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To link to this article: http://dx.doi.org/10.1080/10426914.2015.1037901

Accepted online: 05 Jun 2015. Published online: 05 Jun 2015.

Article views: 101
A Fuzzy Logic-Based Prediction Model for Kerf Width in Laser Beam Machining

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In laser beam machining, the main concern is the machining quality as kerf width of the end product. It is essential for industrial applications to cut the workpiece with minimum kerf width. However, it is difficult to develop a precise functional relationship between input and output variables in laser machining. Therefore, an effort has been conducted to build up an intelligent fuzzy expert system (FES) model to predict the kerf width in CO2 laser cutting. The developed fuzzy model was performed on fuzzy toolbox in MATLAB R2009b by employing Mamdani technique.

INTRODUCTION

The applications of laser in the industry have been growing up significantly because of its outstanding benefits, such as machining quality, productivity, saving of the pre- and post-processing of the materials, and minimum loss of the parent workpiece as well as technical benefits [1, 2]. In laser cutting processes, a kerf width is formed through the relative movement of the workpiece and laser beam. This movement allows creating an intricate shape on a flat workpiece. Mechanism of laser cutting is one kind of thermal process that depends on laser power which is divided into conduction, energy of melting, and vaporization and energy losses to the environment to balance the input laser energy. The objective of this research to investigate kerf width for different input parameters such as assisting gas pressure, laser power, cutting speed, and standoff distance onto 3 mm thickness of polymethylmethacrylate (PMMA) and an effort has been carried out to establish a fuzzy expert system (FES) model in order to predict the kerf width.

Laser machining quality is strongly dependent on input parameters. In laser machining process, laser beam hits on workpiece which can be restrained by the adjustment of the assisting gas pressure, laser power, scanning speed, beam mode (continuous wave (CW) or pulsed), and beam frequency [3]. Goeke et al. [4] presented the application of CO2 laser cutting on carbon fiber-reinforced plastics (CFRPs). They observed that radiation of laser and absorption into material have an indicative influence on heat-affected zone and kerf width. Quintero et al. [5] applied CO2 laser cutting on phenolic resin boards (PRBs) and particleboard plates that were covered with a melamine sheet imitating beechwood. They used a design of experiment (DOE) and found the output laser cutting quality as kerf width and surface roughness. Dubey et al. [6] applied a combined method of Taguchi Method and Response Surface Method (TMRSM) for the optimization of silicon alloy steel’s width of kerf and material removal rate (MRR). They also developed a mathematical model of kerf width and MRR, according to the laser input parameters such as gas pressure, pulse width, cutting speed, and frequency of the pulse. Pandey et al. [7] analyzed the kerf quality of Ti alloy in pulsed laser cutting by an intelligent FES. Sheng et al. [8] proposed an analytical prediction model of kerf width as a relation to erosion front. Pandey et al. [9] developed FES model for duralumin sheet to get minimum kerf width and also deviations on top and bottom sides. Syn et al. [10] analyzed surface quality of Incoloy alloy by an intelligent FES model. It has not been found in literature that artificial intelligent (AI) such as an FES is applied on nonmetal like PMMA to predict kerf width. Therefore, an effort has been carried out to build up a model of kerf width in CO2 laser cutting by FES. The uses of PMMA are being increased not only for as a substitute of glass but also for bone cement, prostheses, intraocular lens, dentures, biosensors, and biomechanical device [11]. This research will be helpful to find the optimum cutting

Keywords Fuzzy; Kerf; Laser; Model; PMMA.
conditions and take decisions for adjusting the input parameters of PMMA material in laser machining industry with minimum loss of the parent material.

**MATERIALS AND METHODS**

All the experiments have been performed by a Zech laser machine that can provide up to 500 W CW CO₂ laser with the mode structure of TEM₀₁. It consists of two parts: a laser generation system (ZLX5) and a laser cutting workstation (ZL 1010). In this experiment, laser machine was associated with a computer system to control the operation. AutoCAD software was used for drawing. C-Cut software was applied to synchronize movements of the laser head with cutting direction. The material used for this experiment was PMMA with thickness of 3.0 mm. A digital caliper, resolution of 0.01 mm and range from 0 to 150 mm, was employed to get the measurement of the lengths H₁ and H₂ as shown in Fig. 1. Corner’s power, nozzle diameter, and delay time were constant in this experiment. Table 1 shows the list of the input parameters with different levels. Linear rectangular cuts were made on a PMMA workpiece. A schematic of the cutting nozzle and optics is shown in Fig. 2. A converging lens (thickness 6 mm) with a fixed focal length (127 mm) was installed along the beam path. Standoff distance was varying from 1 to 10 mm to produce laser spot on the workpiece. To measure the upper kerf width, H₁ and H₂ side lines were picked out three times at several locations and were computed by the following equation:

\[
\text{Kerf width} = \frac{(H_1 - H_2)}{2}
\]  

