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Bimorph Piezoelectric Cantilevered (BPC) actuators have been of increasing interest in micro-manipulation processes during recent years. Due to properties such as transverse vibration, the performance and manoeuvrability have considerably improved, compared with conventional longitudinal piezoelectric actuators. Therefore, dynamic modelling of such actuators has been the centre of attraction. For this purpose, a target point on the actuator, e.g. the cantilever end tip, is usually considered as the actuator output. One degree of freedom lumped and continuous type dynamic models have been considered in prior research works. These types of modelling lead to two significant issues. First, the effect of higher vibrational modes in the actuator output is disregarded. Second, a minimum phase dynamic system is achievable for all target points regardless of position. In this paper, these two issues will be analytically and experimentally investigated. To this end, a linear continuous dynamic model for a general BPC actuator is derived and discretized by attaining exact mode shapes. The Prandtl–Ishlinskii (PI) model is utilized to model and identify the non-linear hysteresis behaviour. In contrast to previous works, dynamic behaviour analysis elaborates on the effect of higher modes in the actuator output response. In addition, the possibility of non-minimum phase behaviour based on the location of the target point is investigated. Simulation studies and experimental results confirm the validity of the proposed dynamic model and its behaviour analysis.

Keywords: bimorph piezoelectric actuator; continuous modelling; hysteresis; dynamic behaviour analysis

1. Introduction

Micro-manipulation by unimorph and bimorph Piezoelectric Cantilevered (BPC) actuators has been an interesting challenge for researchers in current years. There are two significant characteristics in applications using such actuators. One of the elements includes the special properties of piezoelectric ceramics, such as high natural frequency, fine resolution and time response [1,2]. The second regards high manoeuvrability due to the cantilevered type structure and transverse deflection. The latter shows the unique capability of BPC actuators compared with conventional longitudinal piezoelectric stages.

Owing to the direct piezoelectric effect, BPC structures have been utilized as various types of sensors, among which position sensors [3], force sensors [4], mass detection [5]...
and energy harvesting [6,7]. In addition, with respect to the piezoelectric inverse effect, BPC actuators have also been used in micro-manipulation applications, such as micro-assembly [8] and cell characterization [9]. Position and force control by these actuators are considered in such cases. Self-sensing actuation has additionally been an appealing research field for utilizing both capabilities simultaneously in sensor-less manipulation [10]. Therefore, precise dynamic modelling of BPC actuators is an extensive research area. Several investigations that deal with this subject have been presented. Two main modelling methods, namely \textit{lumped} and \textit{continuous}, are addressed in publications.

With regards to lumped dynamic modelling, second-order mass, spring and damper models have been employed. A nonlinear function is also considered to express the hysteresis non-linear effect [11]. This approach is prominent in the modelling of longitudinal piezoelectric actuators as well [12]. However, lumped models are quite restrictive for BPC actuators. In fact, the identified dynamic model is only valid for a specified target point on the actuator. By changing the target point, the output behaviour would alter, which leads to the necessity of new identification. In addition, the effect of higher vibrational modes on output response and dynamic behaviour is ignored. To rectify these issues, continuous dynamic modelling is an alternative solution.

Concerning continuous modelling, Bilgen et al. [13] presented dynamic modelling for a unimorph piezoelectric beam to characterize its actuation. The actuator behaviour in energy harvesting has also been analytically and experimentally investigated [14–16]. To consider bimorph structures, Chen et al. [17] recommended an analytical model for bimorph power generators. The effects of actuator geometry and excitation frequency have been considered too. In another work, the bimorph dynamic model was analysed based on the Euler–Bernoulli and Timoshenko beam theories [18]. Effects of buffer layer and electrodes in the actuator dynamic have further been explored [19]. In all mentioned works, two core concerns exist. First, in the linear dynamic models proposed, the hysteresis non-linear effect has been eliminated as a result of low-amplitude piezoelectric voltage. Second, the higher mode effect in the output behaviour has not been investigated because of sensing applications.

To consider actuator output behaviour, Timoshenko beam theory with the approximate Galerkin approach has been suggested for the dynamic modelling [20]. The main concern still lies in the assumed linear dynamic model along with high input voltages. Yi et al. [21] considered the effect of hysteresis as a disturbance added to the linear dynamic model. Chao et al. imported the effect of hysteresis in the continuous dynamic model [22]. In these works, only the first vibrational mode is considered to present the output dynamic behaviour. This sort of modelling would lead to two significant issues. For one, the effect of higher vibrational modes in the actuator output response would be ignored. Second, the minimum phase dynamic behaviour would be achieved for all target points, regardless of its position on the actuator.

