Optimal placement and sizing of a DG based on a new power stability index and line losses

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ABSTRACT

This paper proposes a new algorithm for Distributed Generator (DG) placement and sizing for distribution systems based on a novel index. The index is developed considering stable node voltages referred as power stability index (PSI). A new analytical approach is adopted to visualize the impact of DG on system losses, voltage profile and voltage stability. The proposed algorithm is tested on 12-bus, modified 12-bus and 69-bus radial distribution networks. The test results are also compared and found to be in close agreement with the existing Golden Section Search (GSS) algorithm.

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1. Introduction

With the Distributed Generation (DG), many power companies are investing in small scale renewable energy resources such as wind, photo-voltaic cells, micro-turbines, small hydro turbines, CHP or hybrid. In England and Wales for example, DG was only 1.2 GW during 1993–1994, however today this figure has increased substantially and reached up to over 12GW [1]. It is also expected with the KYOTO protocol commitment by various countries to reduce CO2 emission, the market for DGs as a “Clean Energy” resource is promising. According to Energy Network Association (ENA) report [2], the UK government is targeting to achieve 15% of electricity from renewable sources by 2015, implying rapid growth in DG and investment in the power infrastructure.

However there are several issues concerning the integration of DGs with existing power system networks; that needs to be addressed [3–5]. The integration of DG changes the system from passive to active networks, which affects the reliability and operation of a power system network [4]. Furthermore, the non-optimal placement of DG can result in an increase of the system losses and thus making the voltage profile lower than the allowable limit [6]. Since utilities are already facing technical and non-technical issues, they cannot tolerate such additional issues. Hence an optimum placement of DG is needed in order to minimize overall system losses and therefore improve voltage profiles.

In the past, different methodologies have been developed to determine the optimum location and optimum size of the DG. These methodologies are either based on analytical tools or on optimization programming methods. Analytical approaches have been proposed by several authors. In [7], the authors presented analytical approach to determine the optimal location for the DG with an objective of loss minimization for distribution and transmission networks. In [8], the author used the loss sensitivity equation to determine the optimum size of DG and the exact loss equation to determine the optimum location of DG based on minimum losses. In [9], the author presented the loss sensitivity factor based on equivalent current injection using two Bus-Injection to Branch-Current (BIBC) and Branch-Current to Bus-Voltage (BCBV) matrix. A simple search algorithm is proposed in [10] for optimal sizing and placement of DG for a network system, based on losses and cost function as an objective function. The method is simple but time consuming for searching both the best location and optimum size. In [11], the author considered the minimum loss and generation cost as a parameter in addition to DG power limits to determine the optimal size and location of the DG. The author developed two programs named “Bloss” and “dpslack” for optimum sizing of DG, considering DG min–max limits also. Later-on, site is decided by considering the minimum total power losses considering DG on each bus. The method is accurate but very tedious and mathematical computation need much computation time. In [12] authors have evaluated the effect of DG placement on network reliability.
power losses reduction and power quality using the simple analytical approach and has shown the optimum DG allocation affects the system reliability and system losses.

Optimization based algorithms have also been proposed by different authors. In [13], ACS (Ant Colony Search Algorithm) is used here to find the optimal placement of DG and reclosers based on system reliability. Particle Swarm Optimization algorithm is used in [14,15] for optimum placement considering the minimum electricity cost for consumers. In [16], authors have presented the dynamic based programming approach to find the best location for DG with maximum profit as an objective function. Genetic Algorithm (GA) based methods are proposed in [17–19] for optimal sizing and placement of DG, considering different objective functions. GA-Fuzzy based optimal placement of DG is discussed in [20], considering multi-objective functions including system losses, system loading as well as the profit for DISCOs (Distribution Companies). In [21], the author has presented the combine GA and PSO based approach for optimal location and capacity of DG, considering multi-objective constraints like voltage stability, losses and improved voltage regulations. GAs based methods are slow in computation and convergence [11], particularly useful when multi-objective conditions are considered.

This paper proposes a new method for DG placement and sizing based on the line voltage stability index. Previously the author in [22] has used the continuation power flow method to determine the most voltage-sensitive bus in the distribution system which could result in voltage instability in the system. DG is placed on the identified sensitive bus and the size of DG on that bus is increased gradually till the objective function (voltage constraints) is achieved. The proposed algorithm is also working on the same objective function for DG allocation. The developed index is used to identify the most critical bus in the system that can lead to system voltage instability when load increase above certain limit. The DG is placed at the identified bus. The search algorithm is used for estimating the size of DG considering minimum network losses. Overall, this proposed method is simpler and requires less computational time for determining the optimum placement and size of DG as compared to classical search algorithms.

