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Effect of Lewis Number on Unsteady Double Diffusive Buoyancy Induced Flow in a Triangular Solar Collector with Corrugated Wall

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Abstract

A numerical investigation has been carried out in this paper to study the effect of varying Lewis number on the nature of heat and mass transfer in a triangular solar collector at unsteady condition. The triangular solar collector consists of two inclined glass covers and an absorber plate. The situation studied here resembles heat transfer phenomena in solar collectors and hence a thorough study on the effect of different parameters dictating this kind of heat transfer is essential. Bottom corrugated wall is subjected to high temperature and high concentration. Finite element method was employed to solve the unsteady dimensionless governing equations of continuity, momentum, energy and concentration of the problem. Calculations were carried out for Raleigh number ranging from 10\(^4\) to 10\(^6\) and for dimensionless time parameter \(= 0.1, 0.5\) and 1. For the conditions mentioned here the effect of varying local and overall Nusselt and Sherwood number at those conditions are discussed. A comprehensive explanation follows each result.

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1. Introduction

Both industrial and non-industrial fields deal with problems related to heat transfer and for this reason different heat transfer problems have been given importance for the last 200 years. Mixed convection is a very common facet of this type of problem. Problems involving mixed convection are experienced widely in many household and industrial application fields like attic space heating, chemical processing, solar systems, food processing, electronics cooling, desalination, thermal and pollution control and so on\cite{1,2}. Consequently thorough investigation is called for as to the

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mixed convection paradigm of heat transfer problems. Among different parameters that dictate such thermal phenomenon Lewis number has a significant impact and this paper aims to study a particular problem in light of variation in Lewis number. The most common shapes which are extensively studied for the mixed and double diffusive condition both in steady and unsteady cases are square, rectangular, trapezoidal shaped enclosures [3–5]. Triangular shaped enclosures get very little attention on this regard although triangular shaped enclosures are seen in so many practical applications. Among the studies on triangular shapes Hasanuzzaman et al.[6] investigated the effect of Lewis number on heat and mass transfer in triangular cavity and observed that when the Lewis number increases the heat transfer rate along with Nusselt number decreases and the mass transfer rate increases. So the increase of Lewis number has adverse effect on heat transfer while positive effect on mass transfer. Rahman et al. [7] studied the different geometrical parameter such as Raleigh number, Prandtl number for a steady double diffusive buoyancy induced flow in a triangular cavity and showed that the parameters have significant effect on heat and mass transfer. For triangular cavity these literatures [8,9] can be followed. Cheng [10] studied both heat and mass transfer in natural convection implementing a vertical wavy surface and concluded that for the wavy surface both the heat and mass transfer rate are higher than the plane surface. So in a corrugated bottom surface there should be an increase in the heat and mass transfer rate. Jang and Yan[11] also reported similar results for the mixed convection along vertical wavy surface. Solar energy is the most secure source of renewable energy. As a result it has been given a huge importance and different countries have taken policies on this regard. Solar energy is also environment friendly and cost effective. Nowadays solar technology has a wide range of applications such as solar thermal collector, solar PV cell, solar water heater, solar cooling and so on. In the solar thermal collector which is used in different solar thermal applications the heat transfer is associated

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$C_p$</td>
<td>specific heat (J kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration (m s$^{-2}$)</td>
</tr>
<tr>
<td>$Gr$</td>
<td>Grashof number</td>
</tr>
<tr>
<td>$H$</td>
<td>enclosure height (m)</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity (W m$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the enclosure (m)</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless pressure</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$T$</td>
<td>fluid temperature (K)</td>
</tr>
<tr>
<td>$t$</td>
<td>dimensional time (s)</td>
</tr>
<tr>
<td>$U$</td>
<td>dimensionless horizontal velocity component</td>
</tr>
<tr>
<td>$V$</td>
<td>dimensionless horizontal velocity component</td>
</tr>
<tr>
<td>$X$</td>
<td>dimensionless horizontal coordinate</td>
</tr>
<tr>
<td>$Y$</td>
<td>dimensionless vertical coordinate</td>
</tr>
<tr>
<td>$Sh$</td>
<td>Sherwood Number</td>
</tr>
<tr>
<td>$Le$</td>
<td>Lewis Number</td>
</tr>
<tr>
<td>$N$</td>
<td>Buoyancy Ratio</td>
</tr>
<tr>
<td>$c$</td>
<td>Concentration of species</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity (m$^2$s$^{-1}$)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>thermal expansion coefficient (K$^{-1}$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>solid volume fraction</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity (kg m$^{-1}$ s$^{-1}$)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity (m$^2$ s$^{-1}$)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>$\theta$</td>
<td>non-dimensional temperature</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>stream function</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>general dependent variable</td>
</tr>
</tbody>
</table>
with mass transfer. To enhance the heat transfer and keep the solar panel free from environmental corrosion glass cover may be used. Due to the unique property of glass cover the bottom part of the collector is heated. In this paper a solar thermal collector is modelled where the inclined glass covers of the collector make a triangular shape and a corrugated bottom have a higher heat transfer rate than the plane walls. This triangular shape solar collector contains air along with different pollutant such as moisture which induces a mass transfer along with heat transfer. The evaluation of heat and mass transfer has been done in light of Nusselt and Sherwood number. And the results are described by related streamline, isotherm and iso-concentration contours along with the graph. As the heat and mass transfer are not widely studied in triangular enclosure so the purpose of this work is to study a triangular type enclosure which has a similarity for triangular solar thermal collector.

