The use of ELM-WT (extreme learning machine with wavelet transform algorithm) to predict exergetic performance of a DI diesel engine running on diesel/biodiesel blends containing polymer waste

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ABSTRACT

In this study, a novel method based on Extreme Learning Machine with wavelet transform algorithm (ELM-WT) was designed and adapted to estimate the exergetic performance of a DI diesel engine. The exergetic information was obtained by calculating mass, energy, and exergy balance equations for the experimental trials conducted at various engine speeds and loads as well as different biodiesel and expanded polystyrene contents. Furthermore, estimation capability of the ELM-WT model was compared with that of the ELM, GP (genetic programming) and ANN (artificial neural network) models. The experimental results showed that an improvement in the exergetic performance modelling of the DI diesel engine could be achieved by the ELM-WT approach in comparison with the ELM, GP, and ANN methods. Furthermore, the results showed that the applied algorithm could learn thousands of times faster than the conventional popular learning algorithms. Obviously, the developed ELM-WT model could be used with a high degree of confidence for further work on formulating novel model predictive strategy for investigating exergetic performance of DI diesel engines running on various renewable and non-renewable fuels.

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1. Introduction

Biodiesel (methyl or ethyl esters of long chain fatty acids) has received increasing attention due to its unique characteristics and similar properties to those of petro-diesel [1–4]. Biodiesel has a higher cetane number compared to the petro-diesel because of its higher oxygen content lead to a more complete combustion and subsequently lower emissions. However, it is well-understood that the heating value of biodiesel is lower than that of the petro-diesel [5]. In an attempt to address this shortcoming, Mohammadi et al. [6,7] dissolved a small amount of EPS (expanded polystyrene) wastes in biodiesel and reported promising improvements in engine performance parameters. In spite of that, advanced engineering analyses like thermodynamic approaches should still be applied to assess the sustainability and renewability of diesel—biodiesel–EPS blends by designating the most eco-friendly fuel and recognizing the best operating conditions.

In recent years, exergy analysis has found a growing interest amongst researchers for improving the performance of gasoline and diesel engines as well as for designing new fuel blends with higher exergy efficiencies [8–11]. Generally, exergy is the maximum amount of useful work which can be attained from a
In our very recent investigation, the exergetic and sustainability parameters of a DI diesel engine was remarkably improved using polymer waste dissolved in biodiesel as a novel diesel additive [20]. It was found that the inclusion of 50 g EPS/L biodiesel was a promising strategy to achieve the maximum exergetic efficiency and sustainability (e.g., 40.21% and 1.67, respectively). However, combustion is a quite dynamic, complex, transient, and uncertain phenomenon, whose undergoing mechanisms are not yet fully discovered. In the case of diesel–biodiesel–EPS blend combustion, the complexity becomes even more pronounced due to the biological nature of biodiesel. Due to these reasons, recognition of relationships between inputs and outputs of an ill-defined system such as internal combustion engine using phenomenological, mathematical, statistical, and analytical techniques is much more difficult [21]. Nevertheless, these relationships can be easily detected by using heuristically-soft computing techniques because of their capability to learn about the combustion process to be modelled without the need for any priori information. In fact, modern computational approaches are applied nowadays for solving the real-world problems and determining the optimal values and functions and are receiving enormous attention by researchers in different scientific disciplines.

Artificial neural network (ANN), as a major computational approach, has been introduced and applied in different engineering fields during the past decades. This method is capable of solving complex nonlinear problems which are difficult to solve by conventional statistical and mathematical approaches. There are many algorithms for training ANN such as back propagation, SVM (support vector machine), HMM (hidden Markov model). The shortcoming of ANN is its learning time requirement. To overcome this, Huang et al. [22] introduced an algorithm for single layer feed forward ANN known as Extreme Learning Machine (ELM). This algorithm is capable of solving problems caused by gradient descent based algorithms like back propagation which applies in ANNs. ELM is also able to decrease the time required for training an ANN. In fact, it has been proved that by utilizing the ELM, the learning process becomes very fast and that it leads to a robust performance [23]. Accordingly, a number of investigations have been successfully carried out on the application of ELM algorithm for solving the problems in various scientific fields [24–29]. On the other hand, Wavelet transform (WT) captures both frequency and location information (location in time) [30] and has some desirable properties compared to the Fourier transform [31]. More specifically, the transform in WT is based on a wavelet matrix, which can be computed more quickly than the analogous Fourier matrix. Therefore, one could anticipate that the coupled version of ELM and WT would be a more powerful algorithm that each of them individually and that the coupled version would possess a faster learning speed compared to the traditional algorithms like BP (back-propagation).

