Boundary control of dual-output boost converter using state-energy plane

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Abstract: A boundary control scheme for dual-output boost converter (DBC) with enhanced performance using second-order switching surface is presented in this study. The derivation of the switching surface is performed in state-energy plane rather than the state-plane to obtain a general geometrical representation that provides a good dynamic response during start-up and sudden load changes, achieving steady state in two switching actions. To illustrate the control characteristic and performance, a detailed mathematical analysis is fully developed and compared with other control methods for the DBC. The proposed control was simulated, implemented in a digital signal processor TMS320F2812, and tested using prototype hardware. The results obtained for the DBC show good performance in terms of fast transient response under 50% load change and respectable output DC-bus balancing.

1 Introduction

To supply voltage, current and frequency needed for the load, and to guarantee the desired dynamics response, DC–DC power converters have to be suitably controlled. Conventionally, classical linear control techniques have been used for the control of these power converters [1, 2]. For the sake of simplicity, many existing control schemes for power electronic circuits are confined to develop small-signal models around the nominal operating point [3]. These methods are used to determine the dynamic behaviour of the system using classical linear system theories [4, 5]. If the injected perturbation has small magnitude, the system behaves as predicted by the small-signal model, and the transients converge to the quiescent operating point [5]. However, controllers failed to satisfactorily perform constrained specifications under large parameter variations and load disturbances [6]. Therefore the steady-state response may diverge from the desired quiescent point. In addition, the control performances may not be optimal. Hence, small-signal modelling is insufficient for the complete analysis of power switching converters [5, 6].

An alternative approach is to use the large-signal state-space geometric analysis which provides a more complete picture of the stability and performance for power electronic systems [7]. Switching boundary control (SBC) is a geometric approach where it was firstly introduced to power electronic systems by Burns and Wilson in 1976 [8]. In boundary control, a state-plane is introduced to analyse the switching boundary of a power electronic system. A switching boundary denoted by σ is used to divide the state-plane into disjointed regions. These regions allow us to visualise what goes on in non-linear system analytically. Detailed investigations into the modelling, design and analysis of these boundaries will lead to achieve desired transient response and ensure stability [9].

Among the well-known theories using SBC are sliding mode control and hysteresis control. These controls use the theory of boundary control with first-order switching surface δ. In general, δ derived boundary controllers offer good large-signal response and stability to the converter system, but the transient dynamics is still non-optimal for that many research works propose adaptive solution, such as the adaptive hysteresis control to enhance the dynamics performance of the system. However, an unstable combination or limit cycle may emerge. Moreover, the hysteresis band causes undesirable output steady-state error [10, 11]. The concept of the second-order switching surface δ is proposed in [12–14]. It is derived by estimating the state trajectory movement after a switching action, resulting in a high-state trajectory velocity along the switching surface. This phenomenon accelerates the trajectory moving towards the target operating point. Converters with δ exhibit better dynamic characteristics than the ones with δ. Instead of directing the state trajectory movement, as in δ, the proposed surface is derived from the natural movement of the state trajectory after a switching action. The goal is to make the converter revert to the steady state in two switching actions under large-signal disturbances [14, 15]. In [16], a new approach was presented by modifying the state-plane by energy-plane based on the theory of energy conservation. The proposed method is applied to boost converter and overcome the problems faced in developing SBC using state-plane. Results show good performance of the converter and can be applied also to other types such as buck–boost or inverters.
In [17], the theory of second-order switching surface is generalised for single-phase multilevel inverter.

In this paper, a dual-output boost converter (DBC) is controlled by extending the concept used in [16] where a second-order switching surface is developed for each region of control. The control algorithm is implemented in digital signal processor (DSP) to overcome the problems faced on analogue circuit design. Apart from providing a stable DC output voltage, the proposed control can also revert back to the steady state in two switching actions after a large-signal disturbance. The theoretical prediction and experimental results are in good agreement. Modeling, design and analysis of the overall system will be given.

