OBSERVATIONS OF MATHEMATICS CLASSROOMS IN SOME COUNTRIES

Ronny Kwan Eu Leong  
Universiti Malaya  
<krl2118@columbia.edu>

Edward Ham  
Bakersfield College, California  
<eddyham@gmail.com>

Abstract
Comparative education is the field that generates lots of interest from educationists around the world. Many countries have their own unique methods in the teaching and learning of mathematics. It is important to observe the teaching of mathematics in the classrooms around the world. Not only one could gauge how mathematics education is implemented in one country as compared to others, but ideas for innovative mathematics teaching in classrooms could also be adapted with educational implications. In this article, the authors share their insight on the teaching and learning pedagogies implemented in the mathematical classrooms in Russia, Singapore, Cambodia and Vietnam.

Introduction
Comparative education has become the field that gained much attention in education settings globally. Many educationists have realized the importance to observe the teaching of mathematics around the world. Not only one could gauge how the mathematics education is implemented in one country as compared to others, but ideas for innovative mathematics teaching in classrooms could also be adapted with educational implications. The Department of Mathematics at Columbia University Teachers College understands the importance of comparative education and provides students the opportunity to attend study tours and to travel to foreign countries to observe the teaching of mathematics with report of findings on comparative. These study tours are taken for credit, with the requirement that a daily journal of classroom observations should be kept along with the writing of a research paper at the end of the study tour. A few examples of the countries visited are: Singapore, Vietnam, Cambodia, Prague, Budapest, Chile, Argentina, Australia, Russia, South Korea, and Japan. Future plans for study tours include India and Kenya. This article will only focus on the authors’ classroom observations and insights gained from the visits to Singapore, Vietnam, and Cambodia during the Spring of 2008 and Russia during the Fall of 2009.

Discussion of findings from observational studies
Russia
The observations in Russia were diverse. Visits to elementary schools, high schools, universities, and specialized schools were arranged. Schools with advanced study in mathematics started in Russia during the end of the 1950’s and early 1960’s due to a reform in education to produce students with specialized abilities (Karp, 2009; Vogeli, 1997). This was the beginning of gifted education in Russia. Russian schools for the mathematically gifted were largely influenced by the Hungarian schools of the 20th century. These Hungarian schools were based on the framework of French models (Vogeli, 1997). Specialized schools
started with a boarding school training the computer programmers in 1959. The specialized schools in mathematics and physics were established under the aegis of leading universities in 1963 (Donoghue, et al., 2000). Andrey Kolmogorov, the famous mathematician, founded a boarding school specializing in mathematics in Moscow (Karp, 2009).

An elementary mathematics classroom was observed in School #207, a public school in St. Petersburg specialized in the advanced teaching of English. Class began with students solving problems using mental mathematics; no pencil and paper was allowed, and students had to explain their answers orally. Questions ranged from basic geometry to basic computations, but the goal was to identify that each question had a specific trick or manipulation necessary to make the computations simpler. According to the teacher, all students in grades 5 and 6 spent the first 5-10 minutes of class working on mental mathematics.

High school algebra classes from three different schools (including one school that specialized in gifted mathematics students) in both St. Petersburg and Moscow were observed. Interestingly enough, each class was covering the same topic, i.e. ‘radicals’. However, the instruction was different at each school. In School #207, students spent time in the class solving ten radical problems. While certain students were working independently on the problems, other students were selected to go to the board to solve a problem. The teacher corrected the students at the board immediately, effectively creating a solution guide for the students to follow. In School #222, students were given a handout of fully solved problems combining knowledge of radicals and exponents in which several mistakes were made in each problem. Students independently worked on identifying the mistakes and offered proper solutions to each problem. After some time, students were asked to answer the questions orally.

In School #30, typically there were five classes per grade and 25 students per class. The 9th Grade Algebra class began with a set of questions on radicals and logarithm. The tasks for the students were to determine whether the statements provided were true or false. One student was called to the board to write his answers. If the statement was false, the student was asked to explain. The teacher would help out if the students were having difficulty in explaining. In the Grade 7th Algebra class, the teacher incorporated some history elements on the Babylonians flood to begin her lesson on quadratic equations. Then each student was given a worksheet where they were supposed to represent the mathematical statements symbolically. One unique feature of the classroom was the blackboards were available in front and at the back of the class. The teacher selected student to write his answers on the board on the back of the class while the others did their work individually. Marks were given for each individual. The teacher had a record of it. In the second task of 8 questions on quadratic
equations, students were supposed to find the mistakes and correct them. To show the answers, the teacher had written the answers on the layered board in front of the class. She could just fold the first layer of the board so no time was wasted writing the answers. Next the teacher discussed the answers with the students. A common feature among the different algebra classes was that the discussion was teacher-centred; as the teacher facilitated the questioning and provided answers rather than giving heuristic hints. However, there were also several students who led discussions to explore answers and the emphasis on explanation of the answers given by the students. It is expected that in the future this school will be a model school for Western Russia for computer science and mathematics.