2. Problem formulation

2.1. Physical modeling

In the figure 1 a triangular solar collector having a corrugated bottom has been presented with the specified boundary conditions. The shape of the cavity is triangular which is similar to the triangular solar collector. The inclined walls are the glass covers which are kept at a low temperature $T_c$ and the corrugated bottom wall is the solar thermal collector which is kept at a constant high temperature $T_h$. The air inside and the moisture of the air create a double diffusive heat transfer phenomenon. The height of the enclosure is $H$ and the length of the enclosure is $L$ where the gravity is working in the negative direction of the coordinate system. No radiation effect has been taken into consideration.

![Fig. 1. Schematic of the problem with the domain and boundary conditions](image)

![Fig. 2. Grid independency study with $\tau = 0.3, Le = 10, N = 10$ and $RaT = 10^5$](image)

2.2. Model Equations

A Newton Fourier fluid is formed inside the solar thermal collector due to the air and different available pollutant of the air such as moisture, dust etc. Mixture density of fluid is a function of temperature and concentration. The Navier stokes equation, energy equation and concentration equation forms the governing equations for the problem under consideration. For the unsteady case the equation can be written in dimensionless form as

$$\zeta = \frac{\partial V}{\partial X} = -\frac{\partial U}{\partial Y} = -\nabla^2 \psi$$

(1)

$$\frac{\partial \zeta}{\partial \tau} + U \frac{\partial \zeta}{\partial X} + V \frac{\partial \zeta}{\partial Y} = Pr \nabla^2 \zeta + Ra_T \frac{\partial \theta}{\partial X} + N \frac{\partial C}{\partial X}$$

(2)
\[
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \nabla^2 \theta \tag{3}
\]

\[
\frac{\partial C}{\partial \tau} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Le} \nabla^2 \theta \tag{4}
\]

Where the dimensionless variables are introduced as:

\[
\tau = \frac{\alpha t}{H^2}, X = \frac{x}{H}, Y = \frac{y}{H}, \Omega = \frac{\zeta H^2}{\alpha}, \psi = \frac{\Psi}{\alpha}, \theta = \frac{T - T_l}{T_h - T_l} and C = \frac{c - c_l}{c_h - c_l} \tag{5}
\]

The variables have their usual meaning as listed in the nomenclature. Prandtl number \((Pr)\), thermal Rayleigh number \((Ra_T)\), Lewis number \((Le)\) and buoyancy ratio \((N)\), are the non-dimensional parameter in the above equations, which are defined respectively as:

\[
Pr = \frac{\nu}{\alpha}, Ra_T = \frac{g \beta_T (T_h - T_l) H^3}{\alpha \nu}, Le = \frac{\alpha}{D} and N = \frac{\beta_c (c_h - c_l)}{\beta_T (T_h - T_l)} \tag{6}
\]

The dimensionless initial and boundary conditions for the present problems become For \(\tau = 0\), Entire domain: \(U = V = \psi = 0, \theta = 0, C = 0\)

For \(\tau > 0\), on the bottom wall: \(U = V = \psi = 0, \Omega = -\left(\frac{\partial^2 \psi}{\partial Y^2}\right), \theta = 1, C = 1\)

On the inclined walls: \(U = V = \psi = 0, \Omega = -\left(\frac{\partial^2 \psi}{\partial X^2}\right), \theta = 0, C = 0\)

Overall Nusselt and Sherwood numbers are defined respectively as

\[
Nu_{av} = -\int_0^1 \frac{\partial \theta}{\partial Y} dX \tag{7}
\]

and

\[
Sh_{av} = -\int_0^1 \frac{\partial C}{\partial Y} dX \tag{8}
\]