### Table 1

<table>
<thead>
<tr>
<th>Fuel type</th>
<th>C (%)</th>
<th>H (%)</th>
<th>O (%)</th>
<th>( q_{\text{low}} ) (kJ/kg)</th>
<th>( \text{ex} ) (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diesel</td>
<td>0.873280</td>
<td>0.126720</td>
<td>–</td>
<td>42400.00</td>
<td>45163.41</td>
</tr>
<tr>
<td>B5P0</td>
<td>0.866932</td>
<td>0.126719</td>
<td>0.006348</td>
<td>42208.34</td>
<td>44980.36</td>
</tr>
<tr>
<td>B5P25</td>
<td>0.866596</td>
<td>0.126565</td>
<td>0.006439</td>
<td>42184.36</td>
<td>44953.61</td>
</tr>
<tr>
<td>B5P0</td>
<td>0.867215</td>
<td>0.126508</td>
<td>0.006277</td>
<td>42186.79</td>
<td>44955.10</td>
</tr>
<tr>
<td>B5P75</td>
<td>0.867429</td>
<td>0.126493</td>
<td>0.006118</td>
<td>42189.29</td>
<td>44956.71</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Engine model</th>
<th>Engine specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>OM314LA EUII</td>
<td>Turbocharged</td>
</tr>
<tr>
<td>Maximum power</td>
<td>81 kW@2800 RPM</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>340N.m@1400–2000 RPM</td>
</tr>
<tr>
<td>Bore</td>
<td>97 mm</td>
</tr>
<tr>
<td>Stroke</td>
<td>128 mm</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>17:1</td>
</tr>
<tr>
<td>Combustion system</td>
<td>4 stroke Direct injection</td>
</tr>
<tr>
<td>Number of cylinders</td>
<td>4, in line, vertical</td>
</tr>
</tbody>
</table>

**Fig. 1.** Process flow diagram of oil extraction from SBE.
Having said all, the present study was aimed at developing and applying an ELM-WT approach for estimating the exergetic performance parameters of a DI diesel engine running on various diesel/biodiesel blends containing EPS. Furthermore, the results obtained through the ELM-WT approach were compared with those of the genetic programming ELM, genetic programming (GP), and artificial neural networks (ANNs). It is worth quoting that to the best of our knowledge there are no published reports on the exergetic modelling of diesel engines running on diesel/biodiesel blends using advanced heuristically-soft computing methods. Moreover, the developed models using advanced soft computing techniques can be employed to determine, monitor, and control the combustion process through diesel engines in order to achieve the most sustainable and efficient combustion conditions.

### 2. Materials and methods

#### 2.1. Fuels preparation and engine tests

The waste oil contained in SBE (spent bleaching earth) was used as feedstock for the production of biodiesel through transesterification reaction with methanol in presence of KOH as catalyst. Then, the EPS used in building materials was dissolved in 1 L of the produced biodiesel at four different inclusion rates i.e., 0, 25, 50, 75 g, at 60 °C. Prior to mixing with diesel fuel to produce B5 blends (i.e. B5P0, B5P25, B5P50 and B5P75, respectively), the biodiesel-polymer solutions were stabilized through the addition of 100 ml acetone.

The lower heating value of diesel, biodiesel, EPS, and acetone were taken into account to be 42,400, 38,828, 29,589, and 40,000 (kJ/kg), respectively. Additionally, the chemical formula of diesel, biodiesel, EPS, and acetone were considered as C_{14.4}H_{24.9}, C_{17}H_{34}O_{2.1}, C_{16}H_{19}O_{2.3}, and C_{4}H_{10}O, respectively.
\((C_8H_8)n, \text{ and } C_3H_6O\). Table 1 tabulates the chemical compositions, lower heating values (q\text{LHV}), and specific chemical exergies of the developed fuel blends. It is worth noting that the addition of EPS into biodiesel increased the lower heating value of the final blend proportional with the EPS content. This is ascribed to the fact that the addition of EPS lowered the density of the biodiesel-polymer blend which in turn resulted in an increased volumetric proportion of diesel in the final B5. In better words, increasing the concentration of EPS in biodiesel marginally increased the mass fraction of diesel in the final B5 proportional with the EPS content. This led to higher q\text{LHV} of the final biodiesel-diesel blend containing EPS. For instance, B5P75 had the highest q\text{LHV} compared with the blends containing lower amounts of EPS, i.e., B5P25 and B5P50.

The fuel blends as well as neat diesel as blank were tested using a four-cylinder, four-stroke and turbocharged DI diesel engine located in IDEM Co. (Tabriz, Iran). The engine specifications are detailed in Table 2. Various loads were exerted on the engine by coupling it to an Eddy current dynamometer (400 hp) manufactured by Pmid Company, model E400. Fuel and intake air flow rates were measured by a flow meter and an air-flow sensor, respectively. Four thermocouples were applied to measure intake air and exhaust gas as well as inlet and outlet water temperatures. Moreover, the pressures of intake air and exhaust gas were measured. Engine tests were performed with a 13 mode test cycle (86.336-79 Diesel engine test cycle) in order to analyse emissions in dynamometer operation tests of heavy-duty diesel engines. An AVL DiCom4000 gas analyser was used to measure NO\textsubscript{x}, CO, and CO\textsubscript{2} using non-dispersive infrared gas analysis (NDIR). Oxygen (O\textsubscript{2}) concentration was evaluated through the exhaust manifold using electrochemical method. The sensor of the analyser was exposed to the exhaust gas and the observations were recorded. Smoke was measured using an AVL 415S smoke meter. It is worth mentioning that the recorded emission data were then used to calculate the chemical exergy of exhaust hot gas in our previous publication [20].

