Fractal dynamics of light scattering intensity fluctuation in disordered dusty plasmas


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Dynamic light scattering (DLS) technique is a simple and yet powerful technique for characterizing particle properties and dynamics in complex liquids and gases, including dusty plasmas. Intensity fluctuation in DLS experiments often studied using correlation analysis with assumption that the fluctuation is statistically stationary. In this study, the temporal variation of the nonstationary intensity fluctuation is analyzed directly to show the existence of fractal characteristics by employing wavelet scalogram approach. Wavelet based scale decomposition approach is used to separate non-scaling background noise (without dust) from scaling intensity fluctuation from dusty plasma. The Hurst exponents for light intensity fluctuation in dusty plasma at different neutral gas pressures are determined. At low pressures, weaker damping of dust motions via collisions with neutral gases results in stronger persistent behavior in the fluctuation of DLS time series. The fractal scaling Hurst exponent is demonstrated to be useful for characterizing structural phases in complex disordered dusty plasma, especially when particle configuration or sizes are highly inhomogeneous which makes the standard pair-correlation function difficult to interpret. The results from fractal analysis are compared with alternative interpretation of disorder based on approximate entropy and particle transport using mean square displacement. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4825141]

I. INTRODUCTION

Dusty plasma is loosely defined as electron-ion plasma with additional charged components of micron-sized dust particles. The interplay between the dust particles’ Coulomb potential energy and thermal energy results in the formation of spatially ordered and disordered particle configurations. Light scattering approach is widely used to study particle transport in dusty plasma due to the simplicity and nonintrusive nature of the technique. For particle image velocimetry (PIV), the dust particles are illuminated using a broad laser beam and the particles are tracked using a high speed CCD video camera. From these images, one can study the structural as well as dynamical behavior of dusty plasma. Alternatively, the dynamic light scattering (DLS) experiment involves analysis of the temporal fluctuations of light intensity scattered by the dust. The light intensity fluctuates due to the net changes in the particles’ positions, which results in time-varying interference patterns. The statistics and correlation behaviors of the light intensity fluctuations provide useful information about the particle dynamics in dusty plasma. For example, Hurd and Ho have used the correlation function of the intensity fluctuation to study the particle transport in glow discharge to demonstrate kinetic to hydrodynamic crossover behavior. Anderson and Radovanov have observed low-frequency oscillatory motion in DLS signal and suggested the formation of glassy amorphous state at the edge of liquid to solid transition.

Recently, Muniandy et al. introduced correlation function for the light intensity fluctuation based on fractional Ornstein-Uhlenbeck process. This study particularly addressed the dynamics of polydisperse particles in dusty plasma and showed that resulting light intensity fluctuation exhibits non-Markovian Gaussian statistics with long-memory correlation. The study of long-memory or power-law processes with fractal characteristics has been widely demonstrated in wide varieties of systems, ranging from physiological signals, semiconductor noises, and financial time series. Fractal analysis of light intensity fluctuation has been explored in many studies involving complex media. Shen et al. have shown that light intensity fluctuation scattered from submicron-sized Brownian colloidal particles exhibits fractal characteristics and the corresponding fractal dimension can be related to mean particle size. Scale separation into fast and slow temporal fluctuations in the scattered light intensity has also been observed in some systems, whereby the power spectral density takes the Lorentzian form. Fluctuation of scattered light intensity during gelation process of a suspension of anisotropic monodisperse colloidal discs in water has been studied using discrete wavelet transformation. From the scaling properties in wavelet energy spectrum, the sol-gel transition can be identified by the scaling exponent of intensity fluctuation. Even though fractal analysis has been proven to be a powerful tool for understanding complex dynamics, its application for determining dusty plasmas properties, particularly those based on DLS fluctuation is still not fully explored.

Structural phase changes from disordered gas to partially ordered liquid and solid-like plasma crystals are

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often studied using the pair-correlation function, \( g(r) \). The spatial correlation function is useful for describing homogeneous dusty plasma system. By noting the limitations of correlation analysis of DLS temporal fluctuation that requires stationarity condition and the noisy characteristic of pair correlation function, \( g(r) \) due to inhomogeneous dust particles spatial distribution, we propose the fractal analysis technique for describing complex dynamics of disordered dusty plasmas. The paper is organized as follows. In Sec. II, basic concepts of fractal analysis for time series modeling are described. Fractional Brownian motion (FBM) with Hurst parameter \( H \) (Ref. 13) is used for describing the fractal behavior in terms of graph regularity of DLS fluctuation. An alternative measure of regularity or orderliness based on approximate entropy (ApEn) is included for comparison. Dynamics light scattering experiment and particle imaging velocimetry are described in Sec. III and the results of various estimated parameters are discussed in Sec. IV before the conclusions are made regarding the advantage of fractal approaches for DLS studies.

