Collapse and revivals in the Jaynes-Cummings model: An analysis based on the Mollow transformation

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Collapse and revivals in the Jaynes-Cummings model are studied within the context of the Mollow transformation for a single-mode coherent-state cavity field. The Mollow-transformed Jaynes-Cummings Hamiltonian has two atom-field terms, one corresponding to a classical field and the other to a quantized field driving the atomic transition. As such it maps onto the complementary problem of an atom in an optical cavity, whose field is initially in the vacuum state, driven by an external classical field. Both problems have the same atomic-state dynamics. It is shown that the revivals can be associated with two distinct properties of the quantized field of the transformed Hamiltonian. Revivals occurring at even (odd) multiples of the revival time are correlated with field states that are close to the initial field state and for which the average energy in the field is a minimum (maximum). Using semiclassical dressed states, we are able to map the problem for the field states onto one involving two uncoupled oscillators driven by off-resonant fields, provided effects related to dispersion are neglected. Analytic expressions for the photon number distribution of the transformed cavity field, including dispersion, are obtained for the even revival times.

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I. INTRODUCTION

The Jaynes-Cummings Hamiltonian [1], which is used to model the interaction of a two-level atom with a single-mode quantized cavity field, plays a central role in quantum optics. In the limit that the cavity field is prepared in a coherent state having average photon number much greater than unity, one might think that the atom-field dynamics could be calculated to a good approximation by replacing the cavity field with a classical one, resulting in undamped Rabi oscillations of the atomic-state probabilities. Although semiclassical approximations of this nature are often valid for atoms interacting with fields in free space, they fail for atom–cavity-field interactions when the interaction time is greater than the inverse of the atom–cavity-field coupling strength, measured in frequency units. As one measure of this failure, the atomic-state probabilities in the Jaynes-Cummings model undergo a series of collapses and revivals, a dynamics that is attributed solely to the quantum nature of the field. An excellent review of the Jaynes-Cummings problem was given by Yoo and Eberly [2].

It is reasonable to believe that just about everything must be known about the Jaynes-Cummings solution. Of course, the mathematical solution and its various limits have been discussed in detail. However, the interpretation of the collapse and revivals is not a closed subject. This question of interpretation becomes even more interesting when one considers a variant of the Jaynes-Cummings Hamiltonian based on a unitary transformation introduced by Mollow [3]. The unitary transformation involves a translation of the field operators and leads to a Hamiltonian that contains two atom-field terms, one associated with a time-dependent classical field interacting with the atom and the other with the quantized field interacting with the atom. This transformation is especially useful when the cavity field is prepared initially in a coherent state since, in that limit, the transformed initial state of the field is the vacuum state. The discussion is limited to a cavity field that is resonant with the atomic transition.

Mollow introduced this transformation to show formally that the problem of light scattering in intense fields can be solved without approximation by using a classical input field rather than a quantized coherent state of the field. Moreover, the scattering calculation can be simplified using such an approach. It might appear that a similar approach could be used to simplify the Jaynes-Cummings problem (which, admittedly, is simple in its standard form). However, this conclusion cannot be further from the truth. For the Jaynes-Cummings problem, the Mollow transformation converts an exactly solvable problem [within the rotating-wave approximation (RWA)] into one that must be solved numerically. The reason for this difficulty arises from an amazing feature of the Mollow transformed Hamiltonian. The classical field in the transformed Hamiltonian never changes and, as such, can provide an infinite source of photons for the quantum field. Although this transformed quantized field is not the true quantized field in the cavity, the atomic dynamics for both the original and transformed Hamiltonians is identical since the unitary Mollow transformation contains field operators only. As a consequence, the atomic dynamics can be calculated using either Hamiltonian.

Although the solution is more difficult in this transformed basis, it might provide new insight or offer an alternative explanation of the collapse and revivals. With this motivation, Knight and Radmore [4] used the Mollow transformed Jaynes-Cummings Hamiltonian to study collapse and revivals. They interpreted the collapse and revivals as resulting from an interference between the Rabi oscillations associated with the classical field and the quantum oscillations associated with the cavity field. Unfortunately, the solutions presented...
II. STANDARD JAYNES-CUMMINGS MODEL

In the RWA, the Jaynes-Cummings Hamiltonian for a single-mode cavity field interacting resonantly with a stationary two-level atom (lower state $|1\rangle$ and upper state $|2\rangle$) can be taken as

$$H = \frac{\hbar \omega}{2} \sigma_\uparrow + \hbar \omega a^\dagger a + \hbar g (\sigma_\uparrow a + a^\dagger \sigma_\downarrow),$$

where $\omega$ is both the atomic transition and cavity-field frequency; $g$ is a coupling constant (assumed real); $\sigma_\uparrow = (|2\rangle\langle 1| - |1\rangle\langle 2|)$, $\sigma_\downarrow = |2\rangle\langle 1|$, and $\sigma_\uparrow$ and $\sigma_\downarrow$ are atomic raising and lowering operators; and $a$ and $a^\dagger$ are field destruction and creation operators. The state vector for the atom-field system in an interaction representation is written as

$$|\psi(t)\rangle = c_{1,0}(t)e^{i \omega t/2}|1;0\rangle$$

$$+ \sum_{n=1}^{\infty} [c_{1,n}(t)|1;n\rangle + c_{2,n-1}(t)|2;n-1\rangle]e^{-i \omega nt} e^{i \omega t/2},$$

where the first index in each ket refers to the atomic state and the second to the field state. We have separated off the state $|1;0\rangle$ since this is an eigenstate of the system in the RWA. For states other than $|1;0\rangle$, the Hamiltonian is $2 \times 2$ block diagonal in the states $|1;n\rangle$ and $|2;n-1\rangle$. For any $n \geq 0$ and with $c_{2,-1}(t)$ set equal to zero, it is then a simple matter to obtain the solutions

$$\begin{pmatrix} c_{1,n}(t) \\ c_{2,n-1}(t) \end{pmatrix} = \begin{pmatrix} \cos(g_n t) & -i \sin(g_n t) \\ -i \sin(g_n t) & \cos(g_n t) \end{pmatrix} \begin{pmatrix} c_{1,0}(0) \\ c_{2,0}(0) \end{pmatrix},$$

with

$$g_n = \sqrt{n}g.$$  

The initial condition at $t = 0$ is taken as

$$|\psi(0)\rangle = |1\rangle |\alpha\rangle,$$

where

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n e^{-\omega^2 t/2}}{\sqrt{n!}}$$

is a coherent state with $\alpha$ assumed to be real. As a consequence, the initial-state amplitudes are

$$c_{1,0}(0) = \frac{\alpha e^{-\omega^2 t/2}}{\sqrt{n!}}, \quad c_{2,0-1}(0) = 0,$$

leading to time-dependent state amplitudes that are given by

$$c_{1,n}(t) = \frac{\alpha^n e^{-\omega^2 t/2}}{\sqrt{n!}} \cos(g_n t),$$

$$c_{2,n-1}(t) = -i \frac{\alpha^n e^{-\omega^2 t/2}}{\sqrt{n!}} \sin(g_n t).$$

The atomic-ground-state probability $P_1(t)$ is

$$P_1(t) = \sum_{n=0}^{\infty} |c_{1,n}(t)|^2 = \frac{1}{2} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{\alpha^{2n} e^{-\omega^2 t}}{n!} \cos(2g_n t).$$