**Table 5**

<table>
<thead>
<tr>
<th>Total cost</th>
<th>Construction material</th>
<th>Construction (Man hours)</th>
<th>Construction (Manpower)</th>
<th>Construction (Indirect)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Purchased Equipment</strong></td>
<td>1,503,500.10</td>
<td>1,503,500.10</td>
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<td>–</td>
</tr>
<tr>
<td><strong>Equipment Setting</strong></td>
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<td>845.00</td>
<td>25,556.70</td>
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<tr>
<td><strong>Piping</strong></td>
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<td>994,945.10</td>
<td>22,664.00</td>
<td>707,336.10</td>
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<tr>
<td><strong>Civil</strong></td>
<td>160,301.50</td>
<td>80,918.90</td>
<td>3327.00</td>
<td>79,382.50</td>
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<tr>
<td><strong>Steel</strong></td>
<td>50,853.30</td>
<td>42,240.50</td>
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<td>8612.80</td>
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<tr>
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<td>859,945.10</td>
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<td>133,872.60</td>
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<tr>
<td><strong>Electrical</strong></td>
<td>519,690.60</td>
<td>446,762.80</td>
<td>2577.00</td>
<td>72,927.80</td>
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<tr>
<td><strong>Insulation</strong></td>
<td>181,676.10</td>
<td>98,728.70</td>
<td>3773.00</td>
<td>82,947.40</td>
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<td><strong>Paint</strong></td>
<td>3460.00</td>
<td>1217.90</td>
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<td><strong>Other</strong></td>
<td>3,915,300.20</td>
<td>412,900.00</td>
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<td><strong>G and A Overheads</strong></td>
<td>220,100.10</td>
<td>89,719.20</td>
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<td>41,796.00</td>
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<tr>
<td><strong>Contract Fee</strong></td>
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<td>97,701.90</td>
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<td>41,796.00</td>
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<tr>
<td><strong>Contingencies</strong></td>
<td>1,748,781.80</td>
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</tr>
<tr>
<td><strong>Total Project Cost</strong></td>
<td>11,464,235.50</td>
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</tr>
<tr>
<td><strong>Adjusted Total Project Cost</strong></td>
<td>11,325,656.83</td>
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<td>–</td>
</tr>
</tbody>
</table>

**Table 6**

<table>
<thead>
<tr>
<th>Total cost</th>
<th>Construction material</th>
<th>Construction (Man hours)</th>
<th>Construction (Manpower)</th>
<th>Construction (Indirect)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Purchased Equipment</strong></td>
<td>1,687,700.10</td>
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</tr>
<tr>
<td><strong>Equipment Setting</strong></td>
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<td>–</td>
<td>870.00</td>
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<td><strong>Piping</strong></td>
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<td>1,331,235.50</td>
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<td>725,647.70</td>
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<td><strong>Civil</strong></td>
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<td>87,661.90</td>
<td>3577.00</td>
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<tr>
<td><strong>Steel</strong></td>
<td>60,037.10</td>
<td>49,880.60</td>
<td>361.00</td>
<td>10,156.60</td>
</tr>
<tr>
<td><strong>Instrumentation</strong></td>
<td>1,042,124.60</td>
<td>89,719.20</td>
<td>3386.30</td>
<td>41,796.00</td>
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<tr>
<td><strong>Electrical</strong></td>
<td>529,512.90</td>
<td>453,321.30</td>
<td>2690.00</td>
<td>76,191.60</td>
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<tr>
<td><strong>Insulation</strong></td>
<td>194,586.10</td>
<td>105,351.40</td>
<td>3773.00</td>
<td>89,234.70</td>
</tr>
<tr>
<td><strong>Paint</strong></td>
<td>3869.10</td>
<td>1379.40</td>
<td>117.00</td>
<td>2489.60</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td>4,064,000.20</td>
<td>144,000.00</td>
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<td>–</td>
</tr>
<tr>
<td><strong>G and A Overheads</strong></td>
<td>220,042.70</td>
<td>142,284.30</td>
<td>3386.30</td>
<td>41,796.00</td>
</tr>
<tr>
<td><strong>Contract Fee</strong></td>
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<td>97,701.90</td>
<td>3386.30</td>
<td>41,796.00</td>
</tr>
<tr>
<td><strong>Contingencies</strong></td>
<td>1,837,995.90</td>
<td>89,719.20</td>
<td>3386.30</td>
<td>41,796.00</td>
</tr>
<tr>
<td><strong>Total Project Cost</strong></td>
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<td>12,049,082.70</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td><strong>Adjusted Total Project Cost</strong></td>
<td>11,903,434.45</td>
<td>11,903,434.45</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
These data along with other exegetic parameters were then used in the present study for modelling the thermodynamics performance parameters of the DI diesel engine.