II. FRACTAL ANALYSIS

Fractal characteristics in time series \( X(t) \) can be described through scale invariance property, namely \( X(at) \equiv a^{-H}X(t) \), where \( a \) is a scale factor, \( 0 < H < 1 \) is the \( H \)-self-similar exponent and \( \equiv \) denotes equality in finite-dimensional probability distribution.\(^{13}\) This property is also known as first-order self-similarity because it depends on one-point function behavior at time \( t \). The local growth of self-similar function exhibits power-law scaling, namely \( E[|X(t + \tau) - X(t)|^2] \sim \tau^{2H} \) as \( \tau \to 0 \) and here the \( H \)-exponent is defined as the Hölder exponent and \( E[\ ] \) denotes expectation value. The fractal dimension of self-similar function is related to the Hölder exponent through relationship \( D = 2 - H \). Meanwhile, second-order self similarity is referred to the scale invariance of the time-shift invariant correlation function

\[
C(t, t + \tau) = E[X(t)X(t + \tau)] = C(\tau) \sim \tau^H,
\]

where \( \beta \) is the correlation exponent. The study described in Muniandy et al.\(^5\) was mainly focused on memory effect of stationary (time-shift invariant) correlation model, thus the time series were detrended to remove very low-frequency trend contamination. In present study, we focus on local regularity of fractal function and low-frequency trend removal preprocessing is not necessary as multi-scale wavelet analysis is a natural tool for the task. Hölder exponent or the related fractal dimension is appropriate for describing local regularity or roughness of a function. Meanwhile, the power-law exponent of the correlation function is used to describe long-range dependence or memory characteristics of a process, which can be linked to \( H \)-exponent once a concrete model is identified.

A unique example of a \( H \)-self-similar Gaussian process with stationary increments is the FBM defined as\(^{13}\)

\[
B_H(t) = \frac{1}{\Gamma(H + 1/2)} \int_{-\infty}^{\infty} \left[ |t - s|^{H-1/2} - |s|^{H-1/2} \right] dB(s)
\]

\[
+ \int_{0}^{t} |t - s|^{H-1/2} dB(s),
\]

with \( H \)-self-similar or Hurst exponent \( 0 < H < 1 \) and \( B(t) \) denotes the Gaussian Brownian motion. For a standard FBM, \( B_H(t) \) has zero mean with covariance

\[
C_{B_H}(t_1, t_2) = \frac{1}{2} |t_1|^{2H} + |t_2|^{2H} - |t_1 - t_2|^{2H}.
\]

The variance of FBM follows power-law relation, \( E[B_H(t)^2] \sim t^{2H} \). The \( H \)-self-similarity of \( B_H(t) \) and the stationarity of its increments ensures that the generalized power spectrum of FBM exhibits power-law type \( S(\omega) \sim \omega^{-\beta} \), where \( 0 < \beta < 3 \). This is another manifestation of fractal characteristic in the spectral representation. We also remarked that the power spectral density of fractional Ornstein-Uhlenbeck process given in Muniandy et al.\(^5\) can be shown to converge to the power-law type power spectral density similar to that of fractional Brownian motion at high frequency limit, where the fractional index \( \beta \) is linked to \( H \) by \( \beta = 2H + 1 \). The discrete version of the increment processes of the FBM is the fractional Gaussian noise (FGN) denoted by \( Y_H(t) \equiv B_H(t + 1) - B_H(t) \). FGN is second-order self-similar due to its correlation function that follows power law form.\(^{14}\) The correlation function of FGN provides a natural interpretation of the long- or short-range dependence in stationary time series, parameterized by Hurst exponent \( H \).