This expression is evaluated easily and exhibits the well-known collapse and revivals associated with the Jaynes-Cummings model, as shown in Fig. 1. In what follows, we assume that $\alpha \gg 1$. 

by Knight and Radmore are valid only for very early times, before the first collapse. The actual solution for later times differs dramatically from those presented in their paper. The reason is simple: they used a truncated basis with at most six states of the field included, while more than fifty states are needed for the parameters they used. This is not meant as a criticism of their work, which was carried out at a time when computer capabilities were more limited. Instead, it can be viewed as providing motivation for a new examination of this problem using the Mollow transformed Hamiltonian. We shall see that a different interpretation of the revivals emerges, based on the quantum state of the field. Moreover, in certain limits, the problem can be mapped onto one involving two uncoupled oscillators. We find that there are two fundamentally different types of revivals, one in which the field returns nearly to its initial vacuum state and one in which the field achieves its maximum average value (the fields referred to are the transformed quantum fields, not the true quantum fields).

Apart from giving the same atomic-state dynamics as the Jaynes-Cummings Hamiltonian, the Mollow transformed Hamiltonian actually corresponds to the complementary problem of an atom in an optical cavity driven by an external classical field, for initial conditions in which the atom is in its ground state and the cavity field in the vacuum state. In other words, the Mollow-transformed Hamiltonian is the appropriate semiclassical Hamiltonian for an atom in a cavity driven by an external classical field. It is within this context that Chough and Carmichael [5] studied the Mollow transformed Jaynes-Cummings Hamiltonian (without referring to it as such) using an approach based on the dynamics of a nonlinear quantum oscillator. They were concerned mainly with the fine structure of the revival dynamics that occurs for very long times. While they use a framework that is related to the one presented in this paper, the spirit and emphasis of the two calculations differ considerably.

In particular, we find it convenient to analyze the problem using a semiclassical-dressed-state basis. In such a basis, the Mollow-transformed Hamiltonian resembles that of coupled oscillators. If effects related to dispersion are neglected, the system reduces to that of two uncoupled oscillators driven by off-resonant fields. Using this approach, we are able to obtain approximate analytic expressions for the photon statistics of the field in certain limits.

Before carrying out the calculation in the transformed basis, we first recall the results of the standard Jaynes-Cummings solution, offering a slightly different interpretation of the collapse and revivals than is given traditionally. In Sec. III, the problem is approached using the Mollow transformation, where some approximate analytic solutions are presented along with numerical results. The paper concludes with some additional discussion of the Mollow transformation and its potential applications.
is sharply peaked about $\alpha g t$ for $\alpha \gg 1$. The distribution
\begin{equation}
W_n(\alpha) = \frac{\alpha^{2n} e^{-\alpha^2}}{n!}
\end{equation}
is sharply peaked about $n = \alpha^2 = \bar{n}$. To provide some interpretation to the collapse and revivals, we expand
\begin{equation}
g_n = \sqrt{n} g = \sqrt{n} g_t = (n - \alpha^2) g
\end{equation}
$$
\approx \alpha g + \frac{1}{2} \frac{(n - \alpha^2)^2}{\alpha^2} g - \frac{1}{8} \frac{(n - \alpha^2)^3}{\alpha^3} g
$$
such that
\begin{equation}
P_1(t) \approx \frac{1}{2} + \frac{1}{2} \sum_{n=0}^{\infty} W_n(\alpha) \cos \left(2\chi t + \frac{n - \alpha^2}{\alpha^2} g t\right) - \frac{1}{4} \frac{(n - \alpha^2)^2}{\alpha^3} g t\),
\end{equation}
where
\begin{equation}
\chi = g \alpha.
\end{equation}
The neglect of higher-order terms in the expansion (10) is justified only for times $t \ll 8\alpha^2/g$. If the second two terms in the argument of the cosine function in Eq. (11) are neglected, the atomic dynamics is that of an atom interacting with a classical field, in which $\chi$ is one-half the Rabi frequency associated with the atom–classical-field interaction. In that limit, $P_1(t) = \cos^2(\chi t)$ exhibits undamped Rabi oscillations.

To arrive at an integral expression for the probability (11), we use the large argument limit of a Poissonian distribution to transform
\begin{equation}
\sum_{n=0}^{\infty} W_n(\alpha) f(n - \alpha^2) \rightarrow \int_{-\infty}^{\infty} W(y) f(y) dy,
\end{equation}
where
\begin{equation}
W(y) = \frac{e^{-y^2/2\alpha^2}}{\sqrt{2\pi\alpha}}.
\end{equation}

In this limit, Eq. (11) reduces to
\begin{equation}
P_1(t) \approx \frac{1}{2} + \frac{1}{2} \int_{-\infty}^{\infty} W(y) \cos \left(2\chi t + \frac{y}{\alpha^2} g t - \frac{1}{4} \frac{y^2}{\alpha^2} g t\right) dy.
\end{equation}
The integral term is non-negligible only for times
\begin{equation}
t \lesssim t_c = 2/g
\end{equation}
and for such times the $1/y^2 g t$ term in Eq. (15) can be neglected. As a consequence,
\begin{equation}
P_1(t) \approx \frac{1}{2} + \frac{1}{2} \int_{-\infty}^{\infty} W(y) \cos(2\chi t) \cos \left(\frac{y}{\alpha^2} g t\right) dy
\end{equation}
\begin{equation}
= \frac{1}{2} \left[ 1 + \cos(2\chi t) e^{-2i\chi t^2} \right].
\end{equation}
It is clear that $t_c$ is a collapse time. Moreover, the integral term in Eq. (17) is identical in form to that encountered for the free precession decay signal of an inhomogeneously broadened sample of atoms. Thus the collapse can be understood in the context of inhomogeneous broadening of an ensemble of radiators.