The stream function is calculated from

\[
U = \frac{\partial \psi}{\partial Y}, V = -\frac{\partial \psi}{\partial X} \tag{9}
\]

3. Numerical Procedure

3.1. Numerical methods

Finite element method has been deployed for finding out the numerical solution and specifically Galerkin weighted residual method has been employed. The entire domain is discretized into several elements and triangular mesh has been used to form these elements. A set of algebraic equations has been formulated by the Bossnique approximation and the iterative process is used to solve this algebraic equation set. It has been confirmed that the numerical solution has converging nature and the converging criteria used for the numerical solution is \(|\Gamma^{m+1} - \Gamma^m| \leq 10^{-5}\) Where \(m\) is number of iteration and is the general dependent variable.

3.2. Grid independency test

A grid independency test has been performed to validate the present work with \(\tau = 0.3, Le = 10, Br = 10\) and \(Ra_T = 10^5\). The details of the grid independency test are presented in the figure 2. The code validation is done in light of Nusselt number and Sherwood number. When the test is performed the entire domain is discretized into element number 1214,2042,3456,4444 and 5824. From the figure it is evident that initially for the lower element number the Nusselt and Sherwood number vary thus the grid is not independent. After reaching element number 3456 the overall Nusselt number and Sherwood number do not vary and the grid also become independent. So the element number 3456 has been considered for the whole numerical solution.
4. Results and Discussion