The paper is organized as follows: In Section 2, the impact of DG placement on system losses, voltage profile and stability is discussed. A new analytical approach is used here. Section 3 presents a new algorithm for optimum placement and sizing of DG based on a novel index. In Section 4, the proposed algorithm is applied on 12, modified 12 and 69 bus radial distribution test systems. The results are shown and discussed in detail. To test the effectiveness of proposed method; results are compared with GSS Algorithm.

2. Impact of DG placement – a new analytical approach

For obtaining the maximum benefit from the placement of DG, it is necessary to consider the impact of DG on a power system. The following factors are considered in the placement and sizing of DG.

1. Reduction in line losses.
2. Improvement in voltage profile and stability.

Consider a simple two bus network shown in Fig. 1a and its corresponding phasor diagram in Fig. 1b.

From the phasor diagram, we can write:

\[ \mathbf{V}_r = \mathbf{V}_s - \mathbf{IZ} \]  

(1)

where \( \mathbf{Z} \) is the \( r + jx \) is the impedance of the line.

If we reduce the \( \mathbf{IZ} \) component in the Eq. (1), the receiving end voltage can be improved. There are three ways to reduce the \( \mathbf{IZ} \) components.

1. Provide active power support to the system locally using renewable energy or distributed resources or FACTS (in the present case we are considering the impact of DG only).
2. Provide reactive power support to the system locally using static condensers or FACTS.
3. Use of Anti \( \mathbf{Z} \) element, which is only possible through series capacitance.

The first two methods will result in reduction in \( \mathbf{I} \) component, while the third method will decrease the \( \mathbf{Z} \).

Fig. 2a shows the scenario when we provide active power support \((-\mathbf{P}_0\) to the system locally. The phasor diagram is shown in Fig. 2b, which shows that the introduction of DG will reduce the active line component of the current from \( I \) to \( I' \) \((I > I')\) as the DG size will increase. This will result in lesser \( Ir \) and \( Ix \) drop (for simplicity \( I' \) and \( I'' \) drops are not shown in the phasor diagram).

DG is mostly used for providing active power support to the system. However results could be seen for partially injecting reactive power support at that bus. Fig. 3a shows the scenario when we provide reactive power to the system. This will reduce the reactive current component from \( I \) to \( I' \) \((I > I')\) shown in phasor diagram Fig. 3b, which will results in lesser \( Ir \) and \( Ix \) drop.

![Fig. 1b. Phasor diagram of a simple two bus network.](image)

![Fig. 2a. Active power support.](image)

![Fig. 2b. Phasor diagram for active power support.](image)
For a simple two bus network, the losses in the line is given by,

\[ P_e + jQ_e = I^2 (r + jx) \]  
(2)

where \( P_e \) is the active power loss; \( Q_e \) the reactive power loss; and \( I \) is the line current, given by:

\[ I = \frac{V_i \angle \delta_i - V_r \angle \delta_r}{r + jx} \]  
(3)

where

\[ V_i \angle \delta_i = V_i (\cos \delta_i + j \sin \delta_i) \]
\[ V_r \angle \delta_r = V_r (\cos \delta_r + j \sin \delta_r) \]

From (3) and (2) results in,

\[ |V_i \angle \delta_i - V_r \angle \delta_r|^2 = (P_e + jQ_e) (r + jx) \]  
(4)

From Eq. (2) and Eq. (3), we can conclude that

1. The line resistance and reactance play a very crucial role for voltage stability.
2. The line losses (or current) should be reduced for stable and improved voltages, which is only possible by providing active and reactive power support to the system. Economically it is not possible to provide support on each bus. Therefore the most suitable site and size of DG should be selected from where the maximum benefits could be achieved.

In [23], the author used the following mathematical formulation which needs to be considered in DG placement.

Minimize total active power losses \( P_L \),

\[ \min \left\{ P_L = \sum_{i=1}^{n} |I_i|^2 R \right\} \]  
(5)

Subject to the following generation and voltage constraints:

\[ 0 \leq P_{dg} \leq \sum P_{load} \]  
(6)

\[ |V_{i \min}| \leq |V_i| \leq |V_{i \max}| \quad i = 1, 2, ..., m \]  
(7a)

\[ |V_i| \leq 1 \pm 0.05 pu \quad i = 1, 2, ..., m \]  
(7b)

where \( n \) is the no. of lines; \( m \) the no. of buses; \( P_{dg} \) the distributed generation power; and \( P_{load} \) is the total connected load.