Equation (17) gives collapse, but no revivals. To obtain the revivals, the discrete nature of the quantum states must be included, at least in the second term of the expansion (10). That is, we must evaluate
\begin{equation}
P_1(t) = \frac{1}{2} + \frac{1}{4} \sum_{n=1}^{\infty} W_n(\alpha) e^{2i\chi t} \exp \left( -i t n^2 q / 4\alpha^4 \right).
\end{equation}
where c.c. stands for complex conjugate, before the replacement (13) is used. It is clear that whenever
\begin{equation}
t = t_{r,q} = 2\pi q\alpha / g = 2\pi q \alpha / g^2, \quad q = 1, 2, 3, \ldots,
\end{equation}
there is a rephasing of the signal since $\exp(i n t_{r,q} / \alpha) = 1$. The rephasing is not complete, however, owing to the fact that the quadratic dispersive term in Eq. (18) is not negligible at $t = t_{r,q}$, even for $q = 1$. Near the revival times, that is, when
\begin{equation}
t = t_{r,q} + \tau_q
\end{equation}
and when $\tau_q$ is of order $\pi q / g$, we can set $e^{i n t_{r,q} / \alpha} = 1$ in Eq. (18), convert the remaining sum to an integral using the prescription (13), and use the fact that $\alpha \gg 1$ to obtain
\begin{equation}
P_1(t = t_{r,q} + \tau_q) = \frac{1}{2} + \frac{1}{2} \frac{1}{(1 + q^2 \pi^2)^{1/4}} \cos \left[ 2\chi \tau_q + 2\pi q \alpha^2 + \frac{q\pi}{2(1 + q^2 \pi^2)} \tau_q^2 - \frac{1}{2} \frac{\pi}{\tan^{-1} (q\pi)} \right] \exp \left( -\frac{\tau_q^2}{2(1 + q^2 \pi^2)} \right).
\end{equation}

The revivals are broadened and reduced in amplitude by the dispersive term. An almost identical expression (lacking only the $-\tan^{-1}(q\pi)/2$ term) was obtained by Yoo and Eberly using the method of steepest descents [2]. Equation (21) is valid only for revival times $t_{r,q} \ll 8\alpha^2 / g$ or, equivalently, for $q \ll \alpha / g$; however, this is not a serious restriction since
the amplitude of the second term in Eq. (19) is small if \( q \ll \alpha/g \) and \( \alpha \gg 1 \). In the graph of \( P_1(t) \) shown in Fig. 1, the collapse and first four revivals can be seen.

The last line is valid only for \( \alpha \gg 1 \) since in that limit only terms having \( n \gg 1 \) contribute significantly to the sum. For times \( t \ll 8\alpha^2/g \) and for \( \alpha \gg 1 \), we can use the expansion (10) and keep only the leading term to obtain

\[
F(t) \approx \alpha \cos \left( \frac{\alpha t}{2\alpha} \right). \tag{23}
\]

The average field amplitude oscillates with a period on the order of the revival time. At odd revival times, \( F(t) = -\alpha \) and at even revival times \( F(t) = \alpha \). As for the average cavity-field energy \( U_f \), one finds

\[
U_f(t) = \hbar \omega (a^\dagger a) = \sum_{n=1}^{\infty} [n|c_{1,n}(t)|^2 + (n-1)|c_{2,n-1}(t)|^2]
\]

\[
= \hbar \omega [\alpha^2 - P_2(t)], \tag{24}
\]

where

\[
P_2(t) = 1 - P_1(t) \tag{25}
\]

is the excited-state probability. At any time, the average field energy is reduced by the average excitation energy of the atom. In the collapse region where \( P_1(t) = 1/2 \), the average field energy is constant and given by \( U_f(t) = \hbar \omega (\alpha^2 - 1/2) \). The average value of the interaction energy in Eq. (1) is equal to zero (the average interaction energy would not vanish if the cavity-field frequency were not resonant with the atomic transition frequency).

### III. JAYNES-CUMMINGS MODEL USING THE MOLLOW TRANSFORM

Following Mollow, we can transform the state vector as

\[
|\psi(t)\rangle' = D(\alpha e^{-iat})|\psi(t)\rangle,
\]

where \( D(\beta) \) is the translation operator defined by

\[
D(\beta) = e^{i(\beta a^\dagger - \beta^* a)} = e^{-\beta^2/2} e^{i\beta a^\dagger} e^{-\beta a} \tag{27}
\]

Under this transformation the Hamiltonian (1) is transformed into

\[
H'(t) = D(\alpha e^{-iat})H D(\alpha e^{iat})
\]

\[
= \frac{\hbar \omega}{2} \sigma_z + \hbar \omega a^\dagger a + \hbar \chi (\sigma_+ e^{-iat} + \sigma_- e^{iat})
\]

\[
+ \hbar g(\sigma_+ a + a^\dagger \sigma_-) \tag{28}
\]

It is also of some interest to calculate the average cavity-field amplitude and energy. The cavity-field amplitude is proportional to \( F(t) = \langle a \rangle e^{iat} \), which can be calculated using Eq. (2)

\[
= \alpha \sum_{n=0}^{\infty} W_n(\alpha) \left[ \cos(g_{n+1}t) \cos(g_nt) + \frac{\sqrt{n}}{n+1} \sin(g_{n+1}t) \sin(g_nt) \right] \approx \alpha \sum_{n=0}^{\infty} W_n(\alpha) \cos((\sqrt{n+1} - \sqrt{n})gt) \tag{22}
\]

and the new state vector evolves according to

\[
i\hbar \frac{d|\psi(t)\rangle'}{dt} = H'(t)|\psi(t)\rangle', \tag{29}
\]

subject to the initial condition

\[
|\psi(0)\rangle' = |1,0\rangle'. \tag{30}
\]

The initial state is now the vacuum state of the field, assuming the initial state of the field was a coherent state for the original Hamiltonian. Of course this is not the true initial state of the cavity field (which is a coherent state) since the field undergoes a transformation as well. On the other hand, any atomic-state probabilities or expectation values must be the same as those for the original Hamiltonian since the operator \( (27) \) acts only on field variables. We shall refer to the quantized field of the transformed Hamiltonian as a pseudofield.

As was pointed out in the Introduction, the Hamiltonian (28) with initial condition (30) actually corresponds to a complementary physical problem in which an atom in an optical cavity is driven by an external classical field. In this complementary problem the atom starts in its ground state and the field in the vacuum state. Thus, although we refer to the cavity field as a pseudofield for our specific problem, it would correspond to the actual field in the complementary problem. Both problems share the same atomic-state dynamics.

