Double-diffusive natural convection in a triangular solar collector

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A B S T R A C T

The current study deals with mathematical study for natural convection flow resulting from the combined buoyancy effects of thermal and mass diffusion inside a triangular shaped solar collector. The solution is performed assuming the isothermal and isoconcentration boundary conditions of absorbers and covers of collector. The two-dimensional governing equations for the physical phenomenon are expressed in a velocity-pressure formulation, along with the energy and concentration balance equations. A finite element method is used to solve these equations. Effects of the thermal Rayleigh number and buoyancy ratio are presented by streamlines, isotherms, isoconcentration as well as local and mean heat and mass transfer rates for the aforesaid parameters. Comparison with the previously published work is made and found to be an excellent agreement. Particularly, the design for enhancing the performance of the collector is determined by examining the abovementioned results.

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1. Introduction

Solar energy is abundant and clean; it is meaningful to substitute solar energy for conventional energy. Solar energy therefore has an important role to play in the building energy system [1–2]. The collectors can be in different shape. They can design rectangular, triangular or trapezoidal, etc. and they are the main piece of the solar energy systems.

Double-diffusive natural convection occurs within the solar collector, owing to the collective thermal and mass diffusion buoyancy effects as well as the temperature difference between cover and absorber. Nowadays, the study of solar collectors is one of the most significant works to progress their performance with a competitive price. Numerical solution has the advantage over an experimental investigation in that the important parameters, such as geometrical dimensions, glass thickness and covenant location may be simply changed. Therefore, its persuading on the overall heat and mass transfer can be studied at a low price. Numerical investigation is also helpful in testing the performance of solar collectors for different components.

In solar collectors, the fresh water manufacture is governed by the natural convection of the bulk moist air enclosed in the area between a transparent cover and water in the collector basin. The inside of the collector is heated by the solar energy when sunlight enters the cell through a glass top. This energy allows the water temperature to rise until the water vaporization is achieved.

Several experiments and theoretic investigations have been undertaken in this region. Boukar and Harmim [3] evaluated the performance of one-sided vertical solar still tested under desert climatic conditions of Algeria, in summer and autumn seasons 2003. The same authors [4] experimentally investigated design parameters of an indirect vertical solar still. Omri et al. [5] examined the thermal exchange by natural convection and effect of buoyancy force on flow structure. The authors provided useful information on the flow structure sensitivity to the governing parameters, the Rayleigh number and the tilt angle on the thermal exchange. Omri [6] numerically studied the flow characteristics inside an asymmetrical triangular still for the configuration optimization. Gao et al. [7] performed a study on natural convection inside the wavy and inclined solar collectors, but they did not consider flow behavior and thermal fields. A numerical experiment is performed for inclined solar collectors by Varol and Öztöp [8].

Nowadays, a small number of mathematical models of the energy balance equations describing the heat and mass transfer in a solar still have been presented. The main envoy of such studies is those proposed by Selcuk [9] and Sodha et al. [10]. Chouikh et al. [11] numerically investigated the heat mass transfer in the inclined glazing cavity for solar brackish water desalination. Joudi et al. [12] numerically investigated the performance of a prism shaped storage solar collector.
with a right triangular cross sectional area. Kalogirou [13] offered an in general review of a huge variety of systems used in desalination processes. The author gave some universal guidelines for choice of desalination systems motorized by renewable energy. The author concluded on the major design parameters that require to be measured in this field. Different types of solar still available in the literature are conventional solar stills, single-slope solar still with passive condenser, double condensing chamber solar still [14], vertical solar still [15], the inverted absorbers solar still [16] and multiple effect solar still [17]. The heat transfer inside the isosceles triangular enclosure is studied by Varol et al. [18–20] for different thermal boundary conditions, including entropy generation.

It is clear from the above literature survey that double diffusive convection is not studied in earlier works for triangular types of solar collectors. Thus, the main aim of the present work is to present numerical results on natural convection heat and mass transfer in a triangular shaped enclosure for different buoyancy ratio and Rayleigh number.

2. Problem statement

2.1. Physical description

The geometry for the configuration is shown by a schematic diagram in Fig. 1. The model considered here is a triangular type solar collector. The enclosed space consists mostly of an absorber plate and two inclined glass covers that form a cavity. The absorber plate is represented by a horizontal bottom wall kept to a constant temperature $T_h$ while the inclined walls are considered transparent and maintained at a constant temperature $T_L$ with $T_h > T_L$.