In this paper, these issues are analytically and experimentally investigated. For this purpose, the actuator dynamic is divided into \textit{linear second-order} and \textit{non-linear hysteresis behaviour}. For the linear part, continuous dynamic modelling for a general BPC actuator based on the Euler–Bernoulli beam theory is presented. Exact mode shapes are achieved to descritize the proposed continuous system. The Prandtl–Ishlinskii (PI) model is also utilized to identify the non-linear hysteresis effect. To evaluate the efficiency of the proposed models, two vibrational modes are considered for the dynamic modelling of fully covered BPC actuators. The effect of higher modes on actuator time response and output behaviour has been investigated. It is analytically demonstrated that the actuator dynamic can be non-minimum phase, based on the actuator target point position. This
may lead to closed-loop instability in output control applications. Experimental results validate the accuracy of the proposed models. In addition, actuator time response and dynamic behaviour demonstrate appropriate consistency with the proposed models.

2. Dynamic modelling of BPC actuators

Figure 1 shows a schematic drawing of BPC actuators.

Two pieces of piezoelectric ceramic symmetrically cover a specified length of the base substrate between \( l_1 \) and \( l_2 \). \( L \) represents the beam total length. \( t_b \) and \( t_p \) are the thicknesses of the beam and piezoelectric actuator, respectively. Both the base substrate and piezoelectric ceramics have the same width, \( b_b = b_p \).

The actuator dynamic would be divided into two parts, linear and non-linear. Initially, continuous dynamic modelling is employed to express the linear behaviour. After which the hysteresis non-linearity would be discussed.

2.1. Linear continuous dynamic modelling for general BPC actuators

Dynamic modelling for general BPC actuators based on the Euler–Bernoulli theory is presented in this section. The dynamic behaviour for an Euler-Bernoulli beam with transverse deflection \( w \) can be considered as follows [23]:

\[
\rho A \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 M}{\partial x^2}
\]

where \( \rho \) and \( A \) are the mass density and cross section area, respectively. \( M \) is the internal bending moment that can be derived by integrating the internal stress through the beam thickness.

The stress–strain relation for the base substrate in the \( x \) direction can be explained by Hook’s law as Equation (2).

\[
\sigma_b = c_b S_b
\]

\( \sigma_b, S_b \) and \( c_b \) are stress, strain and material elasticity modulus, respectively. In addition, for an Euler–Bernoulli beam, strain \( S \) could be considered as a function of transverse deflection as:
where \( z \) is the distance from the neutral axis.

For piezoelectric ceramics, due to the electromechanical behaviour, stress is a function of strain and electric field, as shown in Equation (4) [23].

\[
\sigma_1 = c_{11}^E S_1 - e_{31} E_3
\]

\( c_{11}^E \) and \( e_{31} \) represent elasticity modulus in constant electric field and piezoelectric constant, respectively. \( E_3 \) is the electric field in the \( z \) direction.

In BPC actuators, the direction of the electric field and polarization in upper (U) and lower (L) ceramics should be different to allow the structure to bend. As a result, stress for each ceramic is defined as follows.

\[
\sigma_{1U} = c_{11}^E S_1 - e_{31} E_3
\]

\[
\sigma_{1L} = c_{11}^E S_1 + e_{31} E_3
\]

To achieve a precise dynamic model, the actuator length is separated into three distinct parts, as shown in Figure 2.

Because of the beam structure symmetry, the neutral axis is still located in the middle (Figure 2).

Internal bending moments for parts (I) and (III) can easily be derived.

\[
M_I = M_{III} = \int z \sigma_b dA = \int z^2 c_b \frac{\partial^2 w}{\partial x^2} dA = -c_b I_b \frac{\partial^2 w}{\partial x^2}
\]

where subscript \( b \) denotes the base substrate and \( I_b \) is the base inertia moment that is defined below.

\[
I_b = \int \frac{z^2}{2} dA = \frac{b h l_b^3}{12}
\]

For the second part, the internal moment includes a combination of the base and piezoelectric stresses as mentioned in (9).
A linear relation can be assumed for the electric field and electric voltage as 
\[ E_3 = \frac{V_3}{t_p} \] [23]. By substituting strain, \( S_1 \), and electric field, \( E_3 \), the internal moment would be attained as:

\[ M_{II} = M_b + M_pU + M_pL = \int z \sigma_b dA + \int z \sigma_1 U dA + \int z \sigma_1 L dA \] (9)

\[ = -c_b I_b \frac{\partial^2 w}{\partial x^2} + \int z \left( c_1^E S_1 - e_3 E_3 \right) dA + \int z \left( c_1^E S_1 + e_3 E_3 \right) dA \] (9)

\[ = M_b + M_pU + M_pL = -c_b I_b \frac{\partial^2 w}{\partial x^2} - c_1^E I_p \frac{\partial^2 w}{\partial x^2} - M_p V_3 \] (10)

