t)|^2 = 1 - \frac{4g^2(n_1 + 1)^2(n_2 + 1)^2}{\Omega_{n_1, n_2}^4}\left[1 - \cos \Omega_{n_1, n_2} t\right]
\]

\[
+ \frac{4g^4(n_1 + 1)^2(n_2 + 1)^4}{\Omega_{n_1, n_2}^4} \times \left[\frac{3}{2} - 2\cos(\Omega_{n_1, n_2} t) + \frac{1}{2} \cos(2\Omega_{n_1, n_2} t)\right].
\]

(22)

In what follows we concentrate on the quantities \(|A_t^{n_1, n_2}(t)|^2\) and \(|A_1^{n_1, n_2}(t)|^2\) since \(|A_0^{n_1, n_2}(t)|^2\) is dependent on these quantities through the equation

\[
|A_0^{n_1, n_2}(t)|^2 + |A_1^{n_1, n_2}(t)|^2 + |A_t^{n_1, n_2}(t)|^2 = 1.
\]

III. ATOMIC DYNAMICS

We now consider the atomic dynamics, in particular the dynamics of atomic level-occupation probabilities. For the two field modes initially prepared in a coherent state the probability \(W_{gg}(t)\) of finding both atoms in the ground state can be
written as

$$W_{gg}(t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2)|A^{n_1,n_2}_{+}(t)|^2$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) \frac{4g^2(n_1 + 1)^2(n_1 + 2)^2(n_2 + 1)^2(n_2 + 2)^2}{\Omega^2_{n_1,n_2}} \left[ \frac{3}{2} - 2\cos(\Omega_{n_1,n_2}t) + \frac{1}{2}\cos(2\Omega_{n_1,n_2}t) \right]. \quad (23)$$

The probability $W_{gg}(t)$ of finding one atom in the excited state while the other one in the ground state (at exact resonance) is obtained as

$$W_{eg}(t) = \frac{1}{2} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2)|A^{n_1,n_2}_{-}(t)|^2$$

$$= \frac{1}{2} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) \frac{g^2(n_1 + 1)^2(n_2 + 1)^2}{\Omega^2_R} \left[ 1 - \cos(\Omega_{n_1,n_2}t) \right] \quad (24)$$

and the probability $W_{eg}(t)$ of finding both the atoms in the excited state is

$$W_{eg}(t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2)|A^{n_1,n_2}_{+}(t)|^2 = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) \left[ 1 - \frac{g^2(n_1 + 1)^2(n_2 + 1)^2}{\Omega^2_{n_1,n_2}} \right]$$

$$+ \frac{4g^2(n_1 + 1)^4(n_2 + 1)^4}{\Omega^4_{n_1,n_2}} \left[ \frac{3}{2} - 2\cos(\Omega_{n_1,n_2}t) + \frac{1}{2}\cos(2\Omega_{n_1,n_2}t) \right]. \quad (25)$$

Let us first consider the special case when one of the field modes is initially prepared in a coherent state while the other one is in a vacuum ($\bar{n}_2 = 0$) state. We obtain

$$W_{gg}(t) = \sum_{n_1=0}^{\infty} P_{n_1}(\bar{n}_1) \frac{16g^4(n_1 + 1)^2(n_1 + 2)^2}{\Omega^2_{n_1,0}} \left[ \frac{3}{2} - 2\cos(\Omega_{n_1,0}t) + \frac{1}{2}\cos(2\Omega_{n_1,0}t) \right] \quad (26)$$

where $\Omega_{n_1,0} = \sqrt{\Omega^2_R + \delta_{12}^2}$ and

$$W_{eg}(t) = \frac{1}{2} \sum_{n_1=0}^{\infty} P_{n_1}(\bar{n}_1) \frac{g^2(n_1 + 1)^2}{\Omega^2_R + \left( \frac{\Omega_{n_1,0}}{2} \right)^2} \left[ 1 - \cos \left( 2\sqrt{\Omega^2_R + \left( \frac{\Omega_{n_1,0}}{2} \right)^2} t \right) \right]. \quad (27)$$

