ASSESSMENT OF ALGEBRAIC THINKING AMONG YEAR FIVE PUPILS

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Abstract

Algebraic thinking in arithmetic acts as bridge to connect arithmetic and algebra. Thus, fostering algebraic thinking in primary school level has been emphasised among mathematics researchers and educators. The literature has shown numerous evidence that young students are able to think algebraically. In addition, infusing algebraic thinking in young students will enable a smooth transition from arithmetic to algebra in later stages of education. The main aim of this study is to assess the algebraic thinking of year five pupils (aged 11 years) in one district in Malacca. The sample of this study comprised 263 year five pupils selected from two national primary schools in Malacca. Descriptive study was employed as a methodology for this paper. The data were collected using the Algebraic Thinking Diagnostic Assessment (ATDA) adapted from Ralston (2013). The results of this study showed that both male and female year five pupils in one district of Malacca outperformed in generalised arithmetic strand than modelling and function. Furthermore, there was a significant difference between male and female pupils in generalised arithmetic strand, while no significant difference for modelling and function strands by gender. These findings could shed some light on algebraic thinking in primary school level.

Keywords: early algebra, equal sign, patterns, primary school students, algebraic thinking

INTRODUCTION

Successful completion of secondary school algebra often acts a stepping stone for higher level mathematics (Kaput, 1999). However, algebra mastery among secondary school students is questionable. In Malaysia, poor performance of form two (eighth grade) students in algebra is evident from recent results of an international survey, Trends in Mathematics and Science Study 2011, commonly known as TIMSS. For the domain of algebra, Malaysia was ranked 23rd out of 42 countries. Of even greater concern was that the average score for algebra content among Malaysian students in 2011 was 430, a significant drop from the 2007 score of 454 (Mullis, Martin, Foy, & Arora, 2012). Hence, algebraic thinking of primary
school students should be investigated to identify the root cause of this poor performance. Unfortunately, Malaysia does not participate in TIMSS for the grade four category. Thus, there are no sources to provide the algebraic thinking level of primary school students in Malaysia. This study was motivated by the need to assess year five pupils' algebraic thinking in arithmetic.

LITERATURE REVIEW

The following sections discuss what is algebraic thinking, the framework underlying this study and some relevant studies that have been carried out in primary school level algebraic thinking.

What is Algebraic Thinking?
The transition from arithmetic to algebra has been a major obstacle for many students (Warren, 2003). When moving from primary to secondary school, students' thinking should progress from arithmetic to algebra. This transition involves a shift from knowledge required to solve arithmetic equations to knowledge used when solving algebraic equations. The difference between arithmetic and algebra is that arithmetic requires working from the known to unknown and algebra is working with unknowns (Van Amerom, 2002). Unfortunately, many do not see the link between arithmetic and algebra, as evidenced by the strict separation of these two topics in the mathematics curricula of many countries (Cai & Moyer, 2008; Carraher, Schliemann, Brizuela, & Earnest, 2006). To overcome this problem, many mathematics researchers have proposed encouraging primary school students to think algebraically by infusing underlying algebra elements in arithmetic (Carraher et al., 2006; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Lannin, Barker, & Townsend, 2008).

Algebraic thinking has been viewed from various perspectives in the literature. According to Mason (1996), algebraic thinking consists of identifying similarity and dissimilarities, spotting differences, classifying and labelling including algorithm seeking. Kieran (1996) has described algebraic thinking as "the use of any of a variety of representations that handle quantitative situations in a relational way" (pp. 274-275). To elaborate this, Blanton and Kaput (2003) highlighted that algebraic thinking is also dealing with representations and looking for relationships. Arithmetic thinking focuses on known numbers and calculations while algebraic thinking entails looking for relationships between numbers and the ability to generate the general case. Warren (2003) claimed that encouraging students to do activities involving looking for relationships between quantities and representing the relationships between quantitative situations will foster algebraic thinking. To confine these widespread definitions and perspectives about algebraic thinking, Kaput (2008) has classified algebraic thinking into two aspects, generalisation and symbolisation. Three strands further developed from these two aspects. They are generalised arithmetic, modelling and function. Various perspectives of algebraic thinking fall into one of these strands.

Three Strands of Algebraic Thinking
Kaput (2008) defined generalised arithmetic as "the study of structures and systems abstracted from computations and relations, including those arising in arithmetic (algebra as generalised arithmetic) and in quantitative reasoning" (p. 11). According to this definition he has further classified generalised arithmetic into efficient numerical manipulation and generalisation. Efficient numerical manipulation refers to identifying the easier way to solve an equation without computing step-by-step. For an example, in the equation as such 57 + 38 = __ + 34, the ordinary students would find the total on the left side which is 93. Followed by that they would find how much needs to be added to 34 on the right side to get 93. These students would get the right answer of 59. However, efficient manipulation requires one to solve it by observing the relationship between 36 and 34. Thus, students should get the right answer (59) by using compensation strategy. Compensation strategy refers to the ability to identify the difference between 36 and 34 as two. So students should add 2 to 57
for compensation. Acquisition of this efficient manipulation skill will lead students to excel in manipulation of variables in later years of secondary school and tertiary level mathematics (Jacobs et al., 2007). While learning arithmetic in primary school, students should be taught about fundamental properties of numbers and operations. This is different from relying on algorithms and memorising mathematical properties.