Where

\[ I_p = \frac{b_p \left( (t_b + 2t_p)^3 - (t_b)^3 \right)}{12} \] (11)

\[ M_p = (t_b + t_p) b_p e_3 \]

Finally, the total internal moment can be expressed as a summation of all three part moments.

\[ M = M_I + M_{II} + M_{III} \] (12)

\[ = -\left[ c_b I_b + c_1^E I_p G(x) \right] \frac{\partial^2 w}{\partial x^2} - M_p V_3 G(x) \]

\( G(x) \) denotes the location of piezoelectric actuators and is defined as \( G(x) = H(x - l_1) - H(x - l_2) \), where \( H(x) \) is the Heaviside function.

By considering the total internal moment (12) and dynamic model (1), the continuous dynamic model for the general BPC actuator can be achieved.

\[ \left[ \rho_b A_b + 2\rho_p A_p G(x) \right] \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[ c_b I_b + c_1^E I_p G(x) \right] \frac{\partial^2 w}{\partial x^2} = -M_p V_3 \frac{\partial^2 G(x)}{\partial x^2} \] (13)

By substituting \( G(x) \) in Equation (13), the achieved dynamic would be simplified.

\[ (\rho A) \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left( (cI) \frac{\partial^2 w}{\partial x^2} \right) = -M_p V_3 [\delta'(x - l_1) - \delta'(x - l_2)] \] (14)

\( (\rho A) = \rho_b A_b + 2\rho_p A_p G(x) \)

\( (cI) = c_b I_b + c_1^E I_p G(x) \)

\( \delta(x) \) refers to the Dirac delta function and \( (\prime) \) denotes the differentiation with respect to x.

To take into account the damping effect, two types of damping, i.e. structural and viscous air damping, are added to the dynamic model [16].
\[
\frac{\rho A}{\partial t^2} + c_v(x) \frac{\partial w}{\partial t} + \frac{\partial^2}{\partial x^2} \left( c_s(x) \frac{\partial^2 w}{\partial x^2} \right) + \frac{\partial^2}{\partial x^2} \left( c_l \frac{\partial^2 w}{\partial x^2} \right) = -M_p V_3 \delta'(x - l_1) - \delta'(x - l_2)
\]  
(15)

c_v(x) is the equivalent coefficient of strain rate structural damping and \( c_v(x) \) is the viscous air damping coefficient.

2.2. Dynamic model discretization

To discretize the continuous dynamic model, the separation of variables technique is utilized. For this purpose, the transverse deflection \( w \) would be represented by a uniformly convergent series of eigenfunctions.

\[
w(x, t) = \sum_{i=1}^{n} \varphi_i(x) q_i(t)
\]  
(16)

Therefore, the exact undamped mode shape \( \varphi_i(x) \) should be derived to discretize the dynamic model precisely. The total mode shape, \( \varphi_i(x) \), is a combination of three mode shapes for three different parts. The general mode shape, \( \varphi_j(x) \), for each part is represented as Equation (17) [24].

\[
\varphi_j(x) = A_j \sin(\beta_j x) + B_j \cos(\beta_j x) + C_j \sinh(\beta_j x) + D_j \cosh(\beta_j x); \quad j = 1, 2, 3
\]  
(17)

By inserting boundary conditions, shape mode gains, i.e. \( A_j, B_j, C_j \), and also natural frequencies can be obtained. The boundary conditions are as follows:

\[
x = 0
\]

\[
\varphi_1 \bigg|_{x = 0} = 0 \quad \quad \quad \frac{\partial \varphi_1}{\partial x} \bigg|_{x = 0} = 0
\]

\[
x = l_1
\]

\[
\varphi_1 \bigg|_{x = l_1} = \varphi_2 \bigg|_{x = l_1} \quad \quad \quad \frac{\partial \varphi_1}{\partial x} \bigg|_{x = l_1} = \frac{\partial \varphi_2}{\partial x} \bigg|_{x = l_1}
\]