Both FBM and its derivative FGN are widely used for characterizing local regularity (or roughness) and memory effect in fractal fluctuations in time series. As mentioned earlier, wavelet analysis has been proven to perform better in highlighting fractal scaling characteristics in non-stationary time series. Since we are using FBM as the model for fractal time series, we shall adopt the \( H \)-parameter to mean the Hurst exponent in the context of local regularity exponent and it equally applies to the long-memory parameter.\(^{15}\)

Besides the applications of FBM for geometrical analysis of fluctuation, it is also widely used to describe dynamical properties of anomalous diffusion processes. Diffusion process is characterized by the mean square displacement (MSD) of the Brownian particle defined as \( \langle x^2(t) \rangle = \langle (x(t) - x(0))^2 \rangle \), where \( x(t) \) is the position of the \( i \)th particle at time \( t \) and the angular bracket denotes ensemble average. In a normal or so called Fickian diffusion process, the MSD scales linearly with time, \( \langle x^2(t) \rangle \sim t \). On the other hand, anomalous diffusion\(^{16}\) refers to nonlinear behaviour of MSD, namely \( \langle x^2(t) \rangle \sim t^\alpha \) with scaling exponent \( \alpha \neq 1 \). The system is said to undergo slow or sub-diffusion when \( 0 < \alpha < 1 \), and fast or super-diffusion when \( 1 < \alpha < 2 \). Normal diffusion corresponds to \( \alpha = 1 \) and \( \alpha = 2 \) characterizes ballistic transport. Referring to the variance of FBM described above, one can identify \( \alpha = 2H_{\text{MSD}} \). It is important to note that the Hurst exponent for the DLS fluctuation analysis (denoted as \( H_{\text{DLS}} \)) is not the same as the “Hurst exponent” determined indirectly from the scaling behaviour of the MSD (denoted as \( H_{\text{MSD}} \)) as the
former refers to geometric property (i.e., regularity) of the DLS time series while the latter is linked to particle transport dynamics based on particle trajectories.

A. Wavelet analysis

The Hurst exponent of a time series can be estimated using many different techniques, for example rescaled-range (R/S) analysis,\textsuperscript{17} detrended fluctuation analysis (DFA),\textsuperscript{18} power spectral density approach,\textsuperscript{13,15} and wavelet based estimators.\textsuperscript{19} Wavelet based techniques are relatively more robust and particularly suited for nonstationary and chaotic signals. It can be used to detect local regularity and self-similar scaling feature in time series because of the fact that the wavelet basis functions possess scaling property.\textsuperscript{20}

Wavelet transform decomposes signal into a set of scaled and shifted versions of the mother wavelet.\textsuperscript{20} Continuous wavelet transform of a function $X(t)$ at scale factor $a$ and time-translation factor $b$ is expressed by

$$W(a, b) = \frac{1}{\sqrt{a}} \int \psi\left(\frac{t - b}{a}\right) X(t) dt,$$  

(4)

where $\psi(t)$ is the “mother” wavelet. The scale parameter $a$ is inversely proportional to frequency, whereby small scale refers to high frequency and vice versa. Thus, a wavelet transform is often used for time-scale representation of non-stationary signals analogous to the time-frequency representation, for example using short-time Fourier transform.

In discrete wavelet representation,\textsuperscript{19} a sequence $X(t)$ of length $N = 2^J$ is decomposed in terms of orthogonal bases

$$\varphi_{j,k}(t) = 2^{j/2} \varphi(2^j t - k), \hspace{1cm} \psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k)$$  

(5)

constructed from dilation and shift operation of the “father” scaling function $\varphi(t)$ and mother wavelet $\psi(t)$. The scale and shift parameters $a$ and $b$ are discretized such that $a = 2^j$ and $b = k2^j$, respectively, where $j$ and $k$ are positive integers or indices. Thus, one can write the multi-scale decomposition of $X(t)$ as

$$X(t) = \sum_{k=0}^{2^J-1} U_{j,k} \varphi_{j,k}(t) + \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} W_{j,k} \psi_{j,k}(t),$$  

(6)

where the scaling coarse coefficient $U_{j,k}$ and the wavelet detail coefficient $W_{j,k}$ at particular scale index $j$ and position index $k$ are given by

$$U_{j,k} \equiv \int X(t) \varphi_{j,k}(t) dt$$  

(7)

and

$$W_{j,k} \equiv \int X(t) \psi_{j,k}(t) dt,$$  

(8)

respectively, and the coarsest scale is $J_\text{c}$. To obtain the scalogram, the details coefficient at each scale index $j$ is squared and averaged across index $k$ as given by\textsuperscript{19}

$$S(j) = \frac{1}{N_j} \sum_{k=0}^{2^j-1} |W_{j,k}|^2,$$  

(9)

where $N_j$ is the length of the detail coefficient vector used to normalize the generalized “wavelet energy.” A scalogram is analogous to time-averaged Fourier based spectrogram.