The main constraints as defined in Eqs. (6–7) are to restrain the voltages at each bus along the radial system within the acceptable range and the total active power support should not exceed the system load.

3. Proposed method

The important factor in maintaining the voltage between two nodes is the drop in the line connecting the two nodes, commonly known as voltage regulation. Ideally voltage regulation should be zero, but there are drops due to resistance and reactance of a line. In transmission lines, resistance is much less than the reactance of the transmission lines \( (r \ll x) \); while in overhead distribution system, reactance is much less than the resistance of the line \( (x \ll r) \). There is no anti- resistance element which could improve the voltage regulation. The series capacitor is commonly connected in long transmission lines having high reactance than a distribution network, in order to improve the voltage profile and increasing the system efficiency. However by supporting the active and reactive power demands locally could significantly reduced the voltage drop in the line by reduction in line current and losses and thus improves the system efficiency.

3.1. Development of an index for DG placement

An index is derived for finding the most optimum site of DG based on the most critical bus in the system that can lead to system voltage instability when the load increases above certain limit.

Consider a simple two bus network without and with DG shown in Figs. 1a and 2a, with their phasor diagram also presented in Figs. 1b and 2b.

From Fig. 1 we can write,

\[ S_L = P_L + jQ_L = V_I I_r \]  
(8)

\[ \mathbf{V}_r = \mathbf{V}_i - \mathbf{Z} \]  
(9)

where

\[ I_r = \frac{(P_i) - j(Q_i)}{V_r} \]  
(10)

From Figs. 2a and 3a we can write:

\[ I_r = \frac{(P_L - P_c) - j(Q_L - Q_c)}{V_r} \]  
(11)

Substitute \( I_r \) from Eq. (11) into Eq. (9) and separate into real and imaginary parts will give:

\[ P_L - P_c = \frac{|V_i||V_r|}{V_r} \cos(\theta - \delta_i + \delta_r) - \frac{|V_r|^2}{Z} \cos(\theta) \]  
(12)

\[ Q_L - Q_c = \frac{|V_i||V_r|}{V_r} \sin(\theta - \delta_i + \delta_r) - \frac{|V_r|^2}{Z} \sin(\theta) \]  
(13)

Rearranging Eq. (12) will give:

\[ |V_i|^2 \left( \frac{|V_i|}{V_r} \cos(\theta - \delta) + \frac{Z(P_L - P_c)}{\cos(\theta)} \right) = 0 \]  
(14)

where

\[ \delta = \delta_i - \delta_r \]

The Eq. (14) is a quadratic equation. For stable node voltages, Eq. (14) should have real roots, i.e. discriminant \( B^2 - 4AC > 0 \), which results in the proposed index referred as Power Stability Index (PSI) given by Eq. (15):

\[ |V_i|^2 \left( \frac{|V_i|}{V_r} \cos(\theta - \delta) + \frac{Z(P_L - P_c)}{\cos(\theta)} \right) = 0 \]

where

\[ \delta = \delta_i - \delta_r \]
\[ \text{PSI} = \frac{4 r_j (P_l - P_c)}{|V_j| |\cos(\theta - \delta)|^2} \leq 1 \] (15)

Under stable operation, this value should be less than unity; closer the value of PSI to zero, more stable will be the system.

The above index is used to find the optimum placement of DG. The PSI value is calculated for each line in the given network and sorted from the highest to the lowest value. For the i\(-\)j line having the highest value of PSI, the DG should be placed at j\(-\)bus. For multi DG placement, the location of the second DG will be based on the effect of first DG on PSI using Eq. (15). PSI value for each line will be re-calculated and sorted in the same fashion from highest to lowest. The DG will be placed at the end of the line having the highest value of PSI.

3.2. Optimum sizing of DG

Once the optimum location of DG is identified, the amount of active power from DG changes from 0% to 100% of the total active load, with generation and voltage constraint given in Eqs. (6–7). The main objective in selecting DG size is to minimize total system power losses \( (P_{\text{loss}}) \) by injecting active power \( (P_{\text{dg}}) \), given in Eq. (5). The relation between DG size and losses follow the parabolic curve, first decreases and then starts increases, thus the accuracy of the DG size estimated will depend on the step size selected. In the present case, the step size is maintained 1% of total load. However much smaller size could also be used, but the computation will take much longer time.