Likewise, students should be able to make generalisations. It refers to making a general case which can cover many instances. For example, addition of two odd numbers always gives an even number shows generalisation about addition. In depth understanding of properties of operations and relationships of numbers will foster students in making generalisations (Hunter, 2013). Properties of operations comprised of commutative, associative and distributive. These properties get least consideration in the primary school mathematics classrooms as teachers failed to emphasise them. Understandings of these properties will build a strong foundation for easier understanding of algebra problems in later stages of education.

Similarly, properties of odd, even and zero will provide a foundation for better understanding of algebra. For instance, solving quadratic equations always involve equating it to zero. The property of zero whereby any number multiplied with zero will be a zero initiates a fundamental algorithm for solving quadratic equations.

The second strand of algebraic thinking as classified by Kaput (2008) is modelling. It refers to “the application of a cluster of modelling languages both inside and outside of mathematics” (Kaput, 2008, p. 11). Activities such as open number sentences, equivalence and working with variables are fall in this strand. Solving open number sentences such as $7 \times _{\_} = 42$ is very common in the primary school curriculum. These kinds of activities which require students to find the unknowns are algebraic in nature and create a platform for them to master the properties and relationships between arithmetic operations (Carrarher & Schliemann, 2007).

Equivalence plays an important role in algebraic thinking of young students (Ralston, 2013). It involves three components namely a) the meaning of two quantities being equal, b) the meaning of the equal sign as a relational symbol, and c) the idea that there are two sides to an equation” (Rittle-Johnson & Alibali, 1999, p. 177). Relational understanding of equal sign is important in fostering algebraic thinking (Carpenter, Levi, Berman, & Chilin, 2005; Rittle-Johnson & Alibali, 1999). Unfortunately, students often interpret the equal sign as a symbol to write the result of an arithmetic operation. Carpenter and colleagues (2005) asserted that “Children in the elementary grades generally consider that the equal sign means to carry out the calculation that precedes it; this is one of the major stumbling blocks when moving from arithmetic to algebra” (p. 84). Hence, this conceptual misunderstanding of equal sign leads to procedural error when solving algebra problems at higher education levels.

Subsequently, working with variables is inevitable in algebra. Literature has documented working with literal symbol is the major stumbling block of students. According to Ralston (2013), even though primary school students may not be ready to demonstrate the understanding of meaning of variable, they definitely would be ready to understand that the symbol represents a number. This brief introduction to symbol may help to prevent common misunderstanding of variable in middle and high school later.

Function is the final strand in Kaput’s (2008) classification of algebraic thinking. He claimed function as “the study of functions, relations, and joint variation” (p. 11). At early age, young pupils may not be able to work with functions involving $x$ and $y$. Ability to work with patterns has been emphasised highly in the literature as foundation to work with functions in later stages of education (Cai & Moyer, 2008; Lannin et al., 2006; Ralston, 2013).

As such, recognising and generalising patterns and looking for relationship between quantities potentially enable students to move smoothly from arithmetic to algebraic thinking (Lannin et al., 2005; Warren & Cooper, 2008). The ability to work with patterns is defined as capability to recognise, describe, extend, and create patterns (Ralston, 2013). Patterns can
be numerical and figural. A series of numbers or figures that could be predicted or exhibit certain regularity forms a pattern. Patterns can be repeating patterns or linear and non-linear patterns. Patterns also can be represented in the form of table to enable young students to grasp functional thinking (Warren, Cooper, & Lamb, 2005). Often in-out table is presented in order to introduce function to young children.

Predicting the subsequent number or figure when the first three terms are given is a good exercise for generalisation (Lannin et al., 2006). Students can be exposed to near generalisation and followed by far generalization. Far generalisation is when they are required to predict the 10th or 20th term when the first three terms are given. To perform far generalisation, students may be required to form a 'rule' based on the first three terms. The 'rule' may allow them to predict arbitrary terms easily without the need to find each term in the sequence. This skill is also required in TIMSS 2011: "exploring well-defined number patterns, investigating the relationships between their terms, and finding or using the rules that generate them" (Mullis, Martin, Ruddock, O'Sullivan, & Preuschof, 2009, p. 24).