Fig. 1. Schematic diagram for the problem.

2.2. Mathematical formulation

The salty water on the bottom side of the glazing enclosure is vaporized from the liquid–vapor interface. The vapor moves through the air and condenses at the cooled side walls. Conservation equations are written using the velocity–pressure formulation and considering constant properties except for the density in the body force term. For steady two-dimensional laminar convection the coupled transport equations for $U$, $V$, $\theta$ and $C$ written in Cartesian coordinates in the dimensionless form as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$  \hspace{1cm} (1)

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$  \hspace{1cm} (2)

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra_T Pr (\theta + Br C)$$  \hspace{1cm} (3)

![Fig. 2. Grid independency study for $Ra = 10^6$, $Pr = 0.71$ and $Br = 5$.](image)
\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \]  

(4)

\[ U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Le} \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) \]  

(5)

where the dimensionless variables are introduced as:

\[ X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\nu}, \quad V = \frac{vH}{\nu}, \quad P = \frac{(p + \rho g y)H^2}{\rho \nu^2}, \quad \theta = \frac{T - T_L}{T_h - T_L}, \quad \text{and} \quad C = \frac{c - c_L}{c_h - c_L} \]  

(6)

The variables have their usual sense in fluid mechanics and heat transfer as listed in the nomenclature. The dimensionless parameters appearing in the above equations are the Prandtl number (Pr), the thermal Rayleigh number (Ra), Lewis number (Le) and buoyancy ratio (Br), which are defined respectively as

\[ Pr = \frac{\nu}{\alpha}, \quad Ra = \frac{g \beta \Delta T H^3 \rho \nu^2}{\alpha}, \quad Le = \frac{\alpha}{D}, \quad \text{and} \quad Br = \frac{\beta \Delta C}{\rho \beta (T_h - T_L)} \]  

(7)

The dimensionless boundary conditions corresponding to the considered problem are

on the bottom wall: \( U = V = 0, \theta = C = 1 \)

on the inclined walls: \( U = V = 0, \theta = C = 0 \)

The local heat and mass transfer rates on the surface of heat and contaminant sources are defined respectively as

\[ Nu_x = -\frac{\partial \theta}{\partial Y} \bigg|_{\gamma, \alpha} \quad \text{and} \quad Sh_x = -\frac{\partial C}{\partial Y} \bigg|_{\gamma, \alpha} \]  

The average heat and mass transfer rates on the surface of heat and contaminant sources can be evaluated by the average Nusselt and Sherwood numbers, which are defined respectively as

\[ Nu = -\frac{1}{D} \frac{\partial \theta}{\partial Y} dX \]  

(8)

Fig. 3. Comparison of the (a) present model with (b) the results of Kent [21] for natural convection inside a isosceles triangular enclosure for five different base angles.

Fig. 4. Effect of buoyancy ratio on streamlines for (a) \( Ra = 10^4 \), (b) \( Ra = 10^5 \) and (c) \( Ra = 10^6 \).
3. Numerical Solution

3.1. Solution procedure

The Galerkin weighted residual method of finite element formulation is used as a numerical procedure in this work. The finite element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. In the present study erratic grid size system is considered especially

\[ Sh = -\frac{1}{\alpha} \int_{\alpha} \frac{\partial C}{\partial Y} \, dX \]  

The stream function is calculated from

\[ U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X} \]  

![Fig. 5. Effect of buoyancy ratio on isotherms for (a) \( Ra = 10^4 \), (b) \( Ra = 10^5 \) and (c) \( Ra = 10^6 \).](image)

![Fig. 6. Effect of buoyancy ratio on isoconcentration for (a) \( Ra = 10^4 \), (b) \( Ra = 10^5 \) and (c) \( Ra = 10^6 \).](image)
near the walls to capture the rapid changes in the dependent variables. The coupled governing Eqs. (2)–(5) are transformed into sets of algebraic equations using finite element method to reduce the continuum domain into discrete triangular domains. The system of algebraic equations is solved by iteration technique. The solution process is iterated until the subsequent convergence condition is satisfied: 

$$|\Gamma^{m+1} - \Gamma^m| \leq 10^{-6}$$

where \( m \) is number of iteration and \( \Gamma \) is the general dependent variable.