At two-photon resonance $\delta_{12} = 0$ so that

$$\Omega_{n_1,0} = \Omega_R = g\sqrt{2(n_1 + 1)^2 + 4(n_1 + 2)^2} = g\left[ 2(5n_1^2 + 18n_1 + 17) \right]^{1/2}. \quad (28)$$

The time evolution of $W_{gg}$ and $W_{eg}$ for $\bar{n}_1 = 15, \bar{n}_2 = 0$ for different values of the two-photon detuning $\delta_{12}$ is shown in Fig. 1. We immediately notice the exact periodicity in the collapse and revival of Rabi oscillations. We further note that there are two types of Rabi oscillations present in the dynamics of $W_{gg}$, one at a smaller amplitude than the other. These are attributed to one- and two-photon processes, respectively. To predict the location of revivals, we use the usual arguments [5].

Here, the dominant Rabi frequency is $\Omega_{n_1,0} = \sqrt{2}g(5\bar{n}_1^2 + 18\bar{n}_1 + 17)^{1/2} \approx g\sqrt{10}(\bar{n}_1 + \frac{9}{2}), \bar{n} > 1$, where $\bar{n}_1$ is the average initial photon number. The spread of the probabilities $|P_{n_1}(\bar{n}_1)|^2$ about $\bar{n}_1$ for photon numbers in the range $(\bar{n}_1 \pm \Delta n_1)$, i.e., frequencies in the range $\Omega(\bar{n}_1 - \Delta n_1)$ to $\Omega(\bar{n}_1 + \Delta n_1)$, gives rise to the depinning of the Rabi oscillations. The collapse time $t_c$ can be obtained from the time-frequency uncertainty relation $t_c[\Omega(\bar{n}_1 + \Delta n_1) - \Omega(\bar{n}_1 - \Delta n_1)] \approx 1$.

For Poisson distribution with the initial coherent field, the root-mean-square deviation in the photon number $\Delta n_1 = \sqrt{\bar{n}_1}$ so that $\Omega(\bar{n}_1 \pm \frac{\bar{n}_1}{2}) \approx g\sqrt{10}(\bar{n}_1 + \frac{9}{2}) \pm \frac{1}{\sqrt{10}}$, giving the collapse time $t_c = \frac{1}{2g\sqrt{\bar{n}_1}}$ which depends upon the mean number of photons $\bar{n}_1$. It is observed that the largest amplitude is oscillating at frequency $\Omega_{n_1,0}$.

The time between revivals $t_R$ can be estimated by finding the time when neighboring oscillators at $\bar{n}_1$ and $\bar{n}_1 + 1$ differ by a factor of 2π or some multiple of 2π, i.e.,

$$[\Omega(\bar{n}_1 + 1,0) - \Omega(\bar{n}_1,0)]t_R = 2\pi m, \quad m = 0, 1, 2, 3, \ldots \quad \text{(29)}$$

In the limit $\bar{n}_1 \gg 1$, $gt_R \approx \sqrt{10}(\bar{n}_1 + 1 + \frac{9}{2}) - (\bar{n}_1 + \frac{9}{2}) = 2\pi m$. We obtain $gt_R \approx 2\pi m [\bar{n}_1]^{1/2}$, which is independent of

023810-4
FIG. 1. (Color online) Time evolutions of $W_{gg}(t)$ and $W_{eg}(t)$ for finite detunings: (a) $\delta_{12} = 0$, (b) $\delta_{12} = 10g$, and (c) $\delta_{12} = 20g$. Here, $\bar{n}_1 = 15, \bar{n}_2 = 0$.

The most striking feature characterizing the quantum dynamics of the model (when one of the modes is kept in vacuum while the other one is in a coherent state) with the intensity-dependent interaction terms is the appearance of a strict periodicity in the collapses and revivals. Here, the Rabi frequency $\Omega_{n_1,0} = \sqrt{2g(5\bar{n}_1^2 + 18\bar{n}_1 + 17)^{1/2}} \approx \sqrt{10(\bar{n}_1 + 9/5)}$, $\bar{n} \gg 1$, becomes linear in quantum number so that an exact periodic evolution is observed in Fig. 1.