If $X(t)$ is a fractal process, then $S(j)$ obeys power-law relation

$$S(j) \sim (2^j)^\gamma,$$  

(10)

and for a homogeneous fractal process (i.e., mono-fractal), $\gamma = 2H + 1$. By taking $\log_{2}$ both sides of Eq. (10), one gets $\log_2 S(j) = \gamma j + C$, where $C$ is a constant. The Hurst exponent can be determined from the slope $\gamma$ of the linear scaling region in $\log S(j)$ versus $j$ plot. The Hurst exponent determined from the DLS time series (denoted as $H_{DLS}$) is differentiated from the Hurst exponent determined from the MSD (denoted as $H_{MSD}$) as mentioned earlier. The wavelet scalogram or rather the detail coefficient $W_{j,k}$ is essentially a mathematical “scale microscope” that characterizes local regularity or roughness of a function. Here, the Hurst exponent is treated as identical to Hölder exponent and by relating to fractal dimension, $D = 2 - H_{DLS}$, both $H_{DLS}$ and $D$ can be used to characterize signal complexity. In the following section, an alternative approach based on concept of generalized entropy for quantifying signal complexity is introduced, which will be used to support our claim on the usefulness of fractal analysis vis-à-vis Hurst exponent for analyzing light scattering intensity fluctuation in dusty plasma.

B. Approximate entropy

One of the techniques to quantify irregularity of a given data that includes deterministic chaotic and stochastic process without any model assumption is the ApEn.\textsuperscript{21,22} ApEn quantifies a continuum that ranges from purely periodic signal to purely random white noise. Given a discrete signal with length $N$: $u(n) = \{u(1), u(2), \ldots, u(N)\}$, divided into subsection of length $m$: $X(i) = \{u(i), u(i+1), \ldots, u(i+m-1)\}$, where $i = 1, \ldots, N - m + 1$. Define the distance between subsection $X(i)$ and $X(j)$ as the maximum difference in their respective scalar components,

$$d[X(i), X(j)] = \max_{k=0,\ldots,m-1} |u(i+k) - u(j+k)|.$$  

(11)

All sequence of $X(i)$ is used to construct $C_m^r(r)$ (number of $j \leq N - m + 1$ such that $d[X(i), X(j)] \leq r)/(N - m + 1)$. The $C_m^r(r)$ values measure within a tolerance $r$ the regularity or frequency, of patterns similar to a given pattern of window length $m$. Next, define

$$\phi_m^r(r) = \frac{1}{N - m + 1} \sum_{i=1}^{N-m+1} \ln\left(\frac{C_m^r(i)}{C_m^r(1)}\right),$$  

(12)

where $\ln$ is the natural logarithm. The approximate entropy is then defined as

$$-\text{ApEn}(m, r) = \lim_{N \to \infty} [\phi_m^r(r) - \phi_{m+1}^r(r)].$$  

(13)
Alternatively, negative of ApEn can be expressed as

\[-\text{ApEn}(m, r) = \frac{1}{N-m} \sum_{i=1}^{N-m} \ln \left( \frac{C_{r+1}^m(i)}{C_r^m(i)} \right), \quad (14)\]

which is the average over \(i\) of \(\ln\) [conditional probability that \(|u(j + m) - u(i + m)| \leq r\), given that \(|u(j + k) - u(i + k)| \leq r\) for \(k = 0, 1, \ldots, m - 1\)]. ApEn measures the (logarithmic) likelihood that runs of patterns that are close for \(m\) observations remain close on the next incremental comparison. Greater likelihood of remaining close in the next increment (more regular) will produces smaller ApEn values, and conversely.\(^{21,22}\)

ApEn was constructed along thematically similar lines to the Kolmogorov-Sinai (K-S) entropy which is developed by Kolmogorov\(^{23}\) and expanded by Sinai.\(^{24}\) K-S entropy classifies deterministic dynamical systems by rates of information generation. However, it was pointed out the deficiency of K-S entropy that it is badly compromised by noise, requires vast amount of input data to achieve convergence and usually gives infinite value for stochastic processes.\(^{22,25-27}\) To overcome that, Pincus\(^{22}\) formulated a widely applicable, statistically valid formula, ApEn, which is capable of distinguishing correlated stochastic processes and composite deterministic or stochastic models. ApEn is related to conditional entropy\(^ {28}\) in information theory by

\[\text{ApEn}(m, r) = H(X_m + 1||X_1, \ldots, X_m), \quad (15)\]

where \(H(X_i) = -\sum p_i \log p_i\) is the Shannon entropy, while the rate of entropy, \(\lim_{n \to \infty} H(X_n || X_1, \ldots, X_{n-1})\), is the discrete state analog of the KS entropy.\(^ {22,23}\)