2 DBC system model description

The circuit configuration of the adopted DBC is shown in Fig. 1. It consists of a DC source $V_s$, two power switching devices ($Q_1, Q_2$), one inductor $L$, two fast-recovery diodes ($D_1, D_2$), two DC capacitors ($C_1, C_2$) and two loads ($R_1, R_2$). The proposed configuration has the benefit of having less current ripple about four times if compared with the conventional boost converter, hence four time less inductance than this which means high efficiency and low cost [18]. With reference to Fig. 2, the output voltage of this converter is governed by the switching pair ${({Q_1, Q_2}, (D_1, D_2)})$, and hence four modes can be observed according to switches’ switching state:

- **Mode 1**: Switches ($Q_1, Q_2$) are turned ON and diodes ($D_1, D_2$) are OFF. The inductor current is increasing and the two capacitors are discharging to supply the load. Fig. 2a illustrates this mode and the state-space equation is

$$\begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R_1C_1} & 0 \\ 0 & 0 & -\frac{1}{R_2C_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_s$$

- **Mode 2**: Switch $Q_1$ is turned ON, $Q_2$ is turned OFF, diode $D_1$ is reverse biased and $D_2$ conducts current. The input source is delivering energy to $C_2$. The input slope is positive if $V_s$ is higher than $v_2$ and negative in otherwise. Fig. 2b presents this mode and the state-space equation is

$$\begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R_1C_1} & 0 \\ 0 & 0 & -\frac{1}{R_2C_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_s$$
defined as

\[
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix}' = \begin{bmatrix}
0 & 0 & -\frac{1}{L} \\
0 & -\frac{1}{R_1C_1} & 0 \\
\frac{1}{C_2} & 0 & -\frac{1}{R_2C_2}
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
0 \\
0
\end{bmatrix}v_s
\]

(2)

\[
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix}' = \begin{bmatrix}
0 & 0 & 0 \\
0 & -\frac{1}{R_1C_1} & 0 \\
\frac{1}{C_2} & 0 & -\frac{1}{R_2C_2}
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
0 \\
0
\end{bmatrix}v_s
\]

(3)

\[
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix}' = \begin{bmatrix}
0 & 0 & -\frac{1}{L} \\
0 & -\frac{1}{R_1C_1} & 0 \\
\frac{1}{C_2} & 0 & -\frac{1}{R_2C_2}
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
0 \\
0
\end{bmatrix}v_s
\]

(4)

- **Mode 3:** As shown in Fig. 2c, Q2 is turned ON, Q1 is turned OFF. D2 is reverse biased and D1 conducts current. The input current flows only through the output of C1. If the input voltage is lower than v1, the input current slope is negative, otherwise it is positive. The state-space equation is defined by

\[
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix}' = \begin{bmatrix}
0 & 0 & -\frac{1}{L} \\
0 & -\frac{1}{R_1C_1} & 0 \\
\frac{1}{C_2} & 0 & -\frac{1}{R_2C_2}
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
0 \\
0
\end{bmatrix}v_s
\]

- **Mode 4:** As presented in Fig. 2d, switches (Q1, Q2) are turned OFF and diodes (D1, D2) are ON. The input current flows through both capacitors and the input current slope is negative. The state-space equation is

\[
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix}' = \begin{bmatrix}
0 & 0 & 0 \\
0 & -\frac{1}{R_1C_1} & 0 \\
\frac{1}{C_2} & 0 & -\frac{1}{R_2C_2}
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_1 \\
v_2
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
0 \\
0
\end{bmatrix}v_s
\]

(4)

Based on the operation mode analysis, the DBC can be controlled in two regions depending whether the input voltage is higher or lower than each capacitor’s voltage. As stated in Table 1, a comparison between the input voltages and the output voltages (V1, V2) will determine the region of operation, if the DBC operates in region I the boost ON mode will charge the voltage with Vc and the OFF mode discharge it either by (V1−Vc) or (V2−Vc). On the other hand if the converter is controlled in region II, the boost ON mode will be charged with either (Vc−V1) or (Vc−V2) and the OFF mode discharge it by (V0−Vc).