The Hamiltonian (28) has several interesting features. In addition to the quantized cavity pseudofield-atom interaction, there is now a term corresponding to a classical field interacting with the atom. The expression for the classical field is identified as the expectation value of the field operator in the original basis neglecting any atom-field interactions, assuming that the field is in a coherent state. The amplitude of this classical field in Eq. (28) does not change; consequently, it can provide an unlimited supply of photons for the pseudofield in the cavity. As a result, the problem is now considerably more difficult than in the original Jaynes-Cummings formulation since the combined action of the classical field and the quantized cavity pseudofield lead to an infinite number of coupled-field states. We can see this by expanding the state vector as

\[
|\psi(t)\rangle' = \sum_{n=1}^{\infty} [c_{1,n}(t)|1,n\rangle e^{iat/2} + c_{2,n}(t)|2,n\rangle e^{-iat/2}] e^{-i\omega t} \tag{31}
\]
Substituting this state vector into Eq. (29), we find that the state amplitudes obey the coupled equations
\begin{align}
\dot{c}_{1,n} &= -i\chi c_{2,n} - ig_n c_{2,n-1} \quad (32a) \\
\dot{c}_{2,n} &= -i\chi c_{1,n} - ig_{n+1} c_{1,n+1}, \quad (32b)
\end{align}
subject to the initial conditions
\begin{equation}
\dot{c}_{1,n}(0) = \delta_{n,0}, \quad \dot{c}_{2,n}(0) = 0,
\end{equation}
where \(\delta_{n,n}\) is a Kronecker delta. Equations (32) form an infinite set of coupled equations. Note that, for the initial conditions (33), it follows from Eqs. (32) that \(c_{1,n}(t)\) is purely real and \(c_{2,n}(t)\) is purely imaginary.

Equations (32) can be written in matrix form as
\begin{equation}
\dot{c}' = H' c',
\end{equation}
where \(c'\) is a column matrix using basis states \(c_{10}', c_{20}', c_{11}', c_{21}', c_{12}', c_{22}', \ldots\) and
\begin{equation}
(1n|\mathbf{H}|2n') = (2n'|\mathbf{H}'|1n) = h\chi \delta_{n,n'} + h g_n \delta_{n',n-1}
\end{equation}
with all other matrix elements equal to zero. Thus, the first eight rows and columns of \(\mathbf{H}'\) are
\begin{equation}
\mathbf{H}' = h
\begin{pmatrix}
0 & \chi & 0 & 0 & 0 & 0 & 0 & 0 \\
\chi & 0 & g & 0 & 0 & 0 & 0 & 0 \\
g & 0 & 0 & \chi & 0 & 0 & 0 & 0 \\
0 & \chi & 0 & \sqrt{2}g & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2}g & 0 & \chi & 0 & 0 & 0 \\
0 & 0 & 0 & \chi & 0 & \sqrt{3}g & 0 & \chi \\
0 & 0 & 0 & 0 & \sqrt{3}g & 0 & \chi & 0 \\
0 & 0 & 0 & 0 & 0 & \chi & 0 & 0
\end{pmatrix}
\end{equation}
(36)

The dimensions of the matrix \(\mathbf{H}'\) is \(2n_{\text{max}}\) if Eqs. (32) are truncated at some \(n = n_{\text{max}}\). As we shall see, to properly account for the system dynamics, it is necessary to include values of \(n \lesssim n_{\text{max}} = \max\{gt/2, (2\alpha)^2\}\), that is, to use a matrix having dimension at least of order \(2n_{\text{max}} = \max\{2gt/2, 2(2\alpha)^2\}\). In the example considered by Knight and Radmore [4], the value of \(\alpha\) is equal to 50 and the maximum value of \(gt\) is equal to 10, implying that a minimum matrix dimension of order 50 is needed. Knight and Radmore truncated the matrix at \(2n_{\text{max}} = 6\). In their Fig. 2 [4], the first revival occurs at \(gt = 1.7\), whereas in actuality the first revival occurs at \(gt \approx 314\) for their parameters.

The solution of Eq. (36) is
\begin{equation}
c'(t) = e^{-i\mathbf{H}'t/h} c'(0).
\end{equation}
(37)

This equation can be evaluated numerically. The ground-state population is
\begin{equation}
P_1(t) = \sum_{n=0}^{\infty} P_{1,n}(t),
\end{equation}
where
\begin{equation}
P_{1,n}(t) = |c_{1,n}'(t)|^2
\end{equation}
is the probability to be in state 1 and to have \(n\) photons in the cavity pseudofield.

A. Semiclassical dressed states

To make some headway towards an alternative explanation of the collapse and revivals, it is convenient to introduce semiclassical dressed states [6] that are eigenstates of \(\mathbf{H}'\) in the absence of coupling to the cavity pseudofield. These semiclassical dressed states differ in energy by \(2\chi\). Thus, even though the dressed states are coupled to each other via the cavity pseudofield, this coupling can be negligible if it is small compared with \(2\chi\). Moreover, it might be possible to approximate the off-resonant coupling between the semiclassical dressed states in lowest order as a light shift that each state produces on the other. In this limit, the problem reduces to the analytically soluble problem of an oscillator (the cavity pseudofield) driven by an off-resonant field, where the detuning of the driving field is determined by the light shift. Unfortunately, the reliability of this approach is questionable since the coupling to the cavity pseudofield grows as \(h\sqrt{n}\) and for the largest values of \(n = n_{\text{max}}\) that contribute we will see that \(h\sqrt{n_{\text{max}}} \approx 2h\chi\), invalidating the underlying assumption that the coupling to the pseudofield is much less than the energy separation of the semiclassical dressed states. Nevertheless, the semiclassical-dressed-state approach can provide some interesting insight into the collapse and revivals within the context of the cavity-pseudofield dynamics.

Semiclassical-dressed-state amplitudes are defined by [6]
\begin{equation}
\dot{c}'_{1,n} = \frac{c'_{1,n} - c'_{2,n}}{\sqrt{2}}, \quad \dot{c}'_{1,n} = \frac{c'_{1,n} + c'_{2,n}}{\sqrt{2}},
\end{equation}
(40)

The dressed-state amplitudes obey the coupled equations
\begin{align}
\dot{c}'_{1,n} &= i\chi c'_{1,n} + i \frac{g_n}{2} c'_{1,n-1} + i \frac{g_{n+1}}{2} c'_{1,n+1} \\
&\quad - i \frac{g_n}{2} c'_{2,n-1} - i \frac{g_{n+1}}{2} c'_{2,n+1} \quad (41a) \\
\dot{c}'_{1,n} &= -i\chi c'_{1,n} - i \frac{g_n}{2} c'_{1,n-1} - i \frac{g_{n+1}}{2} c'_{1,n+1} \\
&\quad + i \frac{g_n}{2} c'_{2,n-1} - i \frac{g_{n+1}}{2} c'_{2,n+1}, \quad (41b)
\end{align}
subject to the initial conditions
\[ c_{I,n}^I(0) = c_{I,n}^0(0) = \delta_{n,0}/\sqrt{2}. \] (42)