In practice, estimation of ApEn requires \(a\ priori\) determination of two unknown parameters namely the length of the sequences to be compared, \(m\) and the tolerance threshold for accepting similar patterns between two segments, \(r\). ApEn is more sensitive to \(r\) than \(m\), which can lead to inaccurate estimation.\(^ {29}\) Possible solutions include a modified algorithm of ApEn called sample entropy and the use of maximum of ApEn as the regularity indicator.\(^ {22}\) In this work, we adopt the notion of maximum of ApEn as the indicator of regularity. Before applying the technique to the DLS fluctuation time series, the algorithm by Pincus\(^ {21,22}\) is tested on Gaussian white noise and fractional Brownian motion with \(H = 0.25\), 0.5, and 0.75 synthesized using Wood and Chan algorithm.\(^ {30}\) Obviously, the white noise is a purely random time series without any temporal correlation. FBM with \(H = 0.25\) produces anti-persistent time series and \(H = 0.75\) is persistent time series. ApEn for these times series as a function of \(\log r\) is shown in Fig. 1. As expected, the white noise has the highest entropy (highly irregular) and the FBM time series show decreasing maximum ApEn values as the \(H\) value increases (or the graph becomes smoother).

**III. EXPERIMENTAL SETUP**

The dusty plasma system is created using a 13.56 MHz radio-frequency capacitively coupled discharge in argon gas and using titanium oxide as dust particles. The cylindrical chamber is designed with a diameter of 40 cm and a height of 20 cm and has six window ports fitted to the chamber for optical access and electrical measurement ports. As shown schematically in Fig. 2, a pair of parallel plates with diameter 10 cm is placed inside the chamber and the distance between plates is 8 cm. The upper electrode is grounded, and the lower electrode is connected to RF power through the RF matching unit. The particle cloud is confined using a cylindrical depression on the lower electrode.

The chamber is pumped down until it reaches based pressure \(10^{-2}\) mbar. The argon gas flows continuously to the chamber and the pressure is set by adjusting the valve. After reaching a certain stable pressure, the electrode is energized with 100 W radiofrequency power. Three different gas pressures, namely 0.1 mbar, 0.3 mbar and 0.5 mbar, are chosen to represent different conditions for disordered states of the dusty plasma. Small amount of polydisperse titanium dioxide dust particles are sprinkled using electromechanical dispenser through a small hole on the upper electrode. The floating charged dust particles is then illuminated using a He-Ne laser beam with wavelength \(\lambda = 636.5\) nm. The intensity of the scattered light at an angle of \(60^\circ\) from the primary beam is measured by a photodetector.

**FIG. 1.** Approximate entropies for test time series of different degrees of irregularities (randomness).

**FIG. 2.** Experimental setup for the laser light scattering measurement in dusty plasma system.
beam axis is measured. Due to the limitation of the chamber’s access window, the scattered beam can only be measured around this fixed angle. The scattered light is then filtered from ambient light using a bandpass filter and a polarizer before being focused onto a photodetector. The generated electric signal is recorded using Newport 1936-C single channel optical power meter at sampling rate of 10 kHz.

Light intensity fluctuation in the absence of dust (dust off) and in the presence of dust at gas pressure of 0.1, 0.3, and 0.5 mbar is shown in Fig. 3. Power line interference is removed by applying band-stop filter on the intensity time series. By subtracting the intensity time series with its mean value, one obtains mean-zero intensity fluctuation and then normalized by dividing it with its standard deviation. Intensity data preprocessing procedures are done using signal processing toolbox in MATLAB®.

For visualization purpose, the dust particle motion is recorded using an EoSens CL high-speed-CMOS video camera running at a rate of 100 frames per second and equipped with a macro lens. Particle tracking algorithm implemented on the raw video data to obtain particles trajectories. The particle tracking algorithm used in this work is developed by Blair and Dufresne. The algorithm search for the most probable particle location in the consecutive image frame is based on the closest inter-particle distance (ipd).