The interaction of the laser beam and material is a complex phenomenon. It can be classified as absorption, reflection, conduction, convection, melting, vaporization, radiation, scattering, and transmission [12]. But laser machining of PMMA is different from other metal because CO₂ laser interaction with PMMA begins vaporization while it is subjected to melt shearing [13, 14]. Hence, the statement of the laser machining of PMMA is vague information. The principle of FES has been vindicated as an efficient tool to manage this kind of information [9, 15]. Generally, researchers use statistical analysis for evaluation and investigation of the laser parameters [16, 17] based on experimental result only. On the other hand, the fuzzy logic model is a multi-input multi-output modeling technique which combines different heterogeneous data (such as experimental data, numerical data, mathematical data, recommendation, and suggestions). Also, we have found relative error in regression analysis for the experimental data alone was 9.38% \((R^2 = 0.907)\), which is greater than fuzzy model relative error 3.852% \((R^2 = 0.989)\). This paper concentrates on fuzzy logic because our further implication is fuzzy controller for autolaser cutting process.

Generally the variables are defined as linguistic variables. Each linguistic variable has linguistic values that may be expressed as synthetic language, words, and sentences. In this study, 3 linguistic values were used for input variable (Low, Medium, and High) and 15 linguistic values were used for the kerf width (kw1, kw2, kw3, kw4, kw5, kw6, kw7, kw8, kw9, kw10, kw11, kw12, kw13, kw14, and kw15). PMMA’s kerf width prediction of FES was implemented on MATLAB R2009b software. For getting more precise results, more membership functions were used in output. For fuzzification, two membership functions are common: triangle and trapezoidal membership function [18]. Triangle membership function was used because it is defined as a simplest membership function [19]. In this research, IF–THEN rules have been built up by a set of 64 experimental data, which is called as training data. Rules are built up in the MATLAB rule editor and can be accessed in the rule viewer. In this CO₂ laser study fuzzification of the

![Figure 1.—Kerf width measurement.](image1)

![Figure 2.—A schematic of the cutting nozzle and optics.](image2)

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assist gas pressure: A</td>
<td>0.5</td>
<td>2.5</td>
<td>4.5</td>
<td>Bars</td>
</tr>
<tr>
<td>Laser power: B</td>
<td>100</td>
<td>300</td>
<td>500</td>
<td>Watt</td>
</tr>
<tr>
<td>Cutting speed: C</td>
<td>0.2</td>
<td>0.7</td>
<td>1.2</td>
<td>m/min</td>
</tr>
<tr>
<td>Standoff distance: D</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>Mm</td>
</tr>
</tbody>
</table>
linguistic parameters were made by the following function:

\[ GP(i_1) = \begin{cases} 
  i_1; & 0.5 \leq i_1 \leq 4.5 \\
  0; & \text{otherwise}
\end{cases} \]  
(2)

\[ LP(i_2) = \begin{cases} 
  i_2; & 100 \leq i_2 \leq 500 \\
  0; & \text{otherwise}
\end{cases} \]  
(3)

\[ CS(i_3) = \begin{cases} 
  i_3; & 0.2 \leq i_3 \leq 1.2 \\
  0; & \text{otherwise}
\end{cases} \]  
(4)

\[ SD(i_4) = \begin{cases} 
  i_4; & 1 \leq i_4 \leq 10 \\
  0; & \text{otherwise}
\end{cases} \]  
(5)

\[ KW(o_1) = \begin{cases} 
  o_1; & 0 \leq o_1 \leq 1.9 \\
  0; & \text{otherwise}
\end{cases} \]  
(6)

where \( i_1, i_2, i_3, \) and \( i_4 \) are input variables as assisting gas pressure, laser power, cutting speed, and standoff distance, respectively, and \( o_1 \) is the output variable kerf width.