The most interesting class observed was an 11th grade Geometry class in School #222. The entire period was devoted to teacher going over one homework problem about ‘an isosceles trapezoid’, but introducing different methods to attain the solution. The teacher began by asking a student to solve the problem on the board using the steps suggested from the textbook. As this was occurring, another student went up to the board and put up another method of solving the problem. After discussing these solutions, yet another student was asked to present a different method on the board. Once the textbook solution and student solutions were discussed, the teacher introduced two more methods, bringing the total to five different methods of solving one problem.

Another unique characteristic was found in the geometry lesson on ‘polygons and angles’ in school #57 in Moscow. The teacher prepared a systematic set of geometry problems that were in hierarchical order and similar to the Bloom’s taxonomy for students to solve. Different kinds of representation were used by the students to solve the problems. The teacher encouraged the students to discuss in pairs if they found any difficulties. After the lessons, the observers had a short question and answer session with the Assistant Principal. He mentioned that the students of this school were from Grade 1 to Grade 10. Students were only selected to enter the advanced mathematics of humanities classes in Grade 7. What was fascinating was the rigorous selection process for the students to enter the advanced classes. Students had to be interviewed by teachers about 6 times. During the last interview, they were given mathematics problems that must be solved orally by explanation. The teachers then met with them for another 3 times to select the qualified students.

Even though the Russian mathematics classroom was primarily taught using traditional mode, there were elements of constructivism in the classroom. Every student was asked to participate in the classroom by going to the board to solve a problem. This was done quite efficiently as other students did not sit around watching the student in front. They all worked independently on their own problems. Students were also called on to answer the problems orally and give several explanations to their individual questions. Even in a teacher directed classroom, student participation was required and expected of all students. Furthermore, identifying mistakes in mathematical equations or statements seemed to be apparent in most schools. This must be one of the Russian styles of learning mathematics besides teaching mathematics orally.
While in Russia, a trip was also made to the Pedagogical University of St. Petersburg where several professors presented a discussion on teacher preparation in Russia. Prospective elementary teachers were trained at pedagogical colleges while prospective secondary school teachers were trained in a state university or pedagogical universities. In addition, to be a specialist in mathematics education, there were two models to choose from: the ‘specialty’ or ‘stream’ route. The ‘specialty’ model was a traditional model in which the curriculum specializes in mathematics and took five years. In the ‘stream’ model, teachers usually major in two fields, such as physics and mathematics education. It took six years to complete, culminating with a teaching degree and master’s degree. Mathematics teachers in Russia were expected to have a solid grasp of the mathematical content knowledge.

The observers were also briefed on the general education system in Russia. The elementary schools catered for students from Grade 1 to 4. Then the basic schools covered Grade 5 to 9 (11-15 years old) and finally the senior schools enrolled students in Grade 10 and 11 (16-17 years old). Students must take the Uniform State Exam (USE) at the end of Grade 11. The compulsory subjects were the Russian language and mathematics. The students could select any other 3 subjects.

**Singapore**

Singapore’s top level performance in the Trends in International Mathematics and Science Study (TIMMS) studies in mathematics is highly credited due to the Singapore mathematics curriculum framework (SMCF) that emphasizes mathematical problem solving (Dindyal, 2006). Schools in Singapore focus on more than just academics as the curriculum is divided into three strands, i.e. instructional programs, core curricular programs, and character development. The instructional programs focus on academics and require students to take courses in mathematics, sciences, and the humanities. The core curricular programs focus on extracurricular activities outside of the classroom. Schools also focus on character development with the goal of ‘developing proper citizens’. One way to enforce character development is the implementation of the honour code in which students would take exams without supervision from the teachers who would hold faculty meetings. The idea is to develop students as well-rounded citizens who are academically strong, fully involved in society and have strong character. The Singapore mathematics curriculum is something that most of the educators would both admire and want to emulate. In Singapore, the authors were able to observe both the primary and secondary schools, i.e. Northland Primary School and Yushin Junior College.

Walking around Northland Primary School, one could observe the emphasis placed on mathematics just by looking at school hallways that were strewn with number facts and mathematics games. The mathematics department also created a Making Mathematics
Meaningful Centre to differentiate instruction. The aim of the centre was to complement classroom instruction by providing topics to be taught in a variety of ways throughout several stations. At the centre the students were measuring angles using a protractor, building irregular polygons on GeoBoards and calculating perimeter, posing their own word problems, as well as using computers to review and reinforce geometry concepts.