Note that it follows from Eqs. (40) and (42) that
\[ c_{I,n}^I(t) = c_{I,n}^I(t)^* \] (43)

Equations (41) can be written in matrix form as
\[ i\hbar \mathbf{c}^I_D = \mathbf{H}_D^I \mathbf{c}^I_D, \] (44)

where \( \mathbf{c}^I_D \) is a column matrix using basis states \( c_{I,0}^I, c_{I,1}^I, c_{I,2}^I, c_{I,3}^I, \ldots \)

\[
\begin{pmatrix}
-\chi & 0 & -\frac{\bar{g}}{2} & -\frac{\bar{g}}{2} \\
0 & \chi & \frac{\bar{g}}{2} & \frac{\bar{g}}{2} \\
-\frac{\bar{g}}{2} & \frac{\bar{g}}{2} & -\chi & 0 \\
0 & 0 & -\frac{\sqrt{2}\bar{g}}{2} & \frac{\sqrt{2}\bar{g}}{2}
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

As was stated above, the dressed states differ in frequency by
\[ 2\chi \] and are coupled by an interaction energy of order \( \hbar \bar{g}n. \)

The solution of Eq. (46) is
\[ \mathbf{c}^I_D(t) = e^{-i\mathbf{H}_D^I t/\hbar} \mathbf{c}^I_D(0). \] (47)

This equation can be evaluated numerically. The ground-state population is
\[ P_1(t) = \sum_{n=0}^{\infty} |c_{I,n}^I(t)|^2 = \frac{1}{2} + \text{Re} \left( \sum_{n=0}^{\infty} |c_{I,n}^I(t)|^2 \right). \] (48)

It will prove convenient to define amplitudes corresponding to an interaction representation for the Hamiltonian \( \mathbf{H}_D \), namely,
\[ c_{I,n}^I(t) = c_{I,n}^I(t)e^{i\chi t}, \quad c_{I,n}^I(t) = \tilde{c}_{I,n}(t)e^{-i\chi t}. \] (49)

In terms of these variables, which satisfy the coupled equations
\[
\begin{align*}
\dot{c}_{I,n}^I &= i \frac{\bar{g}_n}{2} c_{I,n-1}^I + i \frac{\bar{g}_{n+1}}{2} c_{I,n+1}^I \\
&\quad - i \frac{\bar{g}_n}{2} c_{I,n-1}^I - i \frac{\bar{g}_{n+1}}{2} c_{I,n+1}^I e^{-2i\chi t}, \quad (50a) \\
\dot{\tilde{c}}_{I,n}^I &= -i \frac{\bar{g}_n}{2} c_{I,n-1}^I - i \frac{\bar{g}_{n+1}}{2} c_{I,n+1}^I \\
&\quad + i \frac{\bar{g}_n}{2} c_{I,n-1}^I - i \frac{\bar{g}_{n+1}}{2} c_{I,n+1}^I e^{2i\chi t}.
\end{align*}
\] (50b)

the ground-state probability is given by
\[ P_1(t) = \frac{1}{2} + \text{Re} \left( \sum_{n=0}^{\infty} |c_{I,n}^I(t)|^2 e^{2i\chi t} \right). \] (51)

B. Collapse

We can understand the collapse of the excited-state probability oscillations by assuming that the amplitudes \( \tilde{c}_{I,n}^I \) and \( \tilde{c}_{I,n}^I \) are slowly varying functions of time compared with \( e^{i\chi t} \).

If Eqs. (50) are coarse-grained over a time interval of order \( 1/2\chi \), the terms varying as \( e^{i2\chi t} \) in Eqs. (50) average to zero and we arrive at
\[
\begin{align*}
\dot{\tilde{c}}_{I,n}^I &= i \frac{\bar{g}_n}{2} \tilde{c}_{I,n-1}^I + i \frac{\bar{g}_{n+1}}{2} \tilde{c}_{I,n+1}^I, \quad (52a) \\
\dot{\tilde{c}}_{I,n}^I &= -i \frac{\bar{g}_n}{2} \tilde{c}_{I,n-1}^I - i \frac{\bar{g}_{n+1}}{2} \tilde{c}_{I,n+1}^I.
\end{align*}
\] (52b)

These equations describe two uncoupled resonantly driven harmonic oscillators, for which the solution is [7]
\[ \tilde{c}_{I,n}^I = |\tilde{c}_{I,n}^I|^* = \frac{1}{\sqrt{2n!}} \left( i g t \right)^n e^{-g^2 t^2/8}. \] (53)

The classical field drives the cavity pseudofield up its ladder of harmonic-oscillator states. For a given time \( t \), the maximum value of \( n \) achieved is of order \( n_{\text{max}} = (gt/2)^2 \). We can check the assumption of slowly varying amplitudes by comparing \( |\tilde{c}_{I,n_{\text{max}}}^I|/\tilde{c}_{I,n_{\text{max}}} \sim g^2 t^2/4 \) with \( 2\chi \).

For times
\[ t \ll \frac{8\chi}{g^2} = \frac{8\bar{g}}{g}, \] (54)

that is, for times much less than the first revival time, the slowly varying approximation is valid for all \( n \) of importance.
In Eq. (51), in this limit,

\[ P_1(t) = \left(1 + \text{Re} \sum_{n=0}^{\infty} \left[ \frac{1}{\sqrt{2\pi n!}} \left( \frac{igt}{2} \right)^n e^{-g^2 t/8} \right]^2 e^{2i\chi t} \right]^{1/2} \]

in agreement with Eq. (17). The origin of the collapse in this picture is the alternating phase between successive contributions from different \( n \). In other words, the probability \( |c_{1,n}(t)|^2 \) appearing in Eq. (51) varies as \((-1)^n\).