2.2. Simulation and economic analysis

2.2.1. SBE oil extraction, biodiesel production and waste polymer dissolution

Biodiesel was produced from the oil extracted from SBE by an extraction process. Fig. 1 illustrates the proposed process for SBE oil extraction including a solid washer and a distillation column for solvent recovery. At the first step, SBE (oil 15%wt. and earth 85%wt.) was transferred to the solid washer and was exposed to hexane for triglycerides extraction. Hexane to oil mass ratio was maintained at 2 during the washing process. As a result, the bottom phase contained 100% earth while the upper fluid phase contained both hexane and triglycerides was transferred into the distillation column. After being preheated, the fluid stream was distilled in order to recover the hexane and the bottom flow of the column, i.e., the extracted oil was used as feedstock for the transesterification process.

The transesterification was conducted using methanol (methanol to oil molar ratio of 6:1), and KOH as catalyst (1% wt.). After 2 h of reaction, a conversion rate of about 97% was achieved. This process scenario comprised of four main parts: reaction, methanol recovery and acid washing, biodiesel purification, and finally polymer dissolution. Methanol recovery was achieved by distillation after the transesterification reactor and the recovered methanol was returned to the main reactor.

Subsequently, the products underwent the decantation and washing processes. Three decanters and one contactor were considered for these objectives. The first decanter separated crude glycerol from biodiesel while the second decanter separated the non-reacted triglyceride and synthesized biodiesel. After recycling the triglyceride, the biodiesel was transferred to the washing section (i.e., contactor and decanter) where its impurities (soap and excess KOH) were removed. At this level, combination of a contactor and a decanter was considered to define a mixer-settler. HCL solution was contacted to biodiesel to neutralize KOH and the soap formed in the main reactor. Afterwards, the biodiesel stream was

<table>
<thead>
<tr>
<th>Fuel blend</th>
<th>B5P0</th>
<th>B5P25</th>
<th>B5P50</th>
<th>B5P75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (USD/kg)</td>
<td>0.7579</td>
<td>0.7589</td>
<td>0.7578</td>
<td>0.7975</td>
</tr>
</tbody>
</table>

Table 7 Final price for different fuel blend calculated by economic date.

Fig. 5. Scatter plots of the actual and predicted values of fuel exergy rate using (a) ELM-WT, (b) ELM, (c) ANN and (d) GP approaches, respectively.

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transferred to a distillation column in order to complete the purification of biodiesel. Free fatty acids, diglyceride, and monoglyceride were separated and the purified biodiesel was sent to the polymer dissolution section. In this section, polystyrene was dissolved into biodiesel and acetone (10%wt.) was added as stabilizer. The final blend was obtained by blending this polymer-biodiesel mixture with petroleum diesel to obtain B5. Fig. 2 illustrates the process flow diagram of the proposed process.

2.2.2. Simulation and economic assessment

In order to simulate the process, Aspen plus V7.2 was used. Since soap was not defined in the Aspen database, therefore, its molecular structure was graphically drawn in the Aspen property section using the Aspen molecular structure tool. In fact, Aspen requires the enthalpy and entropy data of components in order to compute the energy balance in the reactor. More specifically, the Aspen uses the Gibbs free energy and enthalpy of formation for each atom–atom bond of a given component drawn to calculate the overall enthalpy and Gibbs free energy. Hence, only the drawing of the structure of a new component in the Aspen is sufficient to estimate all the unavailable physical data. Triolein and methyl oleate were considered as representative of all triglycerides and methyl esters (biodiesel). HCl was the acidic agent used in the washing process and oleic acid was used to predict the behaviour of free fatty acids in the system. Due to the deviation of the system from ideal properties and the existence of both polar and non-polar substances in the plant, electrolyte non-random two liquid (ELECNRTL) was considered as property package. However, since this global property method is not capable of predicting all conditions, therefore, other packages were incorporated for a number of specific items in the process. More specifically, Redlich–Kwong–Soave (RK–SOAVE) was incorporated for decanters. Heat exchangers were adjusted to use the Renedict–Webb–Rubin–lee–starling (BWR–LS) and Redlich–Kwong–Aspen (RK–Aspen) fluid package.

Economic assessment was conducted using Aspen Economic Analyzer (ICARUS) software, linked to Aspen plus to compute total operating cost, total project capital cost, and product price estimation. North America was selected for plant location and the available market for the materials used. The two plants were assumed to work in three shifts per day and 8000 working h per year. The estimated cost of SBE oil extraction process was considered as feedstock cost for biodiesel production. The biodiesel plant capacity was considered at 5000 kg/h of oil producing 4440 kg/h of pure biodiesel. For oil extraction process, SBE was considered as a priceless substance.