\[
c_b I_b \frac{\partial^2 \varphi_1}{\partial x^2} \bigg|_{x = l_1} = \left( c_{11} I_p + c_b I_b \right) \frac{\partial^2 \varphi_1}{\partial x^2} \bigg|_{x = l_1} \quad \quad \quad c_b I_b \frac{\partial^2 \varphi_1}{\partial x^2} \bigg|_{x = l_1} = \left( c_{11} I_p + c_b I_b \right) \frac{\partial^2 \varphi_1}{\partial x^2} \bigg|_{x = l_1}
\]

\[
x = l_2
\]

\[
\varphi_2 \bigg|_{x = l_2} = \varphi_3 \bigg|_{x = l_2} \quad \quad \quad \frac{\partial \varphi_2}{\partial x} \bigg|_{x = l_2} = \frac{\partial \varphi_3}{\partial x} \bigg|_{x = l_2}
\]

\[
\left( c_{11} I_p + c_b I_b \right) \frac{\partial^2 \varphi_2}{\partial x^2} \bigg|_{x = l_2} = c_b I_b \frac{\partial^2 \varphi_2}{\partial x^2} \bigg|_{x = l_2} \quad \quad \quad \left( c_{11} I_p + c_b I_b \right) \frac{\partial^2 \varphi_2}{\partial x^2} \bigg|_{x = l_2} = c_b I_b \frac{\partial^2 \varphi_2}{\partial x^2} \bigg|_{x = l_2}
\]

\[
x = L
\]

\[
\frac{\partial^2 \varphi_3}{\partial x^2} \bigg|_{x = L} = 0 \quad \quad \quad \frac{\partial^2 \varphi_3}{\partial x^2} \bigg|_{x = L} = 0
\]  
(18)
By achieving exact mode shapes, orthogonality properties can be expressed [24].

\[
\int_0^L (\rho A) \varphi_m(x) \varphi_n(x) \, dx = \delta_{mn}
\]

\[
\int_0^L \varphi_m(x) \frac{\partial^2 \varphi_n(x)}{\partial x^2} \left( cI \right) \frac{\partial^2 \varphi_n(x)}{\partial x^2} \, dx = \omega_m^2 \delta_{mn}
\]

\(\omega_m\) is the \(m\)th undamped natural frequency and \(\delta_{mn}\) is the Kronecker delta function. To discretize the continuous dynamic, Equation (16) is substituted in Equation (15).

\[
\sum_{i=1}^{n} \varphi_i(x) \frac{\partial^2 q_i(t)}{\partial t^2} + c_a(x) \sum_{i=1}^{n} \varphi_i(x) \frac{\partial q_i(t)}{\partial t} + \frac{\partial^2}{\partial x^2} c_s(x) \sum_{i=1}^{n} \varphi_i(x) \frac{\partial q_i(t)}{\partial t} + \frac{\partial^2}{\partial x^2} (cI) \sum_{i=1}^{n} \varphi_i(x) q_i(t) = -M_p V_3 [\delta'(x - l_1) - \delta'(x - l_2)]
\]

By multiplying \(\varphi_j(x)\) to both sides and integrating over the beam length, the discretized dynamic model would be derived.

Using the following equality,

\[
\int_0^L \varphi_j(x) \, dx = \int_0^L \varphi_j(l_i) \, dx = \varphi_j(l_i)
\]

and considering the damping effect as a proportional damping [7], the dynamic model would be simplified as follows:

\[
\ddot{q}_i + (\alpha + \beta \omega_i^2) \dot{q}_i + \omega_i^2 q_i = m_{pi} V(t) \quad i = 1 \ldots n
\]

\[m_{pi} = -M_p \left\{ \varphi_i'(l_2) - \varphi_i'(l_1) \right\}\]

\(\alpha\) and \(\beta\) are proportional damping coefficients, which should be identified experimentally for each actuator.

The entire beam length has been considered to achieve the time domain dynamic. Therefore, it is valid for all points in the three segments. The dynamic model can be equally represented as Equation (23).

\[
\ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_i^2 q_i = m_{pi} V(t) \quad i = 1 \ldots n
\]

To determine the damping coefficients, damping ratios, \(\xi_i\), should be identified for at least two vibrational modes. Then, \(\alpha\) and \(\beta\) would be calculated as follows:

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix}
1 & \omega_i^2 \\
1 & \omega_j^2
\end{bmatrix}^{-1} \begin{bmatrix}
2\xi_i \omega_i \\
2\xi_j \omega_j
\end{bmatrix}
\]

\(c_a(x)\) and \(c_s(x)\) can be obtained subsequently.
2.3. **Non-linear hysteresis effect in high input voltages**

In high-amplitude input voltages, piezoelectric actuators show a rate-independent hysteresis non-linear behaviour. The actuator non-linear dynamic can be expressed as Equation (25).

\[ \ddot{q}_i + 2\xi_i\omega_i \dot{q}_i + \omega_i^2 q_i = m_{pi} H(V(t)) \quad i = 1 \ldots n \]  

Equation (25)

\( H(V(t)) \) is a non-linear function caused by high input voltage to express hysteresis effect. To model hysteresis behaviour, this function should be identified.