**C. Revivals**

Since the slowly varying approximation fails for times greater than or of order of the first revival time, another approach is called for to examine the dynamics of the revivals. We shall see that this approach leads to expressions that require significantly less computer time to evaluate than Eq. (37) or (47). We start with the connection between \( c' \) and \( c \). Using Eqs. (26), (2), and (31), we find

\[ c'_{\mu,n} = \sum_{m=0}^{\infty} D_{mn}^*(\alpha e^{-i\omega t}) e^{i(n-m)\omega t} c_{\mu,m}, \quad \mu = 1, 2, \quad (56) \]

where

\[ D_{mn}(\alpha e^{-i\omega t}) = \langle n | e^{-\alpha^2/2} e^{i\omega t} e^{-i\omega t} | m \rangle \]

\[ = e^{-\alpha^2/2} (-\alpha)^{n-m} e^{i(n-m)\omega t} \frac{\sqrt{n!}m!}{\sqrt{\pi} (\alpha e^{-i\omega t})^{n-m}} \sum_{p=\max(0,m-n)}^{m} \frac{\alpha^{2p}(-1)^p}{(m-p)!p!(n-m+p)!}. \quad (57) \]

The sum represents a series expansion of the confluent hypergeometric function or generalized Laguerre polynomials.

\[ D_{mn}(\alpha e^{-i\omega t}) = \left\{ \begin{array}{ll}
L_m(\alpha^2), & m = n \\
\frac{\gamma^n}{\sqrt{\pi} \sqrt{\alpha^2}} L_{m-n}(\alpha^2), & m > n \\
\frac{\gamma^n}{\sqrt{\pi} \sqrt{\alpha^2}} L_{m}(\alpha^2), & m < n,
\end{array} \right. \quad (58) \]

where \( L_m(\alpha^2) \) is a Laguerre polynomial and \( L_m^p(\alpha^2) \) is a generalized Laguerre polynomial.

Using Eqs. (7), (56), and (57), we then find

\[ c'_{1,n}(t) = \sum_{m=0}^{\infty} D_{mn}(\alpha) \cos(g\sqrt{m}t) \frac{\alpha^m e^{-\alpha^2/2}}{\sqrt{m!}}, \quad (59a) \]

\[ c'_{2,n}(t) = -i \sum_{m=0}^{\infty} D_{mn}(\alpha) \sin(g\sqrt{m}+1)t \frac{\alpha^{m+1} e^{-\alpha^2/2}}{\sqrt{(m+1)!}}. \quad (59b) \]

In general, these expressions are numerically much less time consuming to evaluate than Eq. (37) or (47).

To gain some insight into the revivals, we assume that the combination \( D_{mn}(\alpha)\alpha^m e^{-\alpha^2/2}/\sqrt{m!} \) differs significantly from zero only in a range of order \( \alpha \) centered around \( m = \alpha^2 \) (this assumption was checked numerically). With this assumption, we use the expansion (10) to rewrite Eq. (59a) as

\[ c'_{1,n}(t) \sim \sum_{m=0}^{\infty} D_{mn}(\alpha) \cos \left[ \frac{1}{2} (m - \alpha^2)^2 \right] \frac{\alpha^m e^{-\alpha^2/2}}{\sqrt{m!}}. \quad (60) \]

Although dispersion is known to be important at the revival times, let us neglect it for the moment by dropping the \( \frac{1}{2} (m-\alpha^2)^2 \) term in Eq. (60). Hopefully, use of this assumption will give us a qualitative picture of the revivals. With this approximation,

\[ c'_{1,n}^{DF}(t) \sim \frac{e^{i\lambda t/2}}{2} \sum_{m=0}^{\infty} D_{mn}(\alpha) e^{i\lambda t/2} \frac{\alpha^m e^{-\alpha^2/2}}{\sqrt{m!}} + \text{c.c.}, \quad (61) \]

where the superscript DF stands for dispersion-free. The sum can be carried out analytically using Eq. (57). Setting

\[ \lambda = g/2\alpha = g^2/2\chi, \quad (62) \]

we obtain

\[ c'_{1,n}^{DF}(t) \sim \frac{e^{i\lambda t/2}}{2} \sum_{m=0}^{\infty} \sum_{p=\max(0,m-n)}^{m} \left\{ \frac{(-1)^{n-m+p} \alpha^n \sqrt{n!} \alpha^{2p} e^{i\lambda t}}{(m-p)!p!(n-m+p)!} + \text{c.c.} \right\}. \]

\[ = e^{-\alpha^2} e^{i\lambda t/2} \sum_{p=0}^{\alpha^2} \left\{ \frac{(-1)^{n-m+p} \alpha^n \sqrt{n!} \alpha^{2p} e^{i\lambda t}}{(m-p)!p!(n-m+p)!} + \text{c.c.} \right\}. \]

\[ = e^{-\alpha^2} e^{i\lambda t/2} \sum_{p=0}^{\alpha^2} \left\{ \frac{(-1)^{n-q} \alpha^n \sqrt{n!} \alpha^{2p} e^{i\lambda q t}}{(n-q)!q!(n-m+p)!} + \text{c.c.} \right\}. \]

\[ = e^{-\alpha^2} e^{i\lambda t/2} \sum_{p=0}^{\alpha^2} \left\{ \frac{(-1)^p \alpha^n \sqrt{n!} \alpha^{2p} e^{i\lambda q t}}{n!} + \text{c.c.} \right\}. \]

\[ = \left[ A(t) \right]^{\alpha^2} e^{-\alpha^2} \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \frac{(1-e^{i\lambda t})^n}{(n!)} + \text{c.c.}, \quad (63) \]
where

\[ A(t) = -\alpha(1 - e^{i\lambda t}), \tag{64a} \]
\[ \phi(t) = \frac{\alpha g t}{2} - i\alpha^2(1 - e^{i\lambda t}), \tag{64b} \]

and we have used the fact that \( \chi = \alpha g \).

From Eqs. (40), (43), (53), and (63) we then find that the dressed-state amplitudes are given by

\[ c_{\text{DF}}^{\alpha,n}(t) = \left[ c_{\text{DF}}^{\alpha,n}(t) \right]^* = \frac{[A(t)]^n e^{-i\phi(t)}}{\sqrt{2}\sqrt{n!}}. \tag{65} \]

This is a pretty interesting result since it represents the solution of the decoupled differential equations

\[ \dot{c}_{\text{DF}}^{\alpha,n}(t) = i\hbar \lambda c_{\text{DF}}^{\alpha,n}(t) + \frac{i}{2} \sum_{\delta n} \frac{g_{\delta n}}{2} c_{\text{DF}}^{\alpha,n+\delta}(t), \tag{66a} \]
\[ \ddot{c}_{\text{DF}}^{\alpha,n}(t) = -i\hbar \lambda c_{\text{DF}}^{\alpha,n}(t) - i\frac{g_{\delta n}}{2} c_{\text{DF}}^{\alpha,n+\delta}(t) - i\frac{\delta g_{\delta n}}{2} c_{\text{DF}}^{\alpha,n-\delta}(t), \tag{66b} \]

subject to the initial conditions (42). In an interaction representation, Eq. (66a) corresponds to the dynamics of a quantum oscillator driven by an external classical field that is detuned by \(-\lambda\) from the oscillator’s resonant frequency [7]. As a consequence, the qualitative behavior of the revivals can be understood in terms of the well-known dynamics of that problem. The dressed-state population

\[ P_{\text{DF}}^{\alpha,n}(t) = \left| c_{\text{DF}}^{\alpha,n}(t) \right|^2 \]
\[ = \frac{[2\alpha^2(1 - \cos \lambda t)]^n \exp[-2\alpha^2(1 - \cos \lambda t)]}{2n!} \tag{67} \]

is a periodic function having a period \(2\pi/\lambda = 4\pi\alpha/g\) that is twice the revival time, \(t_{r,1} = 2\pi\alpha/g\). The average occupation number is

\[ \langle n(t) \rangle^{\text{DF}} = \alpha^2[2(1 - \cos \lambda t)] \tag{68} \]

and the standard deviation is

\[ \Delta n(t)^{\text{DF}} = \alpha[2(1 - \cos \lambda t)]^{1/2}. \tag{69} \]