![Flow chart of proposed algorithm.](image)
3.3. Proposed algorithm

For a radial distribution network, load flow analysis is carried out and PSI value is computed for each line using Eq. (15). For $i - j$ line having the highest value of PSI, the DG will be placed at $j$th bus. The search algorithm is used for finding the optimum size of DG at optimum location based on a minimum total power loss, with constraints given in Eqs. (6–7). The complete flow chart for DG allocation and sizing is represented in Fig. 4.

4. Simulations and results

In Section 3, the proposed algorithm for DG placement and sizing is presented. For verification, the proposed algorithm is applied on 12-bus, modified 12-bus and 69-bus radial distribution networks. A computer program has been written in MATLAB 7.12 and run on Core 2 Duo 3.07 GHz processor. Shirmohammadi theorem [24] is used to carry out the load flow analysis. As conventional load flows are not suitable for radial distribution systems because they got diverges, due to high X/R ratio which results in singularity of Jacobian matrix.

4.1. Test systems

The test system of 12-bus [25] and 69-bus [26] radial distribution test systems are shown in Figs. 5 and 6 respectively. In modified 12-bus system, the active load on each bus is multiplied by 5 for better visualization of results, as the actual value of load is very small.

4.2. DG placement based on PSI

The load flow analysis is carried out on 12-bus system and the PSI value is computed for each line using Eq. (15) considering initially no DG. The PSI value for each line is shown in Fig. 7a. It could be observed that the 8th line connecting bus 8 and bus 9 have the highest value than the others. So the installation of DG at bus 9 will be the optimum place. The same approach is carried out for modified 12-bus and 69-bus test system; PSI graph for each system is shown Fig. 7b and c respectively. From Fig. 7b and c, it could be observed that the 8th and 60th line in modified 12-bus and 69-bus test system has the highest value of PSI respectively. Hence, the optimum location of DG is at bus 9 and bus 61 for 12-bus and 69-bus test systems respectively.

4.3. Effect of DG on voltage stability

In order to see the effects of DG on voltage stability on any bus, the load at that bus is increases gradually to find out the maximum load that cause instability. For example, in the modified 12-bus system when there is no DG, bus 7 reaches its maximum value of load at 1.2909 MW, after that voltage collapse could be observed as shown in Table 1. The PSI value calculated using Eq. (15) is also shown in Table 1.

From Table 1, it could be seen that:

1. More the value of PSI approach to the unity, more the link will be weak and additional load could result in voltage collapse
2. Adding DG on bus 7th will result in stable voltage operation.
3. The system capacity has increased, more load could be added.

4.4. Effect of DG on reactive power

Although DGs are mostly used for active power support, reactive power could also be injected to supply reactive load in the system. To see the effect of reactive power support ($-Q_G$) to the system, the modified 12-bus system is used as an example. The summary of the test result is shown in Figs. 8 and 9.
Table 1
Effect of load on PSI.

<table>
<thead>
<tr>
<th>S. no.</th>
<th>Load at bus 7 P_L (MW)</th>
<th>V_L</th>
<th>DG at bus 7 P_DG (MW)</th>
<th>Losses P_L Q_L</th>
<th>PSI P_L Q_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04</td>
<td>0.9553</td>
<td>0</td>
<td>0.0207 0.0081</td>
<td>0.00085</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.8896</td>
<td>0</td>
<td>0.1167 0.0426</td>
<td>0.12700</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>0.7879</td>
<td>0</td>
<td>0.4574 0.1630</td>
<td>0.32151</td>
</tr>
<tr>
<td>4</td>
<td>1.1909</td>
<td>0.7224</td>
<td>0</td>
<td>0.7951 0.2817</td>
<td>0.45323</td>
</tr>
<tr>
<td>5</td>
<td>1.2909</td>
<td>0.6307</td>
<td>0</td>
<td>1.4153 0.4996</td>
<td>0.63961</td>
</tr>
<tr>
<td>6</td>
<td>1.3</td>
<td>NC</td>
<td>0</td>
<td>NC NC NC</td>
<td>NC</td>
</tr>
<tr>
<td>7</td>
<td>1.3</td>
<td>0.8349</td>
<td>0.5</td>
<td>0.2715 0.0974</td>
<td>0.22982</td>
</tr>
<tr>
<td>8</td>
<td>1.3</td>
<td>0.8896</td>
<td>0.8</td>
<td>0.1167 0.0426</td>
<td>0.12700</td>
</tr>
<tr>
<td>9</td>
<td>1.3</td>
<td>0.9476</td>
<td>1.2</td>
<td>0.0268 0.0103</td>
<td>0.02247</td>
</tr>
</tbody>
</table>

NC: No convergence.