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FIG. 3. Scattered light intensity fluctuation time series in dusty plasma at three different pressures and background noise when dust is absent.

FIG. 4. (a)–(c) Screenshot of dusty plasma configurations and (d)–(f) particle trajectories for a duration of 1 s, captured at neutral argon gas pressures 0.1, 0.3, and 0.5 mbar, respectively.
At the end, a time series of particles’ trajectories in two dimension is obtained. Fig. 4 shows the screenshot of spatial configurations and corresponded particles trajectories at three different pressures. Once the particle trajectory is determined, the speed of each particle namely \( v = (v_x^2 + v_y^2)^{1/2} \) is calculated. The distribution of particle speed is asymmetrical and one can use the notion of the most probable (mode) speed to show higher kinetic energy at lower pressure. From the histogram shown in Fig. 5, the approximate values of most probable speed of the particles at pressure 0.1 mbar, 0.3 mbar and 0.5 mbar are estimated to be 12.5 mm/s, 2.5 mm/s and 1 mm/s, respectively.

A standard technique to characterize spatial ordering is to use the particle-particle pair correlation function, \( g(r) = n(r)/(\text{total number of particles} \times \text{annular area enclosed}) \), where \( n(r) \) is the number of particles located inside a concentric annular ring of radius \( r \) from a center particle. The pair correlation function \( g(r) \) measures the translational order of the particle ensemble where series of peaks in the \( g(r) \) plot indicate presence of periodicity. In the \( g(r) \) plot, the radius is normalized by ipd. The corresponding pair-correlation functions for these particle configurations are shown in Fig. 6. The distinctive peak at the first nearest neighbour distance in Fig. 6(c) suggests the presence of short-range order as expected in the disordered liquid state. While at pressures 0.1 mbar and 0.3 mbar, pair correlations show no distinctive peak, which indicates the lack of periodicity in the particle arrangement, typical of disorder gas state. It is difficult to describe the degree of disorder in particles configuration at low pressures using the pair-correlation because the irregular correlation plots of two different kinetic states are very similar. Even in the presence of slightly ordered structures as seen at gas pressure 0.5 mbar, spatial heterogeneity in the particle distribution or inter-particle distances (analogous to polycrystalline domains) also produces highly irregular correlation plot. Since structural complexity is very apparent in the DLS data as well as particle’s configurations, fractal analysis is reckoned to be useful for analyzing degree of irregularity in the time series.

IV. RESULTS AND DISCUSSION

A. Dynamic light scattering analysis

Wavelet multi-resolution analysis is well-suited for characterizing fractal fluctuation phenomena in the presence of nonstationarity or trend. The empirical time series with dust present in the plasma shown in Fig. 3 contain high-frequency background noise and possibly fractal scaling fluctuations originating from dust particles motion due to diffusion or ballistic mechanisms. Light intensity time series without dust is...
analyzed using Daubechies mother wavelet with least regularity (db1 wavelet) because the characteristic resembles that of the white noise. For the rest of the time series, db4 mother wavelet is employed.

The advantage of wavelet approach is apparent with the natural scale separation between background white noise-like time series and scaling time series as seen in Fig. 7. The flat spectrum at small-scale (or high frequency) regime (scale index $j \sim 1, \ldots, 7$) represents the background noise that is present in all the time series.

In the presence of dust, the power-law behavior in log-log plot of the wavelet scalogram is quite obvious (indicating mono-fractal behavior), particularly in the scaling regime with index $j = 8$ to $j = 13$ (corresponding to time scales 0.026 s to 1.6 s). The slopes of the linear scaling regimes are determined using linear least square fitting as shown in the inset of Fig. 7. By assuming the time series can be modeled as FBM, the Hurst exponent is obtained from $H_{DLS} + 1$ relationship. The fractal scaling Hurst exponents are calculated for the three different pressures, using a set of four long segments of the time series at each pressure condition. The values of Hurst exponents determined using wavelet scalogram approach are $0.89 \pm 0.13$, $0.81 \pm 0.15$ and $0.68 \pm 0.22$ for pressure 0.1 mbar, 0.3 mbar and 0.5 mbar, respectively. The background noise is found to be non-scaling time series with relatively flat spectrum as one would expect of the non-scaling white noise. The background noise has been shown to satisfy the Gaussian statistics and temporal uncorrelated.