For the case $\bar{n}_1, \bar{n}_2 \neq 0$, the rapid oscillations in the time records are from the dominant Rabi oscillations at $\Omega_{n_1,0}$ and $2\Omega_{n_1,0}$ but the periodicity of the revivals is maintained.

the mean number of photons. The ratio of the collapse time to the revival time $\frac{t_c}{t_R} = \frac{1}{4\pi \bar{n}_1}$ as observed in the usual Jaynes-Cummings model.

For $\bar{n}_1 = 15$ we obtain $gt_R \approx 1.9$, which is the center of the large-amplitude oscillation of $W_{gg}$ as observed in Fig. 1(a). The other terms in the series are oscillating at the frequency $2\Omega_{n_0,0}$ so that a smaller-amplitude revival at $\frac{t_c}{t_R}$ is observed at $gt_R \approx 0.9$ as expected. On the other hand, for $W_{eg}$ only the frequencies $2\Omega_{n_0,0}$ contribute and the revival occurs as expected at $gt_R \approx 0.9$. It is observed that, with the increase in detuning there is a small gradual decrease in the amplitude of the Rabi frequencies $\Omega_{n_1,0}$ and $2\Omega_{n_1,0}$ but the periodicity of the revivals is maintained.
obtain

\[
\Omega_{n_1,n_2} = \Omega_{\bar{n}_1,\bar{n}_2} + 2g \frac{((\bar{n}_1 + 1)(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)(\bar{n}_2 + 2)^2)}{\sqrt{2((\bar{n}_1 + 1)^2(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2)^2)}} (n_1 - \bar{n}_1)
+ 2g \frac{((\bar{n}_1 + 1)^2(\bar{n}_2 + 1) + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2))}{\sqrt{2((\bar{n}_1 + 1)^2(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2)^2)}} (n_2 - \bar{n}_2) + \cdots.
\]

For the terms oscillating at \(\Omega_{n_1,n_2}\), revivals occur at times \(t_R\) when

\[
2(gt_R) \frac{(\bar{n}_1 + 1)(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)(\bar{n}_2 + 2)^2}{\sqrt{2((\bar{n}_1 + 1)^2(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2)^2)}} = 2\pi k, \quad k = 0, 1, 2, \ldots, \quad (30)
\]

\[
2(gt_R) \frac{(\bar{n}_1 + 1)^2(\bar{n}_2 + 1) + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2)}{\sqrt{2((\bar{n}_1 + 1)^2(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2)^2)}} = 2\pi l, \quad l = 0, 1, 2, \ldots. \quad (31)
\]

Multiplying these equations together, we obtain

\[
(2gt_R)^2 \frac{(\bar{n}_1 + 1)(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)(\bar{n}_2 + 2)^2}{2((\bar{n}_1 + 1)^2(\bar{n}_2 + 1)^2 + (\bar{n}_1 + 2)^2(\bar{n}_2 + 2)^2)} = 4\pi^2 kl, \quad kl = 0, 1, 2, \ldots
\]

For large \(\bar{n}_1\) and \(\bar{n}_2\), we get

\[
(2gt_R)^2 \bar{n}_1 \bar{n}_2 = 4\pi^2 m, \quad m = 0, 1, 2, \ldots, \quad (32)
\]

or

\[
\frac{\pi \sqrt{m}}{\sqrt{n_1} \sqrt{n_2}} = g t_R, \quad m = 0, 1, 2, \ldots. \quad (33)
\]

