Neutrino Oscillations in the Sun and in Cosmology

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1. Stopping Power of Matter on Neutrinos

• Consider Dirac neutrino with helicity = -1.
• The scattering of $\nu_e$ with electrons within the framework of the electroweak theory.
• Consider the effects of two-neutrino oscillation of the type $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ i.e.

$$\nu_e + e^- \rightarrow \nu_i + e^-$$ where $i = \mu, \tau$
Preamble

• We investigate the energy losses of neutrinos.

1. Neutrinos from the sun.
2. Neutrinos in Big-Bang cosmology.
1. Stopping Power of Matter on Neutrinos

- For example, the well known level-crossing of $\nu_e \rightarrow \nu_\mu$

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
$$
1. Stopping Power of Matter on Neutrinos

• Graphically [Bethe],

![Graphical representation of neutrino interactions](image)
1. Stopping Power of Matter on Neutrinos

- The stopping power of matter
[Sulaksono and Simanjuntak]

\[
\frac{dE_{\nu}}{dR} = -m_e N_e \frac{G_F^2}{2\pi} \left[ P(\nu_e \rightarrow \nu_e) \left[ A_e \left( \frac{Q_{m_e}^2 - \eta^2}{2} \right) + B_e \left( \frac{Q_{m_e}^3 - \eta^3}{3} \right) + C_e \left( \frac{Q_{m_e}^4 - \eta^4}{4} \right) \right] \\
+ P(\nu_e \rightarrow \nu_i) \left[ A_i \left( \frac{Q_{m_i}^2 - \eta^2}{2} \right) + B_i \left( \frac{Q_{m_i}^3 - \eta^3}{3} \right) + C_i \left( \frac{Q_{m_i}^4 - \eta^4}{4} \right) \right] \right]
\]
1. Stopping Power of Matter on Neutrinos

- with

\[
A_i = \frac{\left(E_{\nu}^2 + E_{\nu} \left(E_{\nu}^2 - M_{\nu_i}^2 \right)^{1/2}\right) \left[g_{\nu} + g_{A} \right]^2 + \left(g_{\nu} - g_{A} \right)^2 - M_{\nu_i}^2 \left(g_{\nu}^2 - g_{A}^2 \right)}{E_{\nu}^2 - M_{\nu_i}^2}
\]

\[
B_i = - E_{\nu} \left(E_{\nu} + m_e \right) \left(E_{\nu} \left(g_{\nu} - g_{A} \right)^2 + m_e \left(g_{\nu}^2 - g_{A}^2 \right)\right) - \left(g_{\nu} - g_{A} \right)^2 \left(m_e - E_{\nu} \right) - m_e \left(g_{\nu}^2 - g_{A}^2 \right)
\]

\[
\frac{m_e \left(E_{\nu}^2 - M_{\nu_i}^2 \right)^{3/2}}{m_e \left(E_{\nu}^2 - M_{\nu_i}^2 \right)^{1/2}}
\]

\[
C_i = \left(g_{\nu} - g_{A} \right)^2 \left(m_e \left(E_{\nu}^2 - M_{\nu_i}^2 \right)^{1/2} + \left(E_{\nu} m_e + M_{\nu_i}^2 \right)\right)
\]

\[
\frac{m_e \left(E_{\nu}^2 - M_{\nu_i}^2 \right)^{3/2}}{m_e \left(E_{\nu}^2 - M_{\nu_i}^2 \right)^{3/2}}
\]
1. Stopping Power of Matter on Neutrinos

- In the $\nu_e \rightarrow \nu_\tau$ oscillation, $g_V$ and $g_A$ are replaced by $g_V - 1$ and $g_A - 1$ respectively.

- $Q_m = \frac{2m_e \left( E_{\nu_i}^2 - m_{\nu_i}^2 \right)}{m_{\nu_i}^2 + m_e^2 + 2m_e E_{\nu_i}}$ is the maximum energy transferred to the electron.

- $\eta \approx \frac{2N_e e^2}{m_e}$ is the electron plasmon energy.

- $N_e = N_e(R)$ is the electron number density.

- The energy spectrum of a neutrino will be altered.
2. Energy Losses of Solar Neutrinos

- We incorporate the conversion/survival probability \( P(\nu_e \rightarrow \nu_e) \) to