Due to the high natural frequencies of piezoelectric actuators, quasistatic input voltages are usually employed for hysteresis identification to eliminate the effect of dynamic impedances. In such a case, the dynamic model can be simplified to:

\[ \omega_i^2 q_i = m_{pi} H(V(t)) \quad i = 1 \ldots n \]  

Equation (26)

As a result, the rate-independent hysteresis behaviour can be found more easily. To identify the hysteresis function \( H(V(t)) \), a target point \( l_t \) is considered for transverse deflection measurement \( w(l_t) \) by an external position sensor. Based on Equation (16), the deflection can be expressed as follows.

\[ w(l_t) = \varphi_1(l_t) q_1(t) + \ldots + \varphi_n(l_t) q_n(t) \]  

Equation (27)

By substituting the generalized coordinates from Equation (26), the deflection can be found as a function of hysteresis \( H(V(t)) \).

\[ w(l_t) = \varphi_1(l_t) \frac{m_{p1}}{\omega_1^2} H(V(t)) + \ldots + \varphi_n(l_t) \frac{m_{pn}}{\omega_n^2} H(V(t)) \]  

Equation (28)

\[ = \left[ \varphi_1(l_t) \frac{m_{p1}}{\omega_1^2} + \ldots + \varphi_n(l_t) \frac{m_{pn}}{\omega_n^2} \right] H(V(t)) = H'(V(t)) \]

\( H'(V(t)) \) is a new non-linear function, which defines the direct non-linear relation between input voltage and output deflection. By identifying this function through a proper approach, the main hysteresis function, \( H(V(t)) \), would be achievable as:

\[ H(V(t)) = \left[ \varphi_1(l_t) \frac{m_{p1}}{\omega_1^2} + \ldots + \varphi_n(l_t) \frac{m_{pn}}{\omega_n^2} \right]^{-1} H'(V(t)) \]  

Equation (29)

Finally, the identified non-linear behaviour (Equation (29)) should be added to the proposed linear dynamic (23) to attain the total non-linear dynamic model.

2.4. **Hysteresis identification by the Prandtl–Ishlinskii (PI) model**

The PI model is utilized for hysteresis identification. The most important advantages of this model are its simplicity and that its inverse can be calculated analytically. As a result, this approach would be useful for compensating the hysteresis behaviour in control
purposes. In addition, it could be utilized in open-loop systems, where signal feedback is not accessible.

The rate-independent backlash operator (Figure 3) is the primary operator in a conventional PI model.

This operator can be defined as:

\[ y(t) = w_h H_r[x, y_0](t) \]  
\[ y(t) = \max \{x(t) - r, \min \{x(t) + r, y(t - T)\}\} \]  
\[ y_0 = \max \{x_0 - r, \min \{x_0 + r, y_0\}\} \]

where

\[ x(t) \] and \( y(t) \) are the control input and actuator output, respectively. \( r \) is the input threshold value or magnitude of backlash and \( T \) is the sampling time. The weight, \( w_h \), defines the operator slope. The output initial condition is assumed as:

\[ y(0) = \max \{x(0) - r, \min \{x(0) + r, y_0\}\} \]

It can normally be equal to zero.

A real hysteresis loop can be identified by a linear superposition of many primary backlash operators with different thresholds and weights.

\[ y(t) = \sum_{i=0}^{n} w_{hi} H_{ri}[x, y_0](t) = \tilde{w}^T_h H_r[x, \tilde{y}_0](t) \]

where the vector parameters are as follows:

\[ \tilde{w}_h^T = [w_{h1} \ldots w_{hn}] \]
Figure 4 provides a schematic view of hysteresis loop estimation.

As mentioned previously, the inverse PI model can be calculated analytically. The key idea of inverse feedforward hysteresis compensation is to cascade the inverse hysteresis operator $H^{-1}$ with the real hysteresis. Therefore, the effect of non-linear hysteresis would be compensated. In other words, an identity mapping between the desired actuator output, $x_d$, and the actuator response, $x$, would be obtained via hysteresis compensation in quasistatic motions. The structure of the inverse feedforward hysteresis compensation is illustrated in Figure 5.