The price of the blend of petroleum diesel and polymer-biodiesel as the final fuel blend for diesel engines was estimated. More specifically, for sensitivity analysis, different amounts of...
polystyrene in biodiesel were considered and the price of the final fuel blends, i.e., B5P0, B5P25, B5P50, and B5P75 was calculated.

2.3. Input and output variables

The database used for the development of the investigated soft computing models were compiled from our previous report on the exergetic performance of a DI diesel engine running on several biodiesel/diesel blends (B5) containing various quantities of EPS [20]. Table 3 shows the 4 input parameters selected for analysis. These parameters were considered potentially influential in the exergetic performance parameters of the DI diesel (Table 4).

2.4. Extreme learning machine (ELM)

ELM was first introduced by Huang et al. as a learning algorithm tool for SLFN (single layer feed-forward neural network) architecture [32,33]. ELM chooses the input weights randomly and determines the output weights of the SLFN analytically. ELM algorithm has more favourable general capability with faster learning speed. In fact, this algorithm does not require too much human intervention, and can run much faster than the conventional algorithms. Moreover, it is capable of determining all the network parameters analytically, preventing trivial human intervention. Overall, ELM is an efficient algorithm with numerous advantages including ease of use, quick learning speed, higher performance, suitability for many nonlinear activation and kernel functions.

2.5. Single layer feed-forward neural network (SLFN)

SLFN function with \( L \) hidden nodes can be represented as mathematical description of SLFN by incorporating both additive and radial bias function (RBF) hidden nodes in a unified way [34,35]:

\[
\tilde{f}_L(x) = \sum_{i=1}^{L} \gamma_i G(a_i, b_i, x), \quad x \in \mathbb{R}^n, \quad a_i \in \mathbb{R}^n
\]

where \( a_i \) and \( b_i \) represent the learning parameters of hidden nodes. \( \gamma_i \) is the weight connecting the \( i \)th hidden node to the output node. The output value of the \( i \)th hidden node with respect to the input \( x \) is shown by. The additive hidden node with the activation function of \( g(x) : \mathbb{R} \rightarrow \mathbb{R} \) (e.g., sigmoid and threshold), \( G(a_i, b_i, x) \) could be obtained using the following equation [33]:

\[
G(a_i, b_i, x) = g(a_i \cdot x + b_i), \quad b_i \in \mathbb{R}
\]

Fig. 7. Scatter plots of the actual and predicted values of energy transfer rate to cooling water using (a) ELM-WT, (b) ELM, (c) ANN and (d) GP approaches, respectively.
where \( a_i \) denotes the weight vector which connects the input layer to the \( i \)th hidden node. Also, \( b_i \) is the bias of the \( i \)th hidden node \( a_i \). Moreover, \( x \) is the inner product of vector \( a_i \) and \( x \) in \( \mathbb{R}^n \).

\[ G(a_i, b_i, x) = g(b_i \| x - a_i \|) \quad b_i \in \mathbb{R}^+ \] (3)

where \( a_i \) and \( b_i \) represent the center and impact factor of \( i \)th RBF node, respectively. The set of all positive real values is indicated by \( \mathbb{R}^+ \).

The RBF network is a particular case of SLFN with RBF nodes in its hidden layer. For \( N \) arbitrary distinct samples \((x_1, t_1), \ldots, (x_N, t_N)\) \( \in \mathbb{R}^n \times \mathbb{R}^m \), \( x_j \) is \( n \times 1 \) input vector and \( t_j \) is \( m \times 1 \) target vector. If an SLFN with \( L \) hidden nodes can approximate these \( N \) samples with zero error then this implies that there exist \( \beta, a_i \), and \( b_i \) as follows [33]:

\[ f_i(x_j) = \sum_{i=1}^L \beta_i G(a_i, b_i, x_j), \quad j = 1, \ldots, N. \] (4)

The above equation can be written compactly as below:

\[ H\beta = T \] (5)

where,

\[ H \left( \bar{a}, \bar{b}, \bar{x} \right) = \begin{bmatrix} G(a_1, b_1, x_1) & \cdots & G(a_L, b_L, x_1) \\ \vdots & \ddots & \vdots \\ G(a_1, b_1, x_N) & \cdots & G(a_L, b_L, x_N) \end{bmatrix} \quad \text{with} \quad \bar{a} = a_1, \ldots, a_L; \quad \bar{b} = b_1, \ldots, b_L; \quad \bar{x} = x_1, \ldots, x_L \] (6)

\[ \beta = \begin{bmatrix} \beta_1^T \\ \vdots \\ \beta_L^T \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} t_1^T \\ \vdots \\ t_N^T \end{bmatrix} \quad \text{with} \quad L \leq N. \] (7)

\( H \) is the hidden layer output matrix of the SLFN with \( i \)th column of \( H \) being the \( i \)th hidden node’s output with respect to inputs \( x_1, \ldots, x_N \).