In this paper numerical investigation has been carried out using finite element analysis. The results are presented and analyzed in detail with appropriate discussion. Results are obtained for the variation of different significant parameters. Effect of Lewis number on streamlines, isotherms and isoconcentration contours of the fluid in the enclosure at \( \tau = 0.3 \) with \( RaT = 10^3 \) is presented in figure 3. As it can be seen from the figures of column (a), the strength and distribution of vortices inside the enclosure change with variation in Lewis number. For all the values of \( Le \), two major cells form inside the triangular enclosure in a symmetric manner. The fluid inside the enclosure gets heat from the bottom corrugated wall which is maintained at a higher temperature at first. The inclined walls have a lower temperature and are extended in a symmetric manner forming a triangle. As a consequence thermal buoyancy effect is seen inside the enclosure where lighter fluid from the corrugated wall region moves up the enclosure and comes down along the two colder walls. Symmetry of the wall is thus induced in the flow and two vortices with opposite sense of rotation materialize inside the enclosure. Besides the thermal buoyancy effect, due to a gradient of concentration in the fluid, a mass diffusivity effect is seen from the bottom wall to the upper side of the enclosure. These two diffusive effects combined create the vortices inside the triangular enclosure. For \( Le = 0.1 \) and \( Le = 2 \) the strength of the vortices is comparatively higher. For \( Le = 1\psi_{max} = 1.76 \) and \( \psi_{min} = -1.86. \) But as the value of \( Le \) is increased small cells near the bottom wall is seen alongside taming of the strength of vortices. With the increment in the value of \( Le \), the thermal diffusivity has become the more pronounced factor of the two which governs the state of the flow inside the enclosure. The mass diffusion due to concentration gradient isn't much strong at high value of \( Le \). Consequently the strength of the flow decreases. Column (b) shows the isotherms for different values of \( Le \). For \( Le = 0.1 \) the distribution of isotherms is distributed in a dispersed manner inside the enclosure. The density of isotherms is also quite less. Isotherms near the bottom wall are parallel and near the top corner the isotherms are slightly distorted. This distribution shows a retardation of conduction heat transfer at \( Le = 0.1 \). As the value of \( Le \) is increased, the density of isotherms increases and the distribution becomes nearly parallel to one another. This indicates a very good conduction heat transfer at higher value of \( Le \). Column (c) shows isoconcentration contours for varying Lewis number. For lower values of \( Le \), isoconcentration contours are parallel to the bottom wall and concentration of fluid is seen to be decreasing gradually. For \( Le = 10 \) the isoconcentration contours become wavy and severely distorted indicating presence of convection. Figure 4 shows the effect of Lewis number on streamlines, isotherms and isoconcentration at \( \tau = 0.3 \) with \( RaT = 10^5 \). Column (a) exhibits effect of \( Le \) on the streamlines. For \( Le = 0.1 \) two vortices form symmetrically inside the enclosure due to temperature and concentration gradient. As the value of \( Le \) increases this symmetry breaks up and a number of cells are seen inside the enclosure. For \( Le = 2 \), three vortices form at each corner of the enclosure with relatively low flow strength \( (\psi_{max} = 2.80 \) and \( \psi_{min} = -2.25) \). There are two larger cells inside the enclosure with opposite sense of rotation having \( \psi_{max} = 10.10 \) and \( \psi_{min} = -7.17 \). For \( Le = 5 \), the overall flow strength reduces a bit. A very large cell is formed with minimum stream function value of 8.03. Very weak vortices are seen at the bottom two corners and along the left inclined wall. A small cell is seen near the right side of the bottom wall. This distribution indicates a relatively weaker convective heat and mass transfer compared to \( Le = 2 \). For \( Le = 10 \) quite a few cells are formed inside the enclosure. Neither of them has a high value of stream function. This distribution is an evidence of weak local convective heat transfer inside the enclosure. Column (b) shows isotherm contours for different values of \( Le \). For all values of \( Le \) isotherms are found to be closely packed and parallel near the bottom wall. Strong conduction near that region results in this type of isotherm distribution. As the values of \( Le \) becomes higher the isotherms in the middle region becomes distorted indicating a convection region. For \( Le = 10 \), isotherms are parallel near the walls and quite random at the middle. Next column shows isoconcentration contours at different values of \( Le \). As \( Le \) is continuously increased, isoconcentration contours become more and more tinged. Figure 5 depicts effect of Lewis number on streamlines, isotherms and isoconcentration contours for \( RaT = 10^6 \) and at \( \tau = 0.3 \). Streamline variation is seen at column (a). For \( Le = 0.1 \) a large cell forms at the middle of the enclosure having \( \psi_{min} = -181.05 \). Two other vortices are formed beside it having opposite sense of rotation and comparatively weak flow strength. At each corner near the bottom wall, small cells are seen too. As the value of \( Le \) is increased, the cell at the middle becomes weaker but expands gradually. Two cells near the central vortex are replaced by a single vortex of stream function value = 15.15 near the left wall. For \( Le = 5 \) small cells are seen at each corner having a mild strength. For \( Le = 10 \), the cells are the weakest and only two major cells are to be seen formed in a symmetric manner. The next column shows the isoconcentration contours which are actually
quite dense even at $Le = 0.1$. This proves better heat transfer performance for high value of $Ra_T$. With gradually increasing value of $Le$, isotherms orient themselves along the walls and are densely packed there. So there is formation of thermal boundary wall. At the middle the isotherms are tinged indicating convective heat transfer region. Column (c) shows isoconcentration contours distribution for different values of Lewis number. For $Le = 0.1$ isoconcentration contours are parallel to the bottom wall. For higher value of Le, isoconcentration contours are seen to be parallel to the walls of the enclosure and are distorted near the middle. Figure 6 shows the variation of average Nusselt number and Sherwood number for $Ra_T = 10^4, 10^5, 10^6$. This figure helps to compare the heat and mass transfer performances for different Rayleigh number since it is a well-known fact that Sherwood number is the counterpart of Nusselt number in mass transfer phenomenon. As it can be seen there is not much of a variation in Sherwood and Nusselt number for $Ra_T = 10^4$. Both Nusselt number and Sherwood number are relatively higher for $Ra_T = 10^6$. But with increasing $Le$, Nusselt number tends to decrease while Sherwood number increases. This is an indication of better convective mass transfer performance and a retardation of convection heat transfer performance at higher values of $Le$.

5. Conclusions

In this investigation the results of numerical analysis on the effect of varying Lewis number in regard to a double diffusive buoyancy induced flow in a triangular enclosure with corrugated bottom wall has been presented. The followings can be listed as the outcome of this paper:
• At lower value of Lewis number convective heat transfer performance is better while at higher values of Lewis
number convective mass transfer performance becomes better. A compromise can be made at an intermediate value of Lewis number. In this paper Le = 2 showed good performance both in terms of heat and mass transfer.

- Increment of Lewis number decrease the nusselt number as well as the heat transfer rate. And for low thermal Rayleigh number there is no effect of the Lewis number on the heat transfer rate.
- Increment of Lewis number increase the Sherwood number as well as the mass transfer rate. And for Mass transfer Higher Rayleigh number and Lewis number both play significant role for mass transfer.
- More strong vortices as well as strong convection can be achieved for the higher thermal Rayleigh number.

As mentioned earlier problems regarding mixed convection in triangular enclosure have not been explored as much as it should have been. This paper discusses a particular problem regarding such thermal phenomenon and it is hoped that this paper will be able to draw the attention of the researchers on solving various problems of such kind.

References