3. Dynamic behaviour analysis

In this section, the dynamic behaviour of BPC actuators is analysed, this has not been considered in the previous researches. Fully covered actuators are prevalent commercialized BPC actuators in many research works on micro-manipulation [16,18,20,22]. In these actuators, the base substrate is completely covered by piezoceramics. This kind of actuator has been selected for this research to evaluate the dynamic model as well as
dynamic behaviour analysis. The dynamic model of this actuator can be derived by choosing $l_2 = L$ and $l_1 = 0$ in Equation (22). In the previous presented models, the first vibrational mode has only been taken into account. Therefore, the minimum phase dynamic has been achieved for all target points on the actuator. But, it would be shown that the actuator dynamic can be non-minimum phase for some target points by considering the effect of higher vibrational mode.

A linear dynamic system would be minimum phase, provided that all zeros of its transfer function are located in the left half plane (LHP). If the transfer function includes any right half plane (RHP) zero, the system would be non-minimum phase [25]. A similar concept also exists for non-linear dynamic systems by defining the zero dynamic instead of zeros [26].

Non-minimum phase dynamic systems have some significant issues in control applications, with regard to the tracking performance and also asymptotic stability. To be more clear, feedback and feedforward control approaches can be analysed for such systems. In a feedback control method, the closed-loop system may become unstable by increasing the control gains [25]. Therefore, control gains must be inevitably limited in such cases. This conservative condition would increase the tracking error and perfect tracking would not be achievable. On the other hand, in a feedforward approach, the inverse of transfer function should be utilized to achieve the perfect tracking. Although a non-minimum phase system can be generally stable, its inverse would be definitely unstable due to the existence of RHP poles. It may also lead to the closed-loop instability in the control of such systems. To this effect, it is very important to precisely determine the dynamic behaviour of BPC actuators to find the possibility of non-minimum phase dynamic.

Two first vibrational modes have been regarded for dynamic modelling. Due to high piezoelectric natural frequencies, this consideration would be sufficient for many
An optional target point \( (l_t) \) is selected for deflection measurement \( w(l_t) \) by an external position sensor. To analyse the dynamic behaviour, the measured transverse deflection \( w(l_t) \) can be expressed by modal coordinates.

\[
w(l_t) = \varphi_1(l_t)q_1(t) + \varphi_2(l_t)q_2(t) \tag{36}\]

It was mentioned previously that the inverse hysteresis model can compensate the hysteresis effect and linearize the system. Therefore, linear dynamic (23) can serve in behaviour analysis. It can also be shown that this analysis is valid for non-linearized dynamic through zero dynamic concept in non-linear systems \[26\].

Based on Equation (23), the transfer function \( F(s) \), between input voltage and target point deflection, can be achieved.

\[
F(s) = \frac{w(l_t)}{V(t)} = \frac{m_{p1}\varphi_1(l_t)}{s^2 + 2\xi_1\omega_1 s + \omega_1^2} + \frac{m_{p2}\varphi_2(l_t)}{s^2 + 2\xi_2\omega_2 s + \omega_2^2} = \frac{N(s)}{D(s)} \tag{37}\]

\( N(s) \) and \( D(s) \) are the numerator and denominator of the transfer function, respectively. It is clear that the open-loop system is absolutely stable with regard to the stable poles caused by the stable vibrational modes. Nonetheless, the location of zeros may strongly affect the dynamic behaviour as well as the stability of the closed-loop system. The zeros location is achievable by analysing \( N(s) \).

\[
N(s) = m_{p1}\varphi_1(l_t)\left\{ a_1 s^2 + a_2 s + a_3 \right\} \tag{38}\]

\[
a_1 = \left[ 1 + \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} \right], \quad a_2 = \left[ 2\xi_2\omega_2 + \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} 2\xi_1\omega_1 \right] \]

\[
a_3 = \left[ \omega_2^2 + \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} \omega_1^2 \right] \]

The zeros configuration, as roots of \( N(s) \), depends completely on the sign of \( a_i \) as well as the target point position \( (l_t) \). Essentially, the target point position can affect the dynamic system behaviour and the closed-loop stability.

Based on the dynamic model (35), the input gain \( m_{pi} \) depends on the slope of the ith mode shape at the actuator end tip, i.e. \( m_{pi} \propto \varphi_i'(L) \). The output gain \( \varphi_i(l_t) \) also depends on the magnitude of ith mode shape at the target point. It can be seen from Equation (38) that all zeros would be in the LHP, provided that \( \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} > 0 \), i.e. \( m_{pi} \) and \( \varphi_i(l_t) \) have the same sign. In this case, the system would be minimum phase. Otherwise, zeros may go towards the RHP and system would be non-minimum phase.
Two first normalized mode shapes and their slopes are shown in Figure 6 for a dimensionless BPC actuator.