2.6. Principle of ELM

ELM designed as a SLFN with \( L \) hidden neurons is able to learn \( L \) distinct samples with zero error [24]. Even if the number of hidden neurons \( L \) < the number of distinct samples \( N \), ELM can still assign random parameters to the hidden nodes and compute the output weights by pseudo inverse of \( H \) giving only a small error.
The hidden node parameters of ELM $a_i$ and $b_i$ should not be tuned throughout training and can easily be assigned with random values. The following theorems state the same.

**Theorem 1.** (Liang et al. [35]) Let an SLFN with $L$ additive or RBF hidden nodes and an activation function $g(x)$ which is infinitely differentiable in any intervals of $R$ be given. Then, for arbitrary $L$ distinct input vectors $x_i, i = 1, ..., L$ and $(a_i, b_i)^T$ randomly produced by any continuous probability distribution, respectively, the hidden layer output matrix is invertible with probability one, the hidden layer output matrix $H$ of the SLFN is invertible and $\|Hb - T\| = 0.$

**Theorem 2.** (Liang et al. [35]) Given any small positive values $\varepsilon > 0$ and activation function $g(x): R \rightarrow R$ which is infinitely differentiable in any intervals, there exists $L \leq N$ such that for $N$ arbitrary distinct input vectors $x_i \in R^n, i = 1, ..., L$ for any $(a_i, b_i)^T$ randomly produced based upon any continuous probability distribution $\|H_N - H_{N \times m} T_{N \times m}\| < \varepsilon$ with probability one.

Since the hidden node parameters of ELM should not be tuned throughout training and because they are easily assigned with random values, Eq. (5) becomes a linear system and the output weights can be estimated as follows [34]:

$$\beta = H^+ T$$  \hspace{1cm} (8)

where $H^+$ is the Moore-Penrose generalized inverse [36] of the hidden layer output matrix $H$ which can be computed via several approaches consisting orthogonal projection, orthogonalization, iterative, singular value decomposition (SVD), etc. [36]. The orthogonal projection method can be utilized only when $H^T T$ is nonsingular and $H^+ = (H^T T)^{-1} H^T.$ Owing to the use of searching and iterations, orthogonalization method and iterative method have limitations. Implementations of ELM use SVD to compute the Moore-Penrose generalized inverse of $H$, because it can be utilized in all situations. ELM is thus a batch learning method. Fig. 3 shows a single layer feed forward neural network.

Thus, a simple learning method for SLFNs called the extreme learning machine (ELM) can be summarized as follows:

**Algorithm ELM:** Given a training set, activation function, and hidden neuron number:

1. **Step 1:** Assign arbitrary input weight $w_i, i = 1, ..., N.$
2. **Step 2:** Calculate the hidden layer output matrix $H$.
3. **Step 3:** Calculate the output weight $\beta, \beta = H^+ T.$

Fig. 9. Scatter plots of the actual and predicted values of exergy destruction rate using (a) ELM-WT, (b) ELM, (c) ANN and (d) GP approaches, respectively.
2.7. ELM issues

As a synthesis, the ELM algorithm shows faster learning speed over traditional algorithms for SLFNs. Nevertheless, two key aspects should be pointed out.

- Random initialization of hidden node parameters may affect the performances of ELM that also require high complexity of SLFN for improved performance. This may lead to ill-condition, which means that an ELM may not be robust enough to capture variations in data.
- Considering the network complexity issue, it is essential to carefully choose hidden neuron activation functions that show good ability to handle complexity, improve convergence of algorithm and result in a compact structure of network. These aspects are taken into account in the proposed ELM-WT model.

2.8. Discrete wavelet transform

Wavelet transform (WT) represents the basis of the mathematical expression to decompose time series frequency signal into various components [37–39], and is also known as the signal processing algorithm from Fourier transforms. The key superiority of WT over Fourier transform is its ability to perform accurate analysis based on resulting decomposed components with scaled-fit resolution that aids to enhance the size of the study model due to its capability to obtain the required information at various levels [40]. This is ideal for data analysis applications with a time domain and frequency owing to its ability to extract information from transient and non-periodic signals, which makes it very useful in time-frequency localization [41]. WT has many useful basis functions, where one can select depending on the signal being analysed. Recently, these methods have generated enormous interests for engineering applications [42,43].