### 3.2. Grid refinement check

A grid independence analysis is carried out in this section to make sure the accuracy of the numerical results and to find out a proper size of grids for the case of \( Ra_T = 10^6, Pr = 0.71 \) and \( Br = 5.0 \). In these study five different non-uniform grids of triangular elements: 1486, 2808, 3186, 4490 and 5212 are used. The value of average Nusselt and Sherwood numbers inside the cavity are used as a sensitivity measure of the accuracy of the solution and are selected as the monitoring variables for the grid independence study. Fig. 2 shows the convergence of average Nusselt and Sherwood numbers with grid refinement. The grid size of about 3186 elements was therefore, chosen for all cases studied.

### 3.3. Code validation

The study is compared with an earlier work on natural convection in a triangular enclosure was performed by Kent al. [21]. Comparative results are illustrated in Fig. 3 for different inclination angle which is studied by Kent [21]. He used Fluent commercial code as numerical analysis. The comparisional results showed a good agreement with literature on heat transfer and fluid flow. Then, we jumped to next step to add mass transfer to simulate solar collectors.

### 4. Results and discussion

In this work, numerical analysis of convection inside an isosceles solar collector has been made using finite element technique. Air was used as working fluid inside the cavity with \( Pr = 0.71 \). The Lewis number was held fixed throughout this investigation at \( Le = 2.0 \).

Fig. 4 illustrates the effects of the buoyancy ratio on streamlines at different Rayleigh numbers. Rayleigh number is calculated based on the temperature difference and buoyancy ratio denotes the relative strengths of two buoyancy forces namely mass and thermal buoyancy forces. As given from the figures two circulation cells were formed in different rotation directions. The left cell rotates counterclockwise and the right one clockwise. In other words, symmetric flow distribution is observed according to the middle axis of the triangle. Both Rayleigh number and buoyancy ratio enhance the flow strength. The maximum stream function value is obtained for \( Br = 20 \) and \( Ra = 10^6 \). For these cases, isotherms are plotted in Fig. 5. As seen from the figure, conduction mode of heat transfer is dominant inside the cavity for \( Ra = 10^4 \) but plume like temperature distribution has become stronger from \( Br = 0.5 \) to \( 20 \) and \( Ra = 10^6 \). However, temperature distribution showed similar behavior for \( Ra = 10^5 \) and \( Br = 5, 10 \) and \( 20 \) due to strong convection inside the cavity. Isoconcentration is given in Fig. 6. Distribution of isoconcentration exhibits a similar distribution with temperature distribution.

Fig. 7 illustrates the distribution of local heat transfer rate along the bottom wall. It is an interesting result that heat transfer becomes a minimum at \( X = 0.5 \) for all parameters. The reason of this value is the stagnation point and minimum velocity at this point. Trend of local heat transfer rate exhibits a symmetric distribution according to the mid-axis. As an expected result, highest heat transfer is formed for \( Ra = 10^6 \). Heat transfer rate decreases from the left and right edge to the mid-axis. However, it has maximum values around \( X = 0.25 \) and 0.85 for higher values of the buoyancy ratio. If we look at the variation of average heat transfer rate with the buoyancy ratio, it shows
increasing trend with buoyancy ratio and Rayleigh number (Fig. 8). The highest value is formed at $Ra = 10^6$ and $Br = 20$. Local mass transfer rate is given in Fig. 9. Its trend is similar with the local heat transfer rate, and it decreases with decreasing of the buoyancy ratio for all Rayleigh numbers. Finally, average mass transfer is shown in Fig. 10 that a linear increasing is obtained according to the buoyancy ratio. Average value of mass transfer rate increases with increasing of Rayleigh number.

5. Conclusions

This study illustrates a numerical work concerning the fluid flow, heat transfer and mass transfer inside triangular shaped solar collectors. The results are obtained according to different values buoyancy ratio and Rayleigh number. Some important observations can be listed as follows:

- Both heat transfer and mass transfer increase with increasing of buoyancy ratio and Rayleigh number.
- Symmetric flow field, temperature distribution and mass distribution are found according to the middle axis of the triangular cavity.
- Increasing of the buoyancy ratio at the same Rayleigh number enhances the local heat and mass transfer rate.
- Local heat transfer and mass transfer have minimum value at the middle of the bottom wall due to stagnation point and motionless fluid at that point.

References