FIG. 2. (Color online) Time evolutions of \(W_{gg}(t)\) and \(W_{eg}(t)\) for (a) \(\bar{n}_1 = 15, \bar{n}_2 = 5\); (b) \(\bar{n}_1 = 30, \bar{n}_2 = 30\).
Dividing Eqs. (30) and (31), for large \( \bar{n}_1, \bar{n}_2 \) we obtain

\[
\frac{\bar{n}_2}{\bar{n}_1} = \frac{k}{l}.
\]

(34)

For a single atom interacting with an intensity-dependent two-mode quantized radiation field \([10]\) the Rabi frequency \( \Omega_{SA} = 2g(\bar{n}_1 + 1)(\bar{n}_2 + 1) \) and in the limit \( \bar{n}_1 \gg 1, \bar{n}_2 \gg 1 \) we obtain an equivalent expression:

\[
(gt_R)_{SA} = \frac{\pi \sqrt{m}}{\sqrt{\bar{n}_1 \bar{n}_2}}, \quad m = 0, 1, 2, \ldots.
\]

Equation (33) predicts the sequence of revivals at \( gt = 0.36, 0.51, 0.62, 0.72, \ldots \) for \( \bar{n}_1 = 15, \bar{n}_2 = 5 \). The graphs do not always show revivals because of violations of the periodic character present in the time evolution is lost, showing chaotic-like behavior.

IV. MEAN NUMBER OF PHOTONS IN THE MODES

Let us examine the time evolution of the average photon number in each of the modes. The mean number of photons in the modes can be calculated as given below:

\[
\langle a_i^\dagger(t) a_i(t) \rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_1 P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{n_1,n_2}^{n_1,n_2}(t)|^2 + 2 \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + 1) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{n_1,n_2}^{n_1,n_2}(t)|^2
\]

\[+ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + 2) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{n_1,n_2}^{n_1,n_2}(t)|^2, \]

(36)

For \( \bar{n}_1 = 15, \bar{n}_2 = 5 \), Eq. (35) presents the sequence of revivals at \( .18, .26, .31, .36, \ldots \). The graphs do not always show revivals because of violations of the periodic character present in the time evolution is lost, showing chaotic-like behavior.

\[
\langle a_i^\dagger(t) a_j(t) \rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_2 P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{n_1,n_2}^{n_1,n_2}(t)|^2 + 2 \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_2 + 1) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{n_1,n_2}^{n_1,n_2}(t)|^2
\]

\[+ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_2 + 2) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{n_1,n_2}^{n_1,n_2}(t)|^2. \]

(37)

Figure 3(a) depicts the time dependences of the mean number of photons in mode 1 and mode 2 for \( \bar{n}_1 = 15 \) and \( \bar{n}_2 = 5 \). Similar to the case of a single atom interacting with an intensity-dependent quantized bimodal field \([10]\) (keeping one of the modes in the coherent state while the other one in the vacuum state); here also we observe the same strict periodicity in the collapse and revival of the mean number of photons as observed for the atomic probabilities. This is because the time behavior of the mean number of photons is determined by the same harmonic time functions with frequencies \( \Omega_{n_1,n_2} \) and \( 2\Omega_{n_1,n_2} \). Contrary to the case of the single atom, here we observe the presence of two types of Rabi oscillations in the dynamics, one at smaller amplitude than the other. These are attributed to one- and two-photon processes, respectively.

In Fig. 3(b) we plot \( \bar{n}_1(t) \) and \( \bar{n}_2(t) \) for the case when \( \bar{n}_1 = 30, \bar{n}_2 = 30 \). In this case also we see that the periodic character is gradually lost.

V. FIELD STATISTICS

We now study the dynamics of the field statistics of our system, paying particular attention to the production of states of the field exhibiting nonclassical properties. In particular, we examine the possible production of antibunched light, correlation between the two modes, and violation of the Cauchy-Schwarz inequality.