Fig. 7. PSI value for each line in (a) 12-bus system (b) modified 12-bus system, (c) 69-bus system.

Fig. 8. Effect of Q_DG on system losses of modified 12-bus system.
Fig. 8 shows the results when a reactive power equal to the reactive load on each bus is injected by the DG. It could be seen the active and reactive power losses have reduced as compared to without reactive power support on ith bus.

Fig. 9 shows the effect of reactive power support on system voltage profile. When a reactive power is supplied on bus 9 from the DG, the overall voltage profile has improved.

4.5. Optimum DG size

To determine the optimum size of DG, the proposed algorithm is applied on all test systems and the results are tabulated in Table 2. The proposed algorithm is also compared with the GSS Algorithm, implemented using VS&OP power tool [27]. The results are shown in Table 2, from where it could be seen the close agreement of the propose method with the existing one.

From Table 2, it could be observed that:

- The proposed method results are in close agreement with GSS algorithm
- The computation time has been decreased considerably for three test systems (53.6%, 52.39%, 58.45% respectively with the 12-bus, modified 12-bus and 69-bus).

Fig. 9. Effect of $Q_G$ on system voltage profile of modified 12-bus system.

Table 2

<table>
<thead>
<tr>
<th>Bus system</th>
<th>Proposed Algorithm</th>
<th>Golden Section Search algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bus no.</td>
<td>Optimum size (MW)</td>
</tr>
<tr>
<td>12</td>
<td>9th</td>
<td>0.2349</td>
</tr>
<tr>
<td>Modified 12 Bus</td>
<td>9th</td>
<td>1.2000</td>
</tr>
<tr>
<td>69</td>
<td>61th</td>
<td>1.8631</td>
</tr>
</tbody>
</table>

Fig. 10. System losses profile of modified 12-bus with optimum DG size = 1.2 MW at bus-9.

Fig. 11. Effect of DG on system voltage profile of modified 12-bus system. (a) 12 bus network (DG size = 0.2349 MW @ Bus 9th). (b) Modified 12 bus network (DG size = 1.2 MW @ Bus 9th). (c) 69 bus network (DG size = 1.8631 MW @ Bus 61th).
The DG size vs. loss-curves follow a parabolic curve, first decreases and then increases. Thus the size of DG should be carefully selected, above optimum size of DG the system losses increases which results in poor efficiency and voltage regulation.

4.6. Effect of DG on system losses and voltage profile

In Section 2, we have seen the impact of DG on system losses based on analytical analysis. To verify the derived analytical equations, the effect of DG on modified 12-bus system studied by injecting optimum active power of 1.2 MW on each bus. The total active and reactive losses occurring due to different DG location is shown in Fig. 10. It can be observed that the most suitable location of DG in terms of losses is at bus 9, which is in agreement with the result in Fig. 10.

In Section 2, it has also been shown that the reduction in active and reactive losses improve the voltage profile. From Fig. 11, this could be visualized whereby the optimum placement of DG at 9th bus has improved the overall voltage profile.

4.7. Overall findings

From the analysis of the simulation results presented in Section 4, we can conclude the following due to DG placement.

1. The voltage profile has improved and the system voltage stability has increased.
2. Line losses have reduced.
3. The overall system capacity has increased.

The third statement is better presented in Fig. 12 by connecting the most optimum DG size found out in Table 2 to the most optimum location of each system and the nose curve (PV curve) is plotted.

Fig. 12 demonstrates that the system capacity has increased due to the installation of DG at the best location and of optimum size.

5. Conclusion

In this paper, a new analytical approach is presented on the impact of DG in power system. A new algorithm is also proposed for DG location and sizing. The DG allocation and sizing is based on a novel Power Stability Index (PSI) index to determine the most voltage sensitive bus and minimum total power losses. Using the proposed algorithm optimum DG allocation and correct sizing results in an improved voltage profile and minimizes the burden of system losses.

The proposed algorithm has also been tested using three different radial distribution test systems and the results are verified using GSS Algorithm. It has been found that overall the proposed algorithm takes less computation time by 50–60% as compared to Golden Section Search (GSS) algorithm.

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