It is evident that \( \varphi_1(L) > 0 \) and \( \varphi_1(l_t) > 0 \) for any arbitrary target point, \( l_t \). As a result, \( m_{p1}\varphi_1(l_t) \) would be definitely positive. But \( \varphi_2(L) \) is negative and the sign of \( m_{p2}\varphi_2(l_t) \) depends completely on \( \varphi_2(l_t) \). Hence, the beam length is divided into two segments, (I) and (II) in Figure 6.

\( \varphi_2(l_t) \) is negative in part (II), which means that if the target point is located in that region, \( m_{p2}\varphi_2(l_t) \) would be positive, i.e. \( \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} > 0 \). Therefore, the system is still minimum phase. However, if the target is located in part (I), \( \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} \) would be negative and the stability of \( N(s) \) depends on the magnitude of \( \frac{m_{p2}\varphi_2(l_t)}{m_{p1}\varphi_1(l_t)} \) and actually on the sign of \( a_i \) gains.

Different investigations are required for various BPC actuator geometries. As a result, if the target point is considered in part (I), the dynamic system may be non-minimum phase. This phenomenon cannot be observed in lumped dynamic models or in one degree of freedom discretized systems, as reported in the previous literature.

3.1. Simulation study for dynamic behaviour analysis

To investigate the proposed analysis, a simulation case study was carried out. In this analysis, a BPC actuator is considered as a brass substrate fully covered by upper and lower PZT-5A piezoelectric actuators. The material properties of the PZT and substrate are shown in Table 1.

Based on the considered material properties, the BPC actuator natural frequencies listed in Table 2 were achieved by the represented method.

The output frequency response for the endtip target point and also the first two natural frequencies are shown in Figure 7.

Based on the mentioned transfer function analysis, normalized \( a_i \) gains are illustrated in Figure 8.

It is evident that \( a_2 \) and \( a_3 \) are positive for the proposed actuator geometry. However, \( a_1 \) would be negative in the marked region, i.e. \( x_m < 0.737L \). As a result, the system
would be non-minimum phase, if the target point was designated in this region. To clarify this behaviour, frequency responses for four target points are compared in Figure 9.

Obviously, by transferring the target point location through the mentioned point (0.737L), the LHP zeros would be eliminated. In fact, those would be transformed to

| Table 1. Material properties for PZT and substrate. |
|-----------------|-----------------|-----------------|
|                | PZT-5A          | Brass           |
| Length (mm)    | L               | 21.60           | 21.60           |
| Width (mm)     | b_b, b_p        | 3.2             | 3.2             |
| Thickness (mm) | t_b, t_p        | 0.13(each)      | 0.12            |
| Mass density (kg/m³) | ρ_b, ρ_p | 7800           | 9000            |
| Elastic modulus (GPa) | c_b, c_{11} | 59             | 105             |
| Piezoelectric Constant (C/m²) | e_{31} | -12.54         | -               |
| Mass Damping | α               | 1e-3            | 1e-3            |
| Stiffness Damping | β           | 1e-5            | 1e-5            |

Table 2. Actuator natural frequencies.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Natural frequencies (Rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2249</td>
</tr>
<tr>
<td>2</td>
<td>14,094.24</td>
</tr>
<tr>
<td>3</td>
<td>39,464.29</td>
</tr>
</tbody>
</table>

Figure 7. Actuator output frequency response.
Figure 8. Numerator coefficients trend.

Figure 9. Frequency response comparison for different target points.
the RHP zero. Root locus comparison for different target points can better illustrate this phenomenon, as seen in Figure 10.

It can be seen that by changing the target point, RHP zeros make the system non-minimum phase. This phenomenon is an inevitable, intrinsic characteristic of BPC actuators. Nonetheless, this analysis is greatly useful in designing a proper actuator configuration or selecting suitable working target points. In fact, if the actuator is pre-manufactured, the working point should be chosen in such a way as to prevent non-minimum phase behaviour. In contrast, if the working point is pre-specified, the actuator configuration should be designed so as to produce minimum phase behaviour.