On the other hand, continuous wavelet transform (CWT) with signal \( f(t) \), is defined as the time-scale technique of signal processing which is the integral of all the signals over the entire period multiplied by scaled, shifted versions of the wavelet function \( \psi(t) \) given in mathematical expression as

\[
W_f(a,b,\psi) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t-b}{a} \right) dt, \ b \in R, \ a \in R, \ a \neq 0 \quad (9)
\]
where $\psi$ is the mother wavelet function, $\psi^*(t)$ denotes the complex conjugate of $\psi$, $t$ is the time, $b$ describes the time shifting parameter (known as translation) and $a$ is the scale index parameter (i.e. inverse of the frequency). By discretizing Eq. (9), the discrete wavelet transform (DWT) can be found whereby the parameters $a$ and $b$ can be found on the basis of Eq. (10):

$$a = a_0^m, \quad b = na_0^mb_0, \quad a_0 > 1, \quad b_0 \in \mathbb{R}.$$  \hfill (10)

where $n$ and $m$ are integer numbers that control the scale and translation, respectively, $a_0$ is a fixed dilation step, and $b_0$ is a translation factor that depends on the aforementioned dilation step.

DWT of $f(t)$ can be written as:

$$W_f(m, n, \psi) = a_0^{-m/2} \int_{-\infty}^{\infty} f(t)\psi^*(a_0^{-m}t - nb_0)dt \hfill (11)$$

According to Mallat [37], when $a_0 = 2$ and $b_0 = 1$, Eq. (11) becomes binary wavelet transform:

$$W_f(m, n, \psi) = a_0^{-m/2} \int_{-\infty}^{\infty} f(t)\psi^*(a_0^{-m}t - nb_0)dt \hfill (12)$$

$W_f(a, b, \psi)$ or $W_f(m, n, \psi)$ can reflect the characteristics of original time series in frequency ($a$ or $m$) and time domain ($b$ or $n$) at the same time [44]. When $a$ or $m$ is small, the frequency resolution of wavelet transform is low, but the time domain resolution is high. When $a$ or $m$ becomes large, the frequency resolution of wavelet transform is high, but the time domain resolution is low.

In this study, wavelet analysis was employed to decay time series of daily evapotranspiration data to individual components, where the decomposed components can be considered for data inputs into the ELM model. As mentioned in the previous section, ELM is quiet efficient as compared to traditional methods to learn SLFNs. However, issues like parameters initialization, model complexity and choice of activation functions have to be carefully addressed for improved performance. Therefore, the ELM-WT is proposed as shown in Fig. 4. The proposed structure takes advantages of WAVELET and SLFN.

Fig. 11. Scatter plots of the actual and predicted values of sustainability index using (a) ELM-WT, (b) ELM, (c) ANN and (d) GP approaches, respectively.
2.9. Evaluating accuracy of proposed models

Predictive performances of the proposed models were presented as root means square error (RMSE), Coefficient of determination ($R^2$) and Pearson coefficient ($r$). These statistics are defined as follows:

1) Root–mean–square error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (P_i - O_i)^2}{n}} \quad (13)$$

2) Pearson correlation coefficient ($r$)

$$r = \frac{n \sum_{i=1}^{n} O_i \cdot P_i - (\sum_{i=1}^{n} O_i)(\sum_{i=1}^{n} P_i)}{\sqrt{n \sum_{i=1}^{n} O_i^2 - (\sum_{i=1}^{n} O_i)^2} \cdot \sqrt{n \sum_{i=1}^{n} P_i^2 - (\sum_{i=1}^{n} P_i)^2}} \quad (14)$$

3) Coefficient of determination ($R^2$)

$$R^2 = \frac{\left[ \sum_{i=1}^{n} (O_i - \bar{O})(P_i - \bar{P}) \right]^2}{\sum_{i=1}^{n} (O_i - \bar{O}) \cdot \sum_{i=1}^{n} (P_i - \bar{P})} \quad (15)$$

where $P_i$ and $O_i$ are known as the experimental and forecast values of exergetic performance of the DI diesel engine running on various diesel/biodiesel blends containing limited amount of EPS, respectively, and $n$ is the total number of test data.

3. Results and discussion

3.1. Capital investments and final fuel price sensitivity

As revealed by the economic assessment, SBE oil could be extracted at USD 0.21/kg oil. Tables 5 and 6 tabulate the different elements of capital investment for each scenario. Unlike the scenarios B5P25, B5P50 and B5P75, the B5P0 scenario did not include polymer dissolution. This led to the differences in cost parameters. Among the three scenarios involving polymer dissolution (similar capital investment), only the B5P50 which led to the best diesel engine performance was considered in the analyses. The total project cost of the B5P50 was approximately USD 600,000 more than that of the B5P0. This was caused by the mixed reactor which did not exist in the B5P0 plant.

Table 7 presents the final fuel blend price for the four scenarios. As shown, increasing the amount of waste polymer from 50 to 75 g/L biodiesel increased the final price of the fuel blend. As mentioned earlier, the addition of EPS lowered the density of the biodieol-polymer blend which in turn resulted in an increased volumetric proportion of diesel in the final B5. In better words, increasing the concentration of EPS in biodiesel marginally increased the mass fraction of diesel in the final fuel blend and consequently the final price of the fuel blend. Hence, the optimum rate of polymer inclusion was considered at 50 g/L biodiesel which has led to both lower price and best diesel engine performance characteristics.