To analyze the statistical properties of the field modes, we need to calculate the second-order correlation function

\[
G_{ij}^{(2)}(t) = \frac{\langle a_i^\dagger(t) a_j^\dagger(t) a_j(t) a_i(t) \rangle}{\langle a_i^\dagger(t) a_i(t) \rangle \langle a_j^\dagger(t) a_j(t) \rangle}, \quad i, j = 1, 2. \]

(38)

The correlation functions \( G_{11}^{(2)}(t) \) and \( G_{22}^{(2)}(t) \) characterize the second-order coherence in modes 1 and 2, respectively, whereas \( G_{12}^{(2)}(t) \) corresponds to intermode correlation.

The second-order correlation functions \( G_{ii}^{(2)}(t) \) can be represented as

\[
G_{ii}^{(2)}(t) = 1 + \langle [\Delta N_i(t)]^2 \rangle, \quad i = 1, 2 \]

(39)

where \( \langle N_i \rangle = \langle a_i^\dagger(t) a_i(t) \rangle \) is the mean number of photons in the \( i \)th mode and \( \langle [\Delta N_i(t)]^2 \rangle \) is the normally ordered uncertainty of the number of photons in the \( i \)th mode. The light is nonclassical, exhibiting sub-Poission statistics, whenever \( G_{ii}^{(2)} < 1 \) or equivalently whenever \( \langle [\Delta N_i(t)]^2 \rangle < 0 \). Here we are actually calculating...
and \( \bar{n} \) satisfying these conditions are referred to as antibunched. The zero-time-delay coherence function; hence the states satisfying these conditions are referred to as antibunched.

We obtain the following expressions for the quantities \( \langle a_i^\dagger(t)a_i^\dagger(t)a_i(t)a_i(t) \rangle (i = 1,2) \):

\[
\langle a_i^\dagger(t)a_i^\dagger(t)a_i(t)a_i(t) \rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_1(n_1 - 1) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{11-n_1}^{n_2}(t)|^2 + 2 \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + 1) n_1 P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{01-n_1}^{n_2}(t)|^2 \\
+ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + 2)(n_1 + 1) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{11-n_1}^{n_2}(t)|^2,
\]

\[
\langle a_i^\dagger(t)a_i^\dagger(t)a_i(t)a_i(t) \rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_2(n_2 - 1) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{11-n_1}^{n_2}(t)|^2 + 2 \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_2 + 1) n_2 P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{01-n_1}^{n_2}(t)|^2 \\
+ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_2 + 2)(n_2 + 1) P_{n_1}(\bar{n}_1) P_{n_2}(\bar{n}_2) |A_{11-n_1}^{n_2}(t)|^2.
\]

Using Eqs. (36) and (37) for \( \langle a_i^\dagger(t)a_i(t) \rangle (i = 1,2) \) we obtain numerical estimates for the quantities \( : [\Delta N_i(t)]^2 : \). In Fig. 4, we plot the time dependence of the quantity \( : [\Delta N_1(t)]^2 : \) and \( : [\Delta N_2(t)]^2 : \) with the coherent initial state of the cavity field and \( \bar{n}_1 = 8, \bar{n}_2 = 0 \).

We observe a strict periodicity in the collapses and revival pattern as seen in the case of atomic inversion. It is observed that the uncertainty in the number of photons becomes negative in the transient regime and then at regular interval of time \( gt = 1.98, 3.97, 5.94, 7.92, \ldots \). It
corresponds to a nonclassical state with sub-Poisson statistics at the corresponding moments. At the corresponding short-time moments we may assume the antibunching of photons.

For the sake of comparison we refer to Eq. (34) of our earlier paper [10] and write down the following expressions required for the numerical estimation of \( \langle (\Delta N)^2 \rangle \) for single-atom two-mode intensity-dependent processes. For the single-atom case the general state vector for the two atoms initially in the excited state and the field in the two-mode coherent state is expressed as

\[
|\psi(t)\rangle_{SA} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) [a_{n_1,n_2}(t)|a,n_1,n_2\rangle + b_{n_1+1,n_2+1}(t)|b,n_1 + 1,n_2 + 1\rangle],
\]

so that we obtain the following expressions for a single-atom two-mode process:

\[
\langle a_{1}(t)a_{1}(t)\rangle_{SA} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_1 P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |a_{n_1,n_2}|^2 + \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + 1) P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |b_{n_1+1,n_2+1}|^2,
\]

\[
\langle a_{1}(t)a_{1}^\dagger(t)a_{1}(t)a_{1}(t)\rangle_{SA} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_1(n_1 - 1) P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |a_{n_1,n_2}|^2 + \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + 1)n_1 P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |b_{n_1+1,n_2+1}|^2,
\]

\[
\langle a_{2}(t)a_{2}(t)\rangle_{SA} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_2 P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |a_{n_1,n_2}|^2 + \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_2 + 1) P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |b_{n_1+1,n_2+1}|^2,
\]

\[
\langle a_{2}(t)a_{2}^\dagger(t)a_{2}(t)a_{2}(t)\rangle_{SA} = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_2(n_2 - 1) P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |a_{n_1,n_2}|^2 + \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_2 + 1)n_2 P_{n_1}(\tilde{n}_1) P_{n_2}(\tilde{n}_2) |b_{n_1+1,n_2+1}|^2,
\]

where

\[
|b_{n_1+1,n_2+1}|^2 = \sin^2(\sqrt{t}(n_1 + 1)(n_2 + 1)),
\]

\[
|a_{n_1,n_2}|^2 = 1 - \sin^2(\sqrt{t}(n_1 + 1)(n_2 + 1)).
\]

For the single-atom case, we have plotted the numerical estimates for the normally ordered variances defined through Eq. (40) in Fig. 5. It is observed that the uncertainty in the normally ordered variance is always negative except in the transient regime and later at short-time regular intervals. This indicates that in the one-atom case the cavity field mainly exhibits a nonclassical state with sub-Poisson statistics, exhibiting the antibunching of photons.

Also of interest is the degree of intermode second-order coherence determined by

\[
C_{12}(t) = \frac{\langle a_{1}(t)a_{2}^\dagger(t)a_{2}(t)a_{1}(t)\rangle}{\langle a_{1}(t)a_{1}(t)\rangle\langle a_{2}(t)a_{2}(t)\rangle} = 1 + \frac{C(t)}{\langle a_{1}(t)a_{1}(t)\rangle\langle a_{2}(t)a_{2}(t)\rangle},
\]

with \( C(t) \) given by Eq. (40).
are anticorrelated (process [10] and in Fig. 7 present the plot of function $C_t$ for numerical estimates, we use the cross-correlation Hanbury Brown-Twiss type of experiment with two beams is proportional to the excess coincidence counting rate for a cross-correlation function $C_t$ where the cross-correlation function is always positive except for short time intervals. At the corresponding short time intervals, where $C_t$ is negative, the beams become anticorrelated.

We observe the same strict periodicity in the collapse and revival as observed in the case of atomic probabilities. In the model under study, pairs of photons are emitted and absorbed. This leads to a strong correlation of photons. This is verified by the numerical calculation of the cross-correlation function which is always positive except for short time intervals. At the corresponding short time intervals, where $C_t$ is negative, the beams become anticorrelated.

At this point we write down the expression for $\langle a_1(t)a_2(t)a_2(t)a_1(t) \rangle$ for single-atom two-mode two-photon process [10] and in Fig. 7 present the plot of $C_t$ versus $gt$ for and $\bar{n}_1 = \bar{n}_2 = 0$, respectively:

$$\langle a_1(t)a_2(t)a_2(t)a_1(t) \rangle_S$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1+1)(n_2+1)P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) |a_{n_1,n_2}|^2$$

$$+ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1+2)(n_2+2)P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) |a_{n_1,n_2}|^2.$$

Figure 6 depicts the numerically calculated cross-correlation function $C_t$ for the model for $\bar{n}_1 = \bar{n}_2 = 0$. We observe the same strict periodicity in the collapse and revival as observed in the case of atomic probabilities. In the model under study, pairs of photons are emitted and absorbed. This leads to a strong correlation of photons. This is verified by the numerical calculation of the cross-correlation function which is always positive except for short time intervals. At the corresponding short time intervals, where $C_t$ is negative, the beams become anticorrelated.

