Research Article

A Three-Stage Fifth-Order Runge-Kutta Method for Directly Solving Special Third-Order Differential Equation with Application to Thin Film Flow Problem

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In this paper, a three-stage fifth-order Runge-Kutta method for the integration of a special third-order ordinary differential equation (ODE) is constructed. The zero stability of the method is proven. The numerical study of a third-order ODE arising in thin film flow of viscous fluid in physics is discussed. The mathematical model of thin film flow has been solved using a new method and numerical comparisons are made when the same problem is reduced to a first-order system of equations which are solved using the existing Runge-Kutta methods. Numerical results have clearly shown the advantage and the efficiency of the new method.

1. Introduction

A special third-order differential equation (ODE) of the form

\[
\begin{align*}
    y'''(x) &= f(x, y(x)), \quad y(x_0) = \alpha, \\
    y'(x_0) &= \beta, \quad y''(x_0) = \gamma, \quad x \geq x_0
\end{align*}
\]

which is not explicitly dependent on the first derivative \( y'(x) \) and the second derivative \( y''(x) \) of the solution is frequently found in many physical problems such as electromagnetic waves, thin film flow, and gravity driven flows. The solution to (1) can be obtained by reducing it to an equivalent first-order system which is three times the dimension and can be solved using a standard Runge-Kutta method or a multistep method.

Most researchers, scientists, and engineers solve problem (1) by converting the problem to a system of first-order equations. However, there are also studies on numerical methods which solve (1) directly. Such work can be seen in Awoyemi [1], Waelhe et al. [2], Zainuddin [3], and Jator [4]. Awoyemi and Idowu [5] and Jator [6] proposed a class of hybrid collocation methods for the direct solution of higher-order ordinary differential equations (ODEs). Samat and Ismail [7] developed an embedded hybrid method for solving special second-order ODEs. Waelhe et al. [2] developed a block multistep method which can directly solve general third-order equations; on the other hand, Ibrahim et al. [8] developed a multistep method that can directly solve stiff third-order differential equations. All of the methods discussed above are multistep methods; hence, they need starting values when used to solve ODEs such as (1). Senu et al. [9] derived the Runge-Kutta-Nyström method for solving second-order ODEs directly. Mechee et al. [10] constructed new Runge-Kutta methods for solving (1).

In this paper, we are concerned with a one-step method, particularly the three-stage fifth-order Runge-Kutta method, for directly solving special third-order ODEs. Accordingly, we have developed a direct Runge-Kutta method (RKD) which can be directly used to solve (1). The advantage of the new method over multistep methods is that it initialises itself. The method produces \( y_{m+1}, y'_{m+1}, \) and \( y''_{m+1} \) to approximate \( y(x_{m+1}), y'(x_{m+1}), \) and \( y''(x_{m+1}) \), where \( y_{m+1} \) is the computed solution and \( y(x_{m+1}) \) is the exact solution.