**Past studies on early algebraic thinking**

Extensive studies on young students' algebraic thinking have been carried out internationally and a few nationally. Rittle-Johnson and colleagues (1999) investigated the effect of students' mathematical equivalence conceptual knowledge on their solution procedures. The authors focused on the students' conceptual understanding about equal sign and its effect on solving mathematics equivalence. They found that the conceptual knowledge influenced highly than procedural knowledge in solving mathematical equivalence (i.e., \(3 + 4 + 5 = 3 + \underline{3}\)).

Jacob and colleagues (2007) asserted that teachers' knowledge on algebraic thinking plays an important role in fostering student algebraic thinking. Their study investigated the difference in performance between two groups of students where the former was taught by teachers who had participated in professional development on algebraic reasoning, while the latter group was taught by non-participating teachers. The students' relational understanding was studied. The findings showed students in participating teachers group were able to exhibit better understanding of the equal sign and utilised various strategies reflecting relational thinking than the students in non-participating teachers group.

In Malaysia, Gan (2008) has studied the year five pupils' strategies while solving pre-algebraic problems and the elements of algebraic thinking underlying their solution processes. His study involved clinical interview with 13 pupils in year five from Sarawak state of Malaysia. This study found that year five pupils do pose some elements of algebraic thinking such as near generalisation. However, they have yet to exhibit explicit acquisition of algebraic thinking.

**METHODOLOGY**

**Objectives and Research Questions**

The objectives of this study are twofold as follows:

1) To assess year five pupils' algebraic thinking in strands of a) generalised arithmetic, b) modelling, and c) function in one district in Malacca.

2) To analyse if there is difference between male and female pupils' achievement in the Algebraic Thinking Diagnostic Assessment.

In line with the objectives this study aimed at answering the following research questions:

1) What is the year five pupils' algebraic thinking in strands of a) generalised arithmetic, b) modelling, and c) function in one district in Malacca?

2) Is there any significant difference between the male and female year five pupils' achievement in the Algebraic Thinking Diagnostic Assessment?
Participants
This study only involves national primary schools in one district in Malacca. Two national schools were randomly selected. Once the schools were chosen, entire year five pupils from each school were included in the study. The participants numbered 263 (133 males (51%) and 130 females (49%)).

Research Instrument
This study is quantitative in nature using descriptive statistics and Mann-Whitney U test for data analysis. The Algebraic Thinking Diagnostic Assessment (ATDA) was used to collect data. ATDA was adapted from Ralston (2013), which consists of items from three algebraic thinking strands based on Kaput's (2008) framework, as shown in Table 1. Cronbach's alpha was used to determine the reliability in the Statistical Packages for the Social Sciences (SPSS) version 22.0. The total reliability of the 28 test items is .73. This shows the instrument reliability falls in the acceptable range. Some 26 items are dichotomous which are scored correct/incorrect and two items are short-answer items scored with rubrics. Figure 1 shows the main three strands of ATDA together with its sub-strands.

Table 1: ATDA Items According to its Strands

<table>
<thead>
<tr>
<th>Strands</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Generalised arithmetic</td>
<td>10</td>
</tr>
<tr>
<td>2: Modelling</td>
<td>10</td>
</tr>
<tr>
<td>3: Function</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 1: ATDA framework adapted in Ralston (2013).
RESULTS

This section discusses the findings related to the performance of year five pupils in one district in Malacca based on the ATDA. Overall performance score of year five pupils in the ATDA is less than 50% with a sample mean of 43.17 ($M = 43.17$, $SD = 17.96$). The lowest percentage score on this test is 9.68% and the highest is 93.55%. Descriptive statistics were used to analyse the first research question, "What is the year five pupils' algebraic thinking in strands of a) generalised arithmetic, b) modelling, and c) function in one district in Malacca?" Total score for strand 1, 2 and 3 are 12, 11 and 8 respectively. Thus, the total score for ATDA is 31. Table 2 shows the mean score of 263 participants on each strand.

<table>
<thead>
<tr>
<th>Strand</th>
<th>$M$</th>
<th>$SD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalised Arithmetic</td>
<td>47.59</td>
<td>19.47</td>
</tr>
<tr>
<td>Modelling</td>
<td>40.72</td>
<td>24.86</td>
</tr>
<tr>
<td>Function</td>
<td>39.97</td>
<td>24.15</td>
</tr>
</tbody>
</table>

Based on Table 2, strand 3 which deals with patterns as it is the root for function posed great difficulty to students as indicated by the average lowest total percentage score (39.97%). Strand 2 also posed difficulty to students as indicated by the average lowest total percentage score (40.79%). However, it is slightly better than performance in function. Students have outperformed in generalised arithmetic than modelling and function.