At this point we write down the expression for $\langle a_1(t)a_2(t)a_2(t)a_1(t) \rangle$ for single-atom two-mode two-photon process [10] and in Fig. 7 present the plot of $C_t$ versus $gt$ for and $\bar{n}_1 = \bar{n}_2 = 0$, respectively:

$$\langle a_1(t)a_2(t)a_2(t)a_1(t) \rangle_S$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1+1)(n_2+1)P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) |a_{n_1,n_2}|^2$$

$$+ \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1+2)(n_2+2)P_{n_1}(\bar{n}_1)P_{n_2}(\bar{n}_2) |a_{n_1,n_2}|^2.$$

It is observed that the cross-correlation function becomes more positive in the two-atom case as compared to the case of a single atom. We thus obtain evidence of cooperative behavior in the production of correlation of light beams.

Finally, we consider the Cauchy-Schwarz inequality

$$|C_t^{(2)}|^2 \leq G_t^{(1)}G_t^{(2)}.$$
DYNAMICS FOR TWO ATOMS INTERACTING WITH...  

PHYSICAL REVIEW A 86, 023810 (2012)

which is violated by nonclassical states, indicating a nonclassical correlation between the beams. We define the quantity [5]

\[ V(t) = \langle a_1^\dagger(t)a_2^\dagger(t)a_2(t)a_1(t) \rangle^2 - \langle a_1^2(t)a_2^2(t) \rangle \langle a_1^\dagger(t)a_2^\dagger(t) \rangle. \] (54)

Whenever \( V(t) > 0 \), the Cauchy-Schwarz inequality is violated, indicating a nonclassical correlation between the two modes.

In Fig. 8, we plot \( V(t) \) versus \( gt \) for \( \bar{n}_1 = 8, \bar{n}_2 = 0 \). We see that \( V(t) \) is positive except for a very short time interval in the transient regime and later at short regular intervals of time \( gt = 1.98, 3.97, 5.94, 7.92, \ldots \). It can be understood in terms of the quantum nature of the initial vacuum state for one of the cavity modes.

FIG. 7. (Color online) Plot of \( C(t) \) vs \( gt \) for \( \bar{n}_1 = 8, \bar{n}_2 = 0 \) for the single-atom two-mode ladder model with ID coupling.

FIG. 8. (Color online) Plot of \( V(t) \) vs \( gt \) for \( \bar{n}_1 = 8, \bar{n}_2 = 0 \) for the two-atom two-mode ladder model with ID coupling.

FIG. 9. (Color online) Plot of \( V(t) \) vs \( gt \) for \( \bar{n}_1 = 8, \bar{n}_2 = 0 \) for the single-atom two-mode ladder model with ID coupling.

Figure 9 depicts the time dependence of \( V(t) \) for \( \bar{n}_1 = 8, \bar{n}_2 = 0 \) for a single-atom two-mode process with intensity-dependent coupling. From a comparison of Figs. 8 and 9, we see that the Cauchy-Schwarz inequality is less strongly violated in the two-atom case as compared to the one-atom case.

VI. DISCUSSION AND SUMMARY

We have discussed the quantum dynamics for a collective model of two identical two-level atoms interacting with a quantized bimodal field with intensity-dependent coupling terms. We have studied the dynamics of the atomic level populations and have shown the existence of distinct type of revivals in the time records of the Rabi oscillations. The expressions for collapse time and revival time have been obtained and the locations of revivals have been explained. Moreover, some aspects of photon statistics have been examined. We have observed evidence of cooperative behavior in the production of correlations in the light beams. However, we find that the Cauchy-Schwarz inequality is less strongly violated in the two-atom case as compared to the one-atom case. Although the collapse and revival in the inversion and the mean photon number are also found in the single-atom case, the two-atom case serves as the simplest many-body system that shows interesting collective effects in the quantum dynamics.

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