Note that \(\langle n(t) \rangle\) varies from zero (at the even revival times when \(\cos \lambda t = 1\)) to \(4\alpha^2\) (at the odd revival times when \(\cos \lambda t = -1\)), while the standard deviation \(\Delta n(t)\) varies from zero (at the even revival times) to \(2\alpha\) (at the odd revival times).

At first glance, it would appear that revivals occur only at the even revival times

\[ t_{r,q=2p} = 4\pi p\alpha/g = 4\pi p\chi/g^2, \quad p = 1, 2, 3, \ldots \tag{70} \]

since for such times the oscillator returns to its initial state \(c_{\text{DF}}^{\alpha,n}(t_{r,2p}) = \delta_{n,0}\), a result that follows from Eq. (63). That is, the even revivals can be explained by the return of the cavity pseudofield to its initial state.

However, in this model, revivals also occur for the odd revival times

\[ t_{r,q=2p-1} = 2\pi(2p - 1)\alpha/g = 2\pi(2p - 1)\chi/g^2, \quad p = 1, 2, 3, \ldots \tag{71} \]

although they differ qualitatively from the even revivals. To understand the odd revivals we look at Eq. (51) with \(c_{\text{DF}}^{\alpha,n}(t)\) given by Eq. (59a), plotted in Figs. 2–5 for \(\alpha = 10\) as a function of \(n\) for times corresponding to the first four revivals, \(q = 1–4\). The values plotted are the maximum values of \(|c_{\text{DF}}^{\alpha,n}(t)|^2\) at times closest to the revival times [owing to phase factors, \(P_{\text{DF}}^{\alpha,n}(t)\) is not necessarily a maximum at the exact revival times]. In Figs. 2 and 4, corresponding to odd revivals, the dispersion-free result \(P_{\text{DF}}^{\alpha,n}(t_{r,q}) = |c_{\text{DF}}^{\alpha,n}(t_{r,q})|^2\), with \(c_{\text{DF}}^{\alpha,n}(t_{r,q})\) given by Eq. (63), is superimposed on the graphs. In Figs. 3 and 5, corresponding to even revivals, the dispersion-free result is \(P_{\text{DF}}^{\alpha,n}(t_{r,q=2p}) = \delta_{n,0}\). The importance of dispersion is evident from the differences between the exact and dispersion-free results, but the general trend predicted by the dispersion-free model can be seen. The inset in Figs. 3 and 5 is the probability for small values of \(n\) shown on an expanded scale, along with

\[ FIG. 3. \text{(Color online)} \] Maximum value of the probability to find the atom in its ground state and \(n\) photons in the cavity pseudofield as a function of \(n\) near the second revival time, with \(\alpha = 10\). The dispersion-free result is \(P_{\text{DF}}^{\alpha,n}(t_{r,q}) = \delta_{n,0}\) and is not shown in the figure. The inset provides an expanded scale for small-\(n\) values. The solid (red) curve is the exact result and the dashed (blue) curve is an analytic approximation, including effects of dispersion.

\[ \text{given by Eq. (65), namely,} \]
\[ P_{1,\text{DF}}^{\alpha}(t) = \frac{1}{2} + \frac{1}{2} \text{Re} \left[ e^{2it}\lambda^{2} e^{-2i\phi(t)} \sum_{n=0}^{\infty} \frac{[\alpha(1 - e^{i\lambda t})]^{2n}}{n!} \right]. \tag{72} \]

For times other than those near the revival times, the phase of successive terms in the sum is random and the sum is approximately equal to zero. However, at the odd revival times [\(\lambda t_{r,2p-1} = (2p - 1)\pi\)], all terms add in phase and the sum is \(\exp[4\alpha^2]\), exactly canceling the \(\exp(-4\alpha^2)\) in the \(e^{-2i\phi(t)}\) factor. In other words, the odd revival times represent points of stationary phase, allowing states having different \(n\) to interfere constructively in the sum in Eq. (72). Carrying out the sum explicitly in Eq. (72), we obtain

\[ P_{1,\text{DF}}^{\alpha}(t) = \frac{1}{2} + \frac{1}{2} \text{cos}[\lambda t + \alpha^2 \sin(2\lambda t)] e^{-\alpha^2[1 - \cos(2\lambda t)]}. \tag{73} \]
the predictions of a model that includes the effects of dispersion (see below).

In the limit that $\alpha \gg 1$ and $n \ll \alpha^2$, it is possible to include the effects of dispersion in the expression for $P_{1,n}(t)$ given by Eq. (60) in a manner similar to that used to derive Eq. (21); however, the method fails for larger values of $n$, when $n$ approaches $\alpha^2$. Since small values of $n$ dominate $P_{1,n}(t)$ at the even revival times, this approach should work well at the even revival times. From Eqs. (58), (60), and the prescription (13), we find that, near the $p$th even revival time, the probability $P_{1,n}(t)$ varies as [8]

$$P_{1,n}(t \approx t_{r,2p})_{\text{max}} \sim \left\{ \begin{array}{ll}
\frac{n!}{2^n (n/2)!^2 (1+\pi^2 p^2)^{n+1/2}} & n \text{ even} \\
0 & n \text{ odd}.
\end{array} \right.$$  

(74)

In the limit that $\alpha \gg 1$, this function gives a good indication of the distribution of the values of $n$ that are excited at the even revival times; moreover, it conserves probability since

$$\sum_{n=0}^{\infty} \frac{(2n)! (\pi p)^{2n}}{2^n (n!)^2 (1+\pi^2 p^2)^{n+1/2}} = 1.$$  

(75)

The first several values of $P_{1,n}(t \approx t_{r,2p})_{\text{max}}$ are compared with the exact results in the insets of Figs. 3 and 5.