By the implication process, fuzzy rules were evaluated. Mamdani implication was used because there were three membership functions for each input linguistic variables which were taken into consideration. The implication results were analyzed by an aggregation process. In this process, output results of the rules were aggregated into a single fuzzy set. The fuzzification process, membership function, and the linguistic fuzzy sets of assisting gas pressure (GP), laser power (LP), cutting speed (SD), standoff distance (SD), and kerf width (KW) interval can be deduced from Eqs. (2)–(6) as follows:

\[ \mu_{Medium}(GP) = \begin{cases} 
  \frac{i_1 - 0.5}{4.5 - 0.5}; & 0.5 \leq i_1 \leq 2.5 \\
  \frac{4.5 - i_1}{4.5 - 2.5}; & 2.5 \leq i_1 \leq 4.5 \\
  0; & i_1 > 4.5
\end{cases} \]  
(7)

\[ \mu_{Medium}(LP) = \begin{cases} 
  \frac{i_2 - 100}{300 - 100}; & 100 \leq i_2 \leq 300 \\
  \frac{300 - i_2}{300 - 100}; & 300 \leq i_2 \leq 500 \\
  0; & i_2 > 500
\end{cases} \]  
(8)

\[ \mu_{Low}(CS) = \begin{cases} 
  1; & i_3 \leq 0.2 \\
  \frac{0.7 - i_3}{0.7 - 0.2}; & 0.2 \leq i_3 \leq 0.7 \\
  0; & i_3 > 0.7
\end{cases} \]  
(9)

\[ \mu_{Medium}(SD) = \begin{cases} 
  \frac{i_4 - 1}{10 - 1}; & 1 \leq i_4 \leq 10 \\
  \frac{10 - i_4}{10 - 5}; & 5 \leq i_4 \leq 10 \\
  0; & i_4 > 10
\end{cases} \]  
(10)

The output result is obtained as fuzzy variables. By defuzzification process, the fuzzy output can be converted into a relative value. Centroid method was used for defuzzification to find nonfuzzy value \( y \) [19] as expressed in the following equation:

\[ y = \frac{\sum x^i \mu_A(x_i)}{\sum \mu_A(x_i)} \]  
(11)

where \( y \) is the crisp value of the kerf width and \( x^i \) is the position of the singleton in the respective universe.

In the stage of defuzzification, truth degree (\( \mu \)) of rules can be calculated by the aid of minimum for individual rules and then maximum for between two working rules. One example is considered, to perceive fuzzification, input variable GP \( (i_1) = 2.5 \) bars, LP \( (i_2) = 300 \) W, CS \( (i_3) = 0.2 \) m/min. and SD \( (i_4) = 5 \) mm, then the rule 24 is fired. The firing strength \( z \) of the rule 24 is obtained as follows (from Eqs. (7)–(10)):

\[ z_{24} = \min \left\{ \mu_{Medium}(GP), \mu_{Medium}(LP), \mu_{Low}(CS), \mu_{Medium}(SD) \right\} = \min\{1, 1, 1, 1\} = 1 \]

Thus, the membership function for rule 24 is obtained as follows:

\[ \mu_{24}(KW) = \min\{1, \mu_{Kerf}(KW)\} \]

According to the view of Rajasekaran et al. [20], crisp decision is more accurate if output is represented as a single scalar. By using Eq. (11), the crisp output of kerf width found 1 mm that was close to MATLAB result of 0.995 mm.

The mathematical and statistical methods have been used to investigate the accuracy of the prediction ability. Relative error (\( \varepsilon \)) of formation has been established by using the following formula:

\[ \varepsilon = \frac{\sum_{i=1}^{n} |y_i - \bar{y}_i|}{n} \times 100\% \]  
(12)

Another method goodness of fit (\( \eta \)) for the prediction capability of the system is calculated as follows:

\[ \eta = 1 - \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}} \]  
(13)

where \( n \) is the total experiment number, \( y_i \) is the experimental value, \( \bar{y}_i \) is the fuzzy logic predicted value, and \( \bar{y} \) is the average of the experimental value.

RESULTS AND DISCUSSION

The kerf width prediction of the fuzzy logic model has been developed in the present study as an effect of assisted gas pressure, laser power, cutting speed, and standoff distance. Linguistic variables were fuzzified by triangle membership function. Centroid method was used to defuzzification. A total number of 81 experiments were carried out where 64 data were used to train and 17 data were applied to test the validation of the
developed intelligent model. Figure 3 shows the accuracy of FES as the goodness of fit is 0.994 ($R^2 = 0.989$), relative error 3.852% and 0.97 ($R^2 = 0.966$), relative error 5.2% for training and testing, respectively. Relative errors were found less than 10% for both training and testing. Nukman et al. [21] accepted less than 10% prediction error in CO₂ laser cutting process to predict kerf width. Table 2 shows the comparison of prediction error for artificial neural network (ANN), Taguchi ANN, Taguchi ANN with genetic algorithm, and FES model with the experimental values [21].