3.2. Performance evaluation of the proposed ELM-WT model

The average computational time for the ELM-WT modelling was around 456 s using a PC with Intel Core Duo CPU E7600 @ 3.06 GHz and installed memory (RAM) of 2 GB. The average computational time for the ELM, ANN and GP modelling were around 496, 534 and 556 s, respectively, using the same PC with the same performances. The used software with applied codes is available at http://www.ntu.edu.sg/home/egbhuang/elm_codes.html.

Table 9 Outputs of the ELM-WT, ELM, ANN and GP models for fuel exergy, exergy of exhaust gas, exergy transfer to cooling water, exergy transfer to ambient, exergy destruction rate, exergy efficiency and sustainability index of the DI diesel engine running with neat diesel and B5P0 blend at load fraction of 0% and engine speed of 1600 rpm.
The performance of the developed approaches for exergetic modelling of the DI diesel engine running on various diesel/biodiesel blends containing limited amount of EPS are reported and compared herein. Figs. 5–11(a) presents the accuracy of the developed ELM-WT for exergetic performance modelling of the DI diesel engine. Moreover, Figs. 5–11(b–d) illustrate the accuracy of the developed ELM, ANN and GP for exergetic performance modelling of the DI diesel, respectively. It can be seen that most of the points fall along the diagonal line in case of the ELM-WT approach. Consequently, it could be concluded that predicted results are in a very good agreement with the values measured by the ELM-WT method. This observation was also confirmed using the very high coefficient of determination value obtained (Table 8).

Moreover, the number of both overestimated and underestimated values produced was also limited. Consequently, it is obvious that the predicted values enjoy a high level of precision. This is an extra evidence on the applicability of the ELM-WT for simulating and modelling the complex and nonlinear thermodynamic systems such as combustion process. In better words, this demonstrates that the developed ELM-WT approach could precisely track the experimental data and undoubtedly replace the classical soft computing techniques for exergetic performance modelling of DI diesel engines working with renewable and non-renewable fuels. This was ascribed to the fact that the developed ELM-WT method took into account the computed exergetic information better and automatically improved itself during learning. Overall, if a DI diesel is run under known operational condition of engine load and speed as well as using defined fuel components, the exergetic performance of the combustion process could be estimated, controlled, and optimized by one set of the developed ELM-WT approach.

3.3. Performance comparison of the applied approaches

In order to demonstrate the merits of the proposed models on a more definite and tangible basis, the prediction accuracies of the four models were compared. Conventional error statistical indicators, RMSE, r and $R^2$ were used for comparison. Table 8 summarizes the prediction accuracy results for the seven exergetic parameters.

Generally, the developed ELM-WT approach outperformed the ELM, ANN, and GP models according to the results summarized in Table 8. The predictions from the models highly correlate with the actual surface roughness data ($r > 0.95$). Table 9 shows outputs of the ELM-WT, ELM, ANN and GP models for fuel exergy, exergy of exhaust gas, exergy transfer to cooling water, exergy transfer to ambient, exergy destruction rate, exergy efficiency, and sustainability index of the DI diesel engine running with neat diesel and B5P0 blend at load fraction of 0% and engine speed of 1600 rpm. Finally, Fig. 12 compares the prediction errors of the all models for estimating the fuel exergy.

4. Conclusion

In this study, an innovative soft computing approach namely the ELM-WT was developed and applied for modelling the exergetic performance of a DI diesel engine running on various diesel–biodiesel blends containing a limited amount of EPS. Moreover, a comparison of the ELM-WT method with the ELM, ANN and GP was performed in order to assess the prediction accuracy. It was found out that the ELM-WT method used for exergetic modelling of the diesel engine was capable of yielding excellent results compared to the other three techniques. The selected approach successfully predicted the exergetic performance parameters of the DI diesel engine with a Pearson correlation coefficient and a coefficient of determination higher than 0.97 and 0.95, respectively, while a root-mean-square error lower than 2.79 was achieved. Moreover, the developed ELM-WT model possessed many appealing and remarkable features making it distinguishable from the popular traditional gradient-based learning algorithms for feedforward neural networks. More specifically, this consolidated approach was much faster in terms of learning speed compared to the traditional feedforward network learning algorithms such as back-propagation (BP) algorithm. Second, unlike the traditional learning algorithms, the ELM-WT algorithm was able to achieve the smallest training error and also norm of weights. Finally, the ELM-WT approach could be taken into account as an attractive alternative to the classical soft computing techniques for exergetic performance modelling of DI diesel engines. Subsequently, the exergetic performance of the DI diesel engine can be estimated, optimized, and controlled in real-time under a known engine operational conditions and biofuel characteristics by one set of ELM-WT weights and biases.
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