At the odd revival times, values of $n$ of order $4\alpha^2$ make the dominant contribution to the ground-state probability. We have not been able to obtain an approximate analytic expression for $P_{1,n}(t)$ for such times. Neither have we been able to obtain an approximate form of Eqs. (50) that leads to the result (65) for an oscillator driven by an off-resonant field. Looking at Eqs. (50) one might think that it is possible to adiabatically eliminate the terms that are rapidly varying as $e^{\pm 2i\chi t}$ appearing in those equations and get an effective light shift for each of the dressed states. However such a procedure is not justified since the state amplitudes do not vary slowly compared with $e^{\pm 2i\chi t}$ for values of $n$ approaching $4\alpha^2$.

We can also calculate a quantity proportional to the average pseudofield amplitude $F_p(t) = \langle a \rangle e^{i\omega t}$ and the average pseudofield energy $H_{p}(t) = \hbar \omega \langle a^\dagger a \rangle$. Using Eqs. (31), (40), and (65), we find an average pseudofield amplitude

$$F_p(t) = \langle a \rangle e^{i\omega t} = \sum_{n=0}^{\infty} \sqrt{n+1} \left( c_{1,n}^\dagger c_{1,n} + c_{2,n}^\dagger c_{2,n} \right)$$

$$= 2 \text{ Re} \sum_{n=0}^{\infty} \sqrt{n+1} \left( t_{r,1,n}^\dagger c_{1,n} + t_{r,1,n}^\dagger c_{2,n} \right).$$  

(76)

This must be calculated numerically; however, to get an idea of the trend of the result, we can use the dispersion-free result given by Eqs. (64) and (65) and carry out the summation in Eq. (76) to obtain

$$F_p^{DF}(t) = -\alpha [1 - \cos(gt/2\alpha)].$$  

(77)

The average field amplitude varies from 0 at the even revival times to $-2\alpha$ at the odd revival times, neglecting dispersion. The result is similar to Eq. (23) for the average true field amplitude (23); it is simply shifted by $-\alpha$. For $\alpha = 10$, the actual values of $F_p(t)$, including dispersion [Eq. (76)], at the first four revival times are ($-19.9, -0.51, -18.9, -1.82$), in rough agreement with the dispersion-free prediction of $(-20.0, -20.0)$. Similarly, we find the average pseudofield energy

$$U_{p}(t) = \hbar \omega \langle a a^\dagger \rangle = \sum_{n=1}^{\infty} [n(|c_{1,n}^\dagger|^2 + |c_{2,n}^\dagger|^2)],$$  

(78)

which must also be calculated numerically. In the dispersion-free model, we can use Eqs. (62)–(64) and carry out the summation in Eq. (78) to obtain

$$U_{p}^{DF}(t) = 2\hbar \omega \alpha^2 [1 - \cos(gt/2\alpha)].$$  

(79)

In contrast to the true cavity field, the average pseudofield energy now undergoes significant variations, from a value of zero at the even revival times to a value of $4\hbar \omega \alpha^2$ at the odd revival times. For $\alpha = 10$, the actual values of $U_p(t)/\hbar \omega$ from Eq. (79) at the first four revival times are (397.9, 77.3, 38.6, 1), as compared with the dispersion-free prediction of (400, 0, 400, 0); owing to dispersion, values of $n$ different from zero make a contribution to the energy at the even revival times, consistent with Eq. (74). The atom–classical-field and atom-pseudofield average interaction energies both vanish for a resonant field.

**IV. DISCUSSION**

At first glance, the Mollow transform seems to simplify the problem of atoms interacting with coherent states of the radiation field. Mollow showed this to be the case for certain classes of problems. In particular, the Mollow transform is especially useful in problems where there is a coherent-state
field interacting with a single atom and the effect of the atom back on this incident field is negligible. In other words, if we are interested solely in the atomic-state dynamics or the properties of the fields scattered by the atoms in such cases, the results are formally equivalent to those obtained in a simpler problem in which the quantized, coherent-state field is replaced by a classical field. For example, a quasiresonant multimode coherent-state pulse is used to excite a two-level atom and if the pulse duration is much less than the excited-state lifetime, the atomic-state dynamics is determined entirely by the area of a classical pulse whose field is equal to the expectation value of the coherent-state pulse. In other words, the atomic state dynamics does not depend on field fluctuations in this limit.

The situation changes dramatically if the back action of the atoms on the field is significant, as in the cavity problem discussed in this article. In that limit, rather than simplifying the problem, the Mollow transform actually appears to make the problem more difficult. The origin of this difficulty is the unlimited supply of photons that the classical field of the atoms on the field is significant, as in the cavity problem discussed in this paper, the pseudofield statistics that were interpreting the collapse and revivals of the Jaynes-Cummings Mollow transformation, it does afford us the possibility of this manuscript and for his comments and suggestions. P.R.B. would like to thank B. Sanders for pointing out Ref. [5]. This research is supported by High Impact Research MoE Grant UM.C/625/1/HIR/MoE/CHAN/04 from the Ministry of Education Malaysia.

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P.R.B. would like to thank P. Milonni for reading a draft of this manuscript and for his comments and suggestions. P.R.B. would like to thank B. Sanders for pointing out Ref. [5]. Other types of problems using the Mollow transformation may also offer insight into the atomic dynamics. For example, in the Tavis-Cummings model [9], two or more atoms interact with a single-mode cavity field. It is known that this interaction can lead to both spin squeezing and entanglement of the atoms [10]. The Jaynes-Cummings model, we have seen that this leads to an infinite ladder of coupled field states. Although the problem is considerably more difficult to solve using the Mollow transformation, it does afford us the possibility of interpreting the collapse and revivals of the Jaynes-Cummings model in a somewhat different light. Moreover, for the problem discussed in this paper, the pseudofield statistics that were derived correspond to the actual cavity-field statistics for the complementary problem of an atom in a cavity driven by an external classical field.

For $n$ even, the exponential can be expanded as

$$\exp \left\{ -i \left[ \frac{\pi s q^2}{2a^2(1 + i\pi s)} \right] \right\} = \sum_{m=0}^{n/2} \sum_{q=0}^{n/2} \left( -1 \right)^q q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n}$$

since contributions to $c_{1,n}^\dagger(t)$ from terms with higher values of $m$ vanish in the limit $\alpha \to \infty$. Using the fact that

$$\sum_{q=0}^{n/2} \left( -1 \right)^q q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n} q^{2m} q^{2n}$$

when $m < n/2$ and $n$ is even leads to Eq. (74), provided we set $P_{1,n}(t = t_{\Delta}) = |\psi_{1,n}^\dagger(t_{\Delta})|^2$. For $n$ odd, the exponential expansion can be cut off at $m = n/2 - 1$ in the limit $\alpha \to \infty$. The resulting sum over $q$ vanishes for these values of $m$.