The mathematics department spent a great deal of time developing a curriculum for their students to be successful by incorporating ideas such as Journal Writing, Hands-On Lessons, Weak Item Analysis, Scaffolding, Math Trails, Games, and CODER (an acronym for ‘Clone, Opposite, Data Builder, Extension, and Reverse’). During a classroom observation, students were engaged in a CODER lesson, which was a strategy used for students to take word problems and change them into brand new word problems by modifying key words. Not only did the students create their own problems, but they also solved them to ensure the word problem made sense. In doing, the idea of number sense was reinforced in the students’ minds. Students worked in groups to create word problems, and then presented them to the class. The teachers then collected these student-created problems with several samples were selected to be included in the school’s own Thinkerbell book. Thumbing through this book, one could immediately notice the high level of mathematical achievement from elementary students.

Yushin Junior College was attended by high-school age student who would be fast-tracked to attend universities. Interestingly enough, these courses bore similar features with those in the United States because of the more teacher-centred discussion. When asked about the shift in teaching styles from elementary to junior college, the administrators replied that with such an emphasis on analytical and problem solving skills in elementary, as the students got older, they were expected to be able to process material independently using those skills. By focusing on their skills in their early years of schooling, educators were able to develop independent thinkers by their teenage years.

Vietnam
In 1979, Vietnam had an education reform designed to make education more relevant to the nation's economic and social needs. During the trip to Vietnam, schools in both Hanoi and Ho Chi Minh City were observed. The types of schools were observed to be remarkably different at the Math and Science Gifted High School in Hanoi, and the Vietnam American Pacific University (high school) in Ho Chi Minh City. At the Math and Science Gifted High School, instead of observing the classes, roundtable discussions with participants of the International Mathematics Olympiads were held. The interviewers were briefed that these students spent six hours of the school day doing mathematics and science, while devoting only a few hours to the humanities. During the discussion, students’ notebooks were available for use. Each notebook was found to be organized meticulously with the fact that every single step of work was shown. There was immense competition among the students at
the Math and Science Gifted High School, as they were all competing for several scholarships to gain entrance into the top universities in England.

At the high school in the Vietnam American Pacific University, it was found that the curriculum followed the American educational system, thus providing students with the opportunity to attend American universities. Their mathematics curriculum consisted of Algebra, Geometry, and Algebra 2. However, classrooms were typically lecture-based with ‘drill and kill or practice’ exercises. Interaction between students and teachers were also limited.

**Cambodia**

The next destination was Siem Reap, Cambodia. This time a local high school in Siem Reap was observed. The first observation was made on students that were attending school on a Saturday afternoon. After the abolishment of education in 1975 by the Khmer Rouge, the new Cambodian government had to reconstruct the entire education system in 1979 that aimed to guarantee the universal right to basic quality education. Cambodia is a third world country and was reflected in the school environment. The classrooms were equipped with basic facilities; i.e. regular wooden desks and chairs, but windows were unable to be fully closed, thus allowing the rain to enter the classroom during a passing rainstorm. No air conditioning or fans were observed and the temperatures hovered around a humid 100 degrees Fahrenheit. Yet the students were attentive and motivated to learn.

The topic observed was ‘properties of logarithms’. The teacher would call on students to stand and recite the properties of logarithms. If a student was incorrect, he would have to remain standing until he was told to sit. The introduction of the lesson was mainly teacher-centred; students recited the logarithm properties, then the teacher modelled a simple example on the board, thereby giving the students a few sample exercises to work on independently. Students were asked to go to the board and solve the problems. Interestingly enough, several problems had multiple methods of achieving a solution and the teacher made sure to explain all methods. After this, the teacher put up three difficult problems combining several logarithm properties at once. During this time, students broke out into groups and worked collaboratively on solving these problems.

Even with the poor conditions of the school, the knowledge gained by the students and the learning environment, in general, were impressive. There were no classroom management problems and students followed classroom protocol obediently. At a first glance, the class appeared to be quite traditional, focusing on lecture, memorizing properties with ‘drill and kill or practice’ problems. But as the class progressed, elements of constructivism appeared in cooperative learning in the form of scaffolding, with the interaction between the teacher and students were observed.

**Conclusion**

The idea of study tours organized as discussed in this article was to obtain first hand observation on how mathematics education was implemented in foreign countries. Exposing cultural differences in mathematics education to the participants of study tour was illuminating; but an even greater illumination was allowing prospective teachers the opportunity to learn new ideas in the teaching of mathematics, and further adapting these for their own classroom practices. In so doing, teachers were allowed to broaden their horizons and further enhance the idea of globalization in education.
References

Donoghue, E. F., Karp, A., & Vogeli, B. R. (2000). Russian schools for the mathematically and scientifically talented: Can the vision survive unchanged?