The degree of irregularity in the DLS time series is also verified using an alternative (non-fractal) technique namely the approximate entropy described above. In estimating the ApEn, the value of $m$ is chosen arbitrary ($m = 2$) and the results are shown in Fig. 8. The value of the ApEn maxima can be used to characterize degree of irregularity or randomness in the time series with higher value indicates greater irregularity. For example, crude estimation of the ApEn maxima for the DLS data shows that the background noise has the highest value (1.75), followed by the DLS time series at 0.5 mbar (1.5), 0.3 mbar (1.35) and 0.1 mbar (1.30).

**B. Mean squared displacement analysis**

From the particle trajectories, the mean squared displacement of particles is determined and the scaling exponents $\alpha$ are estimated from linear scaling regime of the log-log MSD(t) versus log10 t (see Fig. 9). As mentioned in Sec. II, the MSD scaling exponent $\alpha$ is related to $H_{MSD}$ via the relation $\alpha = 2H$. From the MSD scaling behavior, the most noticeable feature is the bi-scaling behavior that present in all pressures. The transition time, $t_c$ of the two scaling behaviors is rather similar: 0.14 s for 0.1 mbar, 0.13 s for 0.3 mbar, and 0.1 s for 0.5 mbar. The estimated values of Hurst exponents from the MSD plots are summarized in Table I. One can deduce that at disorder gas state (0.1 and 0.3 mbar), particles move ballistically ($H_{MSD} \sim 1$) at short time and able to gain...
large speed and travel greater distance. It is then slowed down to subdiffusive behavior with $H_{\text{MSD}} = 0.33$ (0.1 mbar) and $H_{\text{MSD}} = 0.37$ (0.3 mbar) after time, $t_c$, due to collisions with other dust particles. On the contrary, disorder liquid state at 0.5 mbar moves relatively slower at $t < t_c$ ($H_{\text{MSD}} = 0.78$) as compared to disorder gas states. This might be due to the greater kinetic damping at higher pressure where the dust particles are slowed down as they become strongly coupled due to the greater potential energy compared to thermal energy. At longer time, $t > t_c$ particles follow normal diffusion process ($H_{\text{MSD}} > 0.5$). At much longer time, $t \gg t_c$ superdiffusion is observed ($H_{\text{MSD}} > 0.5$) which could be due to the collective particle-particle interaction (Fig. 9).32,33

C. Connection between Hurst exponents from DLS fractal analysis and MSD analysis

It is noticed that as the dusty plasma state change from gaseous to disorder liquid, particle dynamic has become faster as indicated by the increasing value in $H_{\text{MSD}}$. The change in $H_{\text{MSD}}$ is corresponded to decreasing $H_{\text{DLS}}$ values, which means that the DLS signal is more irregular in the disorder liquid state. This can be explained through the following. The instantaneous intensity of scattered light is proportional to the phase factor $\Phi(t) \sim \exp[j \vec{r}(t) \cdot \vec{q}]$, where $\vec{r}(t)$ is the position of $i$th illuminated particle and $\vec{q}$ is the scattering wavevector.3 Fluxuation in the intensity is due to temporal variation in the interference condition as a result of particle movements. Following that, the variance of phase fluctuation is proportional to the variance of particles’ position, namely $\langle |\Phi(t)|^2 \rangle \sim \exp[q^2 \langle r_i(t)^2 \rangle].$ Thus, we expect that fast diffusion would result in larger variance in intensity/phase fluctuation and hence more irregular DLS time series.

V. CONCLUSIONS

In this work, we have used DLS technique to study complex changes in the particle configurations in homogeneous dusty plasma. The Hurst exponent $H_{\text{DLS}}$ determined from DLS time series using wavelet analysis described the irregularity of scattered light intensity fluctuation, while the Hurst exponent $H_{\text{MSD}}$ calculated from mean-square displacement of an ensemble of dust particles described the individual particle dynamics. It is noted that the standard DLS approach often resorts to correlation analysis of the fluctuation (assuming the time series is stationary). However, using the wavelet based fractal analysis, together with approximate entropy, we have shown that useful information on the structural phases of dust particles can be obtained by treating the time series as non-stationary fractal process. We have shown that the types of particle transport influence the irregularity of fluctuation characteristic in DLS signal. The MSD analysis also showed interesting bi-scaling behavior that can be associated with transition from ballistic-like to sub-diffusive behavior for disordered dusty plasma gaseous state. Meanwhile in the disorder liquid state at higher pressure, the particles appear to undergo even more complicated scaling. This will be addressed further in future study.

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