4. Experimental case study
The proposed dynamic model was experimentally evaluated using a bimorph piezoelectric actuator (T215-A4-103X) and voltage amplifier (EPA-104-230) from Piezo System Company (Woburn, MA, USA). National Instruments data acquisition card (NI-PCI 6229) captured data with a frequency of 20 KHz. Keyence laser displacement sensor (LK-H020) measured the deflection of piezoelectric cantilever target point with a resolution of 10nm for performance evaluation. Figure 11 shows the experimental set-up.
The actuator properties are listed in Table 1, as mentioned in the simulation study, except damping coefficients that are achieved experimentally. To validate the proposed dynamic model, the dynamic is divided into two parts, linear two degrees of freedom system and non-linear hysteresis behaviour.

4.1. Linear dynamic identification and dynamic analysis

First, the target point is considered as $l_i = 20.5 \text{ mm} = 0.95 L$. Frequency response was utilized for linear dynamic validation. A low-amplitude chirp type input voltage was employed for frequency response analysis to eliminate the hysteresis and non-linearity effects. The amplitude considered was $0.3 V$. Figure 12 indicates both actuator frequency responses for the experiments and proposed dynamic model.

Figure 11. Experimental set-up.

Figure 12. Frequency response for $l_i = 0.95 L$ and $V = 0.3 V$. 
Damping coefficients have been identified in a way such that the simulation result coincides with the experimental response. Figure 12 illustrates the correct consistency between the experiment and simulation results. But it seems that the effect of zero in experimental frequency response is not evident. This fact is caused by the laser sensor accuracy limitation. The sensor accuracy is $1.2 \mu m$ and it cannot correctly depict the zero effect. To eliminate this problem, the input voltage should be increased. In such instance, the non-linear behaviour would affect the frequency response, but the zero effect would become more obvious. Figure 13 shows the frequency response for $V = 10V$.

It is obvious that the proposed model properly shows the zero effect.

One advantage of continuous dynamic modelling over lumped modelling is its validity for any target point. In other words, the proposed model is applicable for any other target points. To show this fact, identification was evaluated for another target point, $l_i = 0.7L$ as shown in Figure 14.

The modelling and experimental results demonstrate good consistency. As discussed previously, at the point $l_i = 0.7L$, the system should be non-minimum phase and no zero should be seen. To confirm this phenomenon, a comparison was done with high input voltage, e.g. $V = 10V$, for three target points. Figure 15 demonstrates the frequency response analysis.

It is obvious that the system behaviour would change to non-minimum phase by altering the target point. The result is compatible with the dynamic behaviour analysis in Figure 9. The LHP zero effect was eliminated and the system became non-minimum phase.
Figure 14. Frequency response for $I_t = 0.7L$ and $V = 1V$.

Figure 15. Experimental frequency response results for different target points.
4.2. Non-linear hysteresis identification and output time response

The PI model is used to identify the direct non-linear hysteretic behaviour, $H'(V(t))$, between input voltage, $V(t)$, and transverse deflection of the target point, $w(t)$. Then, the main hysteresis function, $H(V(t))$, would be expressed as follows:

$$H(V(t)) = \left[ \varphi_1(l_t) \frac{m_1}{\omega_1^2} + \varphi_2(l_t) \frac{m_2}{\omega_2^2} \right]^{-1} H'(V(t))$$  \hspace{1cm} (39)

To this end, a multi amplitude quasistatic input was utilized for hysteresis identification to properly cover the full input range of the actuator. The input voltage and

Figure 16. Input, output and hysteresis loop for BPC actuator.
actuator output measured by the position sensor and hysteresis loop are illustrated in Figure 16.

By gaining the input–output hysteretic loop, the PI model can identify the hysteresis non-linear behaviour. Figure 17 shows the identification result.

The hysteresis could be appropriately identified as seen in Figure 17. To validate the hysteresis identification result, Figure 18 shows the actuator output time response for multi-amplitude quasistatic and harmonic input voltages. In addition, simulated dynamic responses by one and two vibrational modes are also shown.

It is clear that the proposed dynamic model with two vibrational modes is in greater agreement with the experimental results. Consequently, the dynamic model realization is completed by the identification of the linear and non-linear parts.

5. Conclusion

In this paper, dynamic modelling and behaviour analysis of BPC actuators have been investigated. For this purpose, a linear continuous type dynamic model for a general bimorph cantilever was proposed. Precise mode shapes were achieved to discretize the continuous system. To consider the hysteresis non-linearity effect, the PI model was applied to identify the hysteresis. Two vibrational modes were taken into account to express the dynamic behaviour. It was analytically demonstrated that the system behaviour may change from minimum phase to a non-minimum dynamic system, based on the target point position. Experimental results validate the efficiency of the proposed models in linear and non-linear behaviour. In addition, actuator time response and dynamic behaviour indicate adequate consistency with the analytical results.
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**References**


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Figure 18. Actuator time response.