### Table 1 Working principle of the DBC

<table>
<thead>
<tr>
<th>Satisfied conditions</th>
<th>Boost states</th>
<th>Switches states (Q1,Q2)</th>
<th>Mode of operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>region I V1 &lt; V0 and V2 &lt; V0</td>
<td>ON</td>
<td>(1,1)</td>
<td>mode 1</td>
</tr>
<tr>
<td>region II V1 &gt; V0 and V2 &gt; V0</td>
<td>OFF</td>
<td>(0,0)</td>
<td>mode 3</td>
</tr>
</tbody>
</table>

The design of the switching boundary and the system performance is based on the trajectories evolution. The state equation is used to formulate a family of system trajectories. The most straight forward approach to construct the system trajectories is to solve the system state equation by varying the initial condition in time-domain. Therefore to formulate the DBC system trajectories in state-plane (iL, V0), the four operation modes described in (1)–(4) are solved with different initial conditions. However as discussed in [16], it is difficult to formulate a simple mathematical function for the optimal switching surface as in [12] because of the spiral shape of the OFF-state. Hence a proposed solution is to use the state-energy plane (iL, W) of the DBC rather than the state-plane (iL, V0) where the instantaneous energy W is defined as

\[
W = \frac{1}{2}Li_L^2 + \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2
\]

(5)

Fig. 3 illustrates the geometrical representations of the switching trajectory in the state-plane, (iL, V1), (iL, V2), (iL, V0) and the state-energy plane (iL, W) using MATLAB software where the DBC has the parameters stated in Table 3 and with initial condition set to zero for each variable. Observing the results presented in Fig. 3 gives that the ON and OFF trajectories in state-energy plane are not parallel, hence a slower response to the reference point. The OFF-state is slower than the ON-state which yield a slower time to reach the intersection point with the detection curve δ. Therefore to improve the response of the system, a possible solution is to multiply the OFF-state coefficient with a desired positive value.

### 3.1 Formulation of switching surface in state-energy plane

As shown in Fig. 4, the control algorithm based on state-energy plane tries to express the energy error ΔW, defined by ΔW = W−Wref, as a quadratic function of the current error by imposing the use of certain surface forms deduced from the four operation modes. Choosing C1 = C2, R1 = R2 and Ceq is the total equivalent capacitance, the instantaneous and reference energies obtained can be formulated based on the total output voltage V0 as follows

\[
W = \frac{1}{2}Li_L^2 + \frac{1}{2}C_{eq}V_0^2
\]

(6)

\[
W_{\text{ref}} = \frac{1}{2}Li_L^{\text{ref}} + \frac{1}{2}C_{eq}V_0^2
\]

(7)
Derivation both of (6) and (7), then replacing both $\frac{di}{dt}$ and $\frac{dV_0}{dt}$ by their respective equivalent value from each operation mode will lead to the following energy error $\Delta W$

$$
\Delta W = W - W_{\text{ref}} = \int_{t_1}^{t_2} (dW - dW_{\text{ref}}) \, dt = \int_{t_1}^{t_2} (P_{\text{in}} - P_{\text{ref}}) \, dt
$$

(9)

Applying trapezoidal method to solve the integral yield

$$
\Delta W = \int_{t_1}^{t_2} P_{\text{in}} \, dt + \int_{t_1}^{t_2} P_{\text{ref}} \, dt = (t_2 - t_1) \frac{P_{\text{in}}(t_2) + P_{\text{in}}(t_1)}{2} - P_{\text{ref}} 	imes (t_2 - t_1)
$$

(10)

Given that

$$
(t_2 - t_1) = \frac{P_{\text{in}}(t_2) - P_{\text{in}}(t_1)}{dP_{\text{in}}}
$$

(11)
Therefore
\[
\Delta W = \frac{1}{2} \frac{1}{dP_{in}} \left\{ (P_{in}(t2) - P_{ref})^2 - (P_{in}(t1) - P_{ref})^2 \right\} \tag{12}
\]
d\(P_{in}\) is obtained for each operation mode from the equation that describes the inductance current variation, for example, if you are in Mode 1
\[
L \frac{di}{dt} = v_s \Rightarrow V_s L \frac{di}{dt} = v_s^2 \Rightarrow dP_{in} = \frac{v_s^2}{L} \tag{13}
\]
From (12), the ON and OFF trajectories can be derived. Hence the criteria for switching (\(Q_1, Q_2\), ON and OFF are established. The ideal switching surface should pass through the operating point (\(i_{\text{ref}}, W_{\text{ref}}\)), and along the ON-state trajectory when the state is above the load line; along the OFF-state trajectory when the state is below the load line as described in Fig. 4.

### 3.2 Derivation of second-order switching surface

Assuming that \(v_1 = v_2\), the DBC operates in two regions which depend whether the input voltage is lower or higher than half of the output voltage \(V_o\). If the converter operates in region I, \((V_s < V_o/2)\), Mode 1 with Mode 2 or Mode 3 are used, otherwise Mode 4 with Mode 2 or Mode 3 are used when the converter operates in region II \((V_s > V_o/2)\). Simplifying (12) for each mode yields the following second-order switching surface:

- **Region I**: The combination of Mode 1 with Mode 2 or 3 will be similar as a normal boost converter. The DC-bus balancing is achieved by comparing the voltage of each capacitor then enabling the discharge through the lower or upper boost converter to compensate the voltage difference between the two capacitors. \(S_1\) is the derived switching surface and is defined as (see (14))

- **Region II**: The grouping of Mode 4 with Mode 2 or 3 will allow the voltage input to be higher than one capacitor voltage, the voltage balancing is achieved in the same manner by enabling Mode 2 or 3. The derived switching surface \(S_2\) is defined as (see (15))

In the case where the converter work as normal boost converter, Modes 1 and 4 are grouped and the switching surface \(S_3\) is described by (see (16))

The coefficients \(k_1, k_2\) and \(k_3\) are set to be
\[
k_1 = \frac{L}{2} \cdot \frac{L}{v_s - v_1}, \quad k_2 = \frac{L}{v_s - (v_1 + v_2)}
\]

### 4 Steady-state characteristics

The control scheme in the boundary control theory does not differentiate between steady state and transient operation. However, the steady-state operation requires a detailed analysis to establish the relation between ripples and the selected switching surface boundaries. Assuming two points \((x_1, x_2)\) on the ON and OFF trajectories with the desired coordinate \((W_1, i_{\text{max}})\) and \((W_2, i_{\text{min}})\), it can be shown from Fig. 4 and based on each control region that:

- **In region I**: At the steady state, the ripple current \((i_{\text{max}} - i_{\text{ref}})\) and \((i_{\text{min}} - i_{\text{ref}})\) have the same value. Points \((x_1, x_2)\) are both on the ON- and OFF-state trajectories, hence \(W_1 = W_2\). Based on (14), the ON and OFF trajectories are defined as follows

\[
\begin{align*}
W_1 - W_{\text{ref}} &= k_2(i_{\text{max}} - i_{\text{ref}})^2 - \Delta W = 0 \\
W_2 - W_{\text{ref}} &= k_1(i_{\text{min}} - i_{\text{ref}})^2 + \Delta W = 0
\end{align*}
\tag{17}
\]

Therefore it is easy to demonstrate that the ripple current is equal to
\[
\Delta i_{\text{min}} = \Delta i_{\text{max}} = \frac{2}{\sqrt{k_1 - k_2}} \Delta W
\tag{18}
\]

In the same manner, it is clear that the energy at both points is equal to
\[
W_1 = W_2 = W_{\text{ref}} + \frac{k_1 + k_2}{k_1 - k_2} \Delta W
\tag{19}
\]

Using (6) and (17) the ripple voltage can be demonstrated to

\[
S_1(t) = \begin{cases} 
0 & \text{if } \alpha_1^2(t) = \Delta W(t) - k_2[i_L(t) - i_{\text{ref}}]^2 > 0 \quad \text{and} \quad i_L(t) > i_{\text{ref}}(t) \\
1 & \text{if } \alpha_2^2(t) = \Delta W(t) - k_1[i_L(t) - i_{\text{ref}}]^2 < 0 \quad \text{and} \quad i_L(t) < i_{\text{ref}}(t)
\end{cases}
\tag{14}
\]

\[
S_2(t) = \begin{cases} 
0 & \text{if } \alpha_1^2(t) = \Delta W(t) + k_3[i_L(t) - i_{\text{ref}}]^2 > 0 \quad \text{and} \quad i_L(t) > i_{\text{ref}}(t) \\
1 & \text{if } \alpha_2^2(t) = \Delta W(t) + k_2[i_L(t) - i_{\text{ref}}]^2 < 0 \quad \text{and} \quad i_L(t) < i_{\text{ref}}(t)
\end{cases}
\tag{15}
\]

\[
S_3(t) = \begin{cases} 
0 & \text{if } \alpha_1^2(t) = \Delta W(t) - k_1[i_L(t) - i_{\text{ref}}]^2 > 0 \quad \text{and} \quad i_L(t) > i_{\text{ref}}(t) \\
1 & \text{if } \alpha_2^2(t) = \Delta W(t) - k_1[i_L(t) - i_{\text{ref}}]^2 < 0 \quad \text{and} \quad i_L(t) < i_{\text{ref}}(t)
\end{cases}
\tag{16}
\]
be

\[
 v_{0\text{min}} = \sqrt{\frac{4}{C} W_{\text{ref}} + \frac{4}{C} k_1 + \frac{k_2}{C} \Delta w - \frac{2L}{C} (i_{\text{ref}} + \Delta i_{\text{max}})^2} \\
v_{0\text{max}} = \sqrt{\frac{4}{C} W_{\text{ref}} + \frac{4}{C} k_1 + \frac{k_2}{C} \Delta w - \frac{2L}{C} (i_{\text{ref}} - \Delta i_{\text{min}})^2}
\]

(20)

The switching frequency is the reciprocal of the time that the converter needs to perform one switching action in steady state. Therefore the switching frequency is equal to

\[
f_s = \frac{1}{T_{\text{on}} + T_{\text{off}}} = \frac{V_{\text{in}}(V_0 - 2V_{\text{in}})}{2LV_0\sqrt{2/(k_1 - k_2)\Delta w}}
\]

(21)

- **In region II:** Following the same procedure in region I, the ON and OFF trajectories at points \( (x_1, x_2) \) is defined as

\[
\begin{align*}
W_1 - W_{\text{ref}} + k_1(i_{\text{max}} - i_{\text{ref}}) = -\Delta w &= 0 \\
W_2 - W_{\text{ref}} + k_2(i_{\text{min}} - i_{\text{ref}}) = \Delta w &= 0
\end{align*}
\]

(22)

Thus, the current ripple can be expressed as

\[
\Delta i_{\text{min}} = \Delta i_{\text{max}} = \sqrt{\frac{2}{k_2 - k_3} \Delta w}
\]

(23)

which yields an energy equal to

\[
W_1 = W_2 = W_{\text{ref}} - \frac{k_2 + k_3}{k_2 - k_3} \Delta w
\]

(24)

Using (6) and (22), the voltage ripple can be easily determined by

\[
\begin{align*}
v_{0\text{min}} &= \sqrt{\frac{4}{C} W_{\text{ref}} - \frac{4}{C} k_2 + \frac{k_3}{C} \Delta w - \frac{2L}{C} (i_{\text{ref}} + \Delta i_{\text{max}})^2} \\
v_{0\text{max}} &= \sqrt{\frac{4}{C} W_{\text{ref}} - \frac{4}{C} k_2 + \frac{k_3}{C} \Delta w - \frac{2L}{C} (i_{\text{ref}} - \Delta i_{\text{min}})^2}
\end{align*}
\]

(25)

The ON and OFF times are determined by the inductor ripple current, thus the switching frequency is equal to

\[
f_s = \frac{1}{T_{\text{on}} + T_{\text{off}}} = \frac{(2V_{\text{in}} - V_0)(V_0 - V_{\text{in}})}{2LV_0\sqrt{2/(k_2 - k_3)\Delta w}}
\]

(26)

5 Large-signal stability analysis

Based on the trajectory behaviour at the boundary, the point along the switching surface \( \sigma^2 \) is classified into three categories: refractive point, reflective point and rejective point. These three possibilities govern the stability of a system. The type of point can be determined by analysing \( \sigma^2 \) and \( \dot{\sigma}^2 \). The dynamics of the system will exhibit differently in the respective modes. In the rejective mode, the trajectories on both sides of the switching surface move away from the switching surface. The converter will be in unstable operation. In the reflective mode, the converter will be in the sliding mode, hence the trajectory will move along the surface to the operating point through several switching actions. In the refractive region, the state will move around the operating point. Therefore by differentiating the switching surfaces \( (S_1, S_2) \) and evaluating the ON and OFF trajectories in terms of \( (\sigma^2, \dot{\sigma}^2) \) polarities, the coefficients \( (k_1, k_2, k_3) \) will govern the system’s stability. The derivation of \( \sigma^2 \) for the ON and OFF trajectories in region I can be written as (see (27))