According to the statistical $p$ values presented in Table 3; it's clear that standoff distance, laser power, and cutting speed are significant for upper kerf width. The beam diameter is approximately proportional to standoff distance. From the surface plot (Fig. 4(a)), it can be said that the kerf width becomes wider with the increase of standoff distance. Because with the increase of standoff distance, the area of the beam spot also increases, so that it leads to the area of enhanced thermal energy resulting in wider kerf width. On the other hand, at higher speeds, kerf width decreases, which is in agreement with Tamrin et al. [13] and Cenna et al. [22]. From the surface plot (Fig. 4(b)), it shows that with increasing of laser power the kerf width also increases because of this thermal energy is also agreement with Fu et al. [23].

### Conclusions

In this study, a Mamdani FES model has been proposed to predict kerf width for the influence of gas pressure, laser power, standoff distance, and cutting speed on laser beam machining of PMMA sheet. All the experiments were performed by varying the input parameters with different levels. Experimental results were used for training and testing of the developed fuzzy model. The fuzzy logic model was performed on fuzzy toolbox in MATLAB R2009b. Prediction ability of the model was satisfactory in terms of relative error and goodness of fit. Hence, the proposed model can be used in PMMA laser machining simulation.

From the study, the following conclusions can be drawn:

1. The prediction capability of the developed model has been found better than Taguchi ANN model [21].

### Table 2.—Comparison of prediction error with experimental values [21].

<table>
<thead>
<tr>
<th>Predicted check</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assist gas pressure</td>
<td>4.5</td>
<td>0.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Laser power</td>
<td>100</td>
<td>100</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>Cutting speed</td>
<td>0.2</td>
<td>0.7</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>Standoff distance</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Normal ANN</td>
<td>7.78</td>
<td>4.93</td>
<td>-9.21</td>
<td>-36.84</td>
</tr>
<tr>
<td>Taguchi ANN</td>
<td>8.89</td>
<td>6.17</td>
<td>-7.90</td>
<td>-21.05</td>
</tr>
<tr>
<td>Taguchi ANN + GA</td>
<td>-7.78</td>
<td>0</td>
<td>5.26</td>
<td>5.26</td>
</tr>
<tr>
<td>Fuzzy Model</td>
<td>3.39</td>
<td>0.18</td>
<td>7.08</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 3.—Regression analysis

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard error</th>
<th>t Stat</th>
<th>P-value</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.458091017</td>
<td>0.041880916</td>
<td>10.93794173</td>
<td>0.374257256</td>
<td>0.541924777</td>
</tr>
<tr>
<td>Assist gas pressure</td>
<td>-0.01101022</td>
<td>0.007512731</td>
<td>-1.46554171</td>
<td>0.002604858</td>
<td>0.000261247</td>
</tr>
<tr>
<td>Laser power</td>
<td>0.000898931</td>
<td>0.004028144</td>
<td>2.05554063</td>
<td>0.000730596</td>
<td>0.000987999</td>
</tr>
<tr>
<td>Cutting speed</td>
<td>0.28477947</td>
<td>0.031977372</td>
<td>8.90565826</td>
<td>1.89836E+12</td>
<td>0.220769805</td>
</tr>
<tr>
<td>Standoff distance</td>
<td>0.059464585</td>
<td>0.003558002</td>
<td>16.71291554</td>
<td>7.65538E+12</td>
<td>0.265869099</td>
</tr>
</tbody>
</table>
3. The statistical analysis of the fuzzy result for kerf width has shown the relative error (3.852%) and goodness of fit 0.994 ($R^2 = 0.989$) in training, whereas relative error (5.2%) and goodness of fit 0.97 ($R^2 = 0.966$) were obtained in testing.

4. When cutting a 3.0 mm sheet of PMMA with a CW CO$_2$ laser, kerf width becomes wider with the increase of standoff distance and power but at higher speed kerf width decreases.

5. The minimum kerf width was 0.3167 mm at gas pressure 0.5 bar, laser power 100 W, standoff distance 1 mm and cutting speed 1.2 m/min.

**ACKNOWLEDGMENTS**

The Faculty of Engineering, University of Malaya, Malaya, is gratefully acknowledged by the authors.

**FUNDING**

Financial and technical support from the University of Malaya under the Grant RP010A-13AET/RP010B-13AET/RP010C-13AET (UMRG) and the faculty of engineering are gratefully acknowledged by authors.

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