| Table 3: The Algebraic Thinking Strands’ Mean Score of Male and Female Year Five Pupils |
|-------------------------------------|--------|-------|--------|-------|--------|-------|--------|-------|-------|
| Strands    | Male    | $M$   | $SD$  | $n$   | Female | $M$   | $SD$  | $n$   |
| Generalised Arithmetic              | 44.99  | 19.16 | 133   |       | 50.26  | 19.49 | 130   |
| Modelling                            | 40.05  | 25.15 | 133   |       | 41.40  | 24.65 | 130   |
| Function                             | 39.38  | 25.17 | 133   |       | 40.58  | 23.14 | 130   |

As shown in Table 3, the data on year five pupils' algebraic thinking from a sample of 133 male and 130 female, with male sample mean of 44.99 ($M = 44.99$, $SD = 19.16$) and female sample mean of 50.26 ($M = 50.26$, $SD = 19.49$) for the generalised arithmetic strand. Female pupils had outperformed male pupils in the generalised arithmetic strand. As for the modelling strand, the male pupils had a sample mean of 40.05 ($M = 40.05$, $SD = 25.15$) while the female pupils obtained a sample mean of 41.40 ($M = 41.40$, $SD = 24.65$). Female pupils again had performed better than male pupils in this strand. Finally, as for the function strand male pupils had sample mean of 39.38 ($M = 39.38$, $SD = 25.17$), while the female pupils had sample mean of 40.58 ($M = 40.58$, $SD = 23.14$). Female pupils' performance was better than male pupils in this strand. When comparing the three strands, male and female pupils had performed better in generalised arithmetic compared to modelling and function.

To answer the second research question on the mean difference in algebraic thinking by gender, the researcher intended to use the independent $t$-test. For this, the assumption of normality of the data was checked using Shapiro-Wilk test. In Shapiro-Wilk test it is found that $P$ values were 0.00 for each strand. This shows the data failed to fulfill the normality assumption (Field, 2012). Therefore, nonparametric Mann-Whitney $U$ test was chosen to analyse gender effect on algebraic thinking strands. If there is a significant effect, test outcomes’ effect sizes also were calculated to decide if the effect is substantial. According to Pallant (2013), if absolute value of $r$ (abs[r]) is 0.1 then the effect size is small, if (abs[r]) is 0.3 then medium, and if (abs[r]) is 0.5 then the effect size is large.
The outcome of the Mann-Whitney $U$ test showed that there is statistically significant difference between the male and female pupils in generalised arithmetic strand ($z = -2.22, p = 0.03$). This result suggested that the gender does have an effect on the generalised arithmetic strand. While, there is no significant difference between male and female pupils in modelling ($z = -0.55, p = 0.58$) and function ($z = -0.32, p = 0.75$) strands. This showed there is no difference between both male and female pupils' performance in modelling and fractions strands. To find the significant difference between male and female pupils, the effect size for Mann-Whitney $U$ test was calculated. For the generalised arithmetic strand, the effect size ($\text{abs}[r] = 0.17$) was computed and it is found to be small.

**DISCUSSION AND CONCLUSION**

The present study investigated year five pupils’ algebraic thinking in three strands; a) generalised arithmetic, b) modelling, and c) function in one district of Malacca. The findings showed some similarities and differences in between these three strands.

The results showed that year five pupils in one district in Malacca outperformed in generalised arithmetic strand than modelling and function for both male and female pupils. It shows that year five pupils exhibit some algebraic thinking elements such as properties of zeroes and relational thinking. They exhibit the understanding that both sides of the equal sign should be equal when the item require them to determine $89 + 44 = 87 + 46$ is true or false. The pupils also demonstrated good understanding of properties of zero by providing conceptual reason that any number multiplied with zero must be zero too.

However, the particular district’s year five pupils’ weakest strand was function. It showed they were unable to work with patterns. Not many were able to perform near and far generalisation by finding the ‘rule’ in both numerical and figural patterns. This results also create an awareness to focus into Malaysian students’ algebraic thinking starting from primary school level. As mentioned previously, Malaysian form two students’ performance in algebra in TIMSS 2011 was very poor. Pattern is one of the most important elements focused in TIMSS 2011 (Mullis et al., 2009). Does the poor foundation in primary school level causes the poor performance of form two students in TIMSS? This question deserves further investigation.

When comparing the mean, female year five pupils had outperformed male students in all three strands. However, the inferential statistics results showed that there is significant difference in by gender in generalised arithmetic strand, while there is no significant difference between gender in modelling and function strands. Results indicated that overall year five pupils in the particular district in Malacca exhibit better performance in the generalised arithmetic strand which involves efficient numerical manipulation and generalisation.

In future, studies should be carried out using bigger samples covering wider geographic area within Malaysia to get comprehensive results of year five pupils’ algebraic thinking. The study could be extended to comparison of urban and rural area students’ algebraic thinking in all of Malaysia. This may allow us to identify the root cause of Malaysian students’ poor performance in international mathematics assessments.
References


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