In region II, the derivation of \( \sigma^2 \) for the ON and OFF trajectories is as follows (see (28) at the bottom of the next page)

Assuming that the converter is in region I, the condition for reflective mode is attained when the product of \( \sigma^2 \) and \( \dot{\sigma}^2 \) is negative. Therefore for the \( S_1 \) ON-state the condition is

\[
\begin{align*}
&k_1 > \frac{L}{2} \quad \text{when } i_1(t) < i_{1\text{off}}(t) \\
&k_2 < 0 \quad \text{when } i_1(t) > i_{1\text{off}}(t)
\end{align*}
\]

(29)

Similarly for \( S_1 \) OFF-state the condition is

\[
\begin{align*}
&k_1 > 0 \quad \text{when } i_1(t) < i_{1\text{on}}(t) \\
&k_2 < \frac{L(V_s - V_1)}{2(V_s - V_1)} \quad \text{when } i_1(t) > i_{1\text{on}}(t)
\end{align*}
\]

(30)

By combining the two conditions stated in (29) and (30), the required values of \( k_1 \) and \( k_2 \) that ensure the converter is in...
the reflective mode is

\[
\begin{aligned}
    k_1 &> \frac{L}{2} \\
    k_2 &< \frac{LV}{2(V_s - V_0)}
\end{aligned}
\]

(31)

In the same way the coefficients \(k_1, k_2\) and \(k_3\) condition that ensures the operation in the three modes, refractive, reflective, and rejective, for the dual output boost converter are as follows:

- **Reflective**: The condition for reflective mode is satisfied when

\[
\begin{aligned}
    k_1 &> \frac{L}{2} \\
    k_2 &< \frac{LV}{2(V_s - V_0/2)}
\end{aligned}
\]

in region I

(32)

- **Rejective**: The condition for this mode is satisfied when

\[
\begin{aligned}
    k_1 &< -\frac{L}{2} \\
    k_2 &< \frac{LV}{2(V_0/2 - V_s)}
\end{aligned}
\]

(33)

If the switching surface is chosen to be along the boundary of the reflective and refractive regions, the converter will exhibit fast dynamic response. Fig. 5 shows the operations’ regions of the predefined coefficients. Moreover the variation of \(k_3\) has an influence on the time taken to reach the steady state when the DBC is controlled in region I as illustrated in Fig. 6. Similarly if the DBC is controlled in region II, \(k_3\) has the same influence.

6 Control algorithm implementation

With reference to the flowchart shown in Fig. 7, the control algorithm which expresses the energy error \(\Delta W\) as a quadratic function of the current error is implemented in DSP TMS320F2812. A timer interrupt is used to calculate the reference current, the instantaneous energy and determine the switching surface of each mode by imposing the use of certain surface forms, \(S_1\) or \(S_2\), deduced from the four operation modes. Then it starts by verifying the control
region where the converter operates. If the voltage of each capacitor is higher than the input voltage, the DBC can be seen as simple boost where the charging voltage in mode 1 is used to charge either capacitor $C_1$ or $C_2$ and achieves a DC-bus balancing. Thus the control uses either modes 1 and 2 or mode 1 and mode 3.

On the other hand, if the voltage of each capacitor is lower than the input voltage, mode 2 or 3 has a positive slope, and hence the inductor current is increasing. Then mode 4 is used with them to discharge the voltage. As consequence the control mode in this region uses either modes 4 and 2 or

![Flowchart](image)

**Table 2** Switching actions of the DBC

<table>
<thead>
<tr>
<th>Region</th>
<th>$V_s &gt; V_{o2}$</th>
<th>Flip-flop</th>
<th>$V_1 &gt; V_2$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>region I</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>region II</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Fig. 6** Output voltage with different values of $k_2$ in region I

**Fig. 7** Control algorithm flowchart
modes 4 and 3. At the end, the pulse patterns of the two switches are generated according to the switching stated in Table 2. A simplification of the control signal can be made using Karnaugh map to express the relation between the switches \((Q_1, Q_2)\) state and the three other criteria (region, flip-flop output and the DC-bus balancing). However, in DSP implementation an RS flip-flop function with a decoder is used to generate the desired output based on Table 2.

<table>
<thead>
<tr>
<th>Components</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>load</td>
<td>( R_1 = R_2 = 250 , \Omega )</td>
</tr>
<tr>
<td>capacitor</td>
<td>( C_1 = C_2 = 200 , \mu F )</td>
</tr>
<tr>
<td>inductance</td>
<td>( L = 1 , mH )</td>
</tr>
<tr>
<td>reference voltage for each C</td>
<td>( V_{1} = V_{2} = 150 , V )</td>
</tr>
<tr>
<td>input voltage</td>
<td>( V_s = 100 ) then ( 180 , V )</td>
</tr>
</tbody>
</table>

Fig. 8  Steady-state simulation results in region I

a  DBC output voltages and inductor current  
b  State-plane and state-energy plane

Table 3  Simulation and hardware parameters

Fig. 9  Simulation results under 50% load change in region I
7 Simulation and experimental results

The adopted DBC has been investigated by software simulation and hardware tests. The proposed control algorithm with DBC parameters stated in Table 3 is simulated using MATLAB software. The DBC is controlled in region I. The steady-state results of the voltage outputs and the inductor current with the state trajectories (state-plane and state-energy plane) are shown in Fig. 8. The results including the output voltages $V_0$ and $v_1$, load current with 50% load change and the switching signals ($Q_1$, $Q_2$) when $V_s < v_1$ are shown in Fig. 9. When the current load is increased, the voltage output reaches the steady state at about 2 ms with an increasing of the ripples. On the other hand if the load is increased, which means the load current is reduced the time of reaching the steady state is lower since the inductor current has enough energy at the moment of the drop. The system has a fast transient response and the two capacitors have almost the same voltage within about two switching actions as stated by the theory.

To verify the effectiveness of the control algorithm in region I, a comparison with hysteresis control mode and current control mode are carried out. As shown in Fig. 10, the hysteresis control mode with a single reference $W_{ref}$ has a switching frequency different than the second-order switching surface with significant ripple. On the other hand, as illustrated in Fig. 11, the current control mode with a single reference $I_{ref}$ has a slower transient response compared with the desired control.

A laboratory prototype of the DBC has been developed to test the circuit and the control strategy. Since the control algorithm can be used to control the DBC in two regions, Fig. 12 shows the output voltages $V_0$ and $v_1$ with the pulse of one switch when the converter is subject to input voltage variation from 100 to 180 V. The difference in pulse width and switching frequency is well observed to achieve the desired output voltage. To investigate the dynamic response of the proposed control scheme, a 50% load change with $V_s < v_1$ is tested and shown in Fig. 13. As it can be seen, the load variation did not affect much the DC-bus balancing. The output voltages track the desired output voltage and recover to steady state after a small switching.
time around 2 ms. The voltage ripple is balanced between 14 and 8 V. Recall that the control algorithm had been modified slightly in mode 1 to limit the current from exceeding the inductance limited current design. This modification is adopted because of the higher amount of energy needed to reach the reference voltage output in the initial stage.

Another important point in the analysis is the behaviour of the converter at start-up; Fig. 14 shows that during start-up the converter reaches the steady state about 6 ms without overshoot problem. It takes more than two switching actions because of current limitation condition set to overcome inductance current limitation. The time of reaching the steady state can be improved by increasing the sampling time of the system.

8 Conclusion

A boundary control technique with second-order switching surface formulated for a DBC has been proposed. Switching surfaces formed from energy plane have been derived to control the DBC. The proposed control algorithm tries to express the energy error as a quadratic function of the current error by imposing the use of certain surface forms deduced from the four operation modes of the DBC. These derived switching surfaces lead to an output voltage tightly regulated with a balanced voltage between the two capacitors. The proposed control exhibits fast transient response; the output voltage is generally reverted to the steady state in two switching actions during large-signal disturbances. Moreover, the control method does not require any sophisticated analogue circuitry because it is implemented in DSP. The obtained results show a good agreement between the theoretical prediction and experimental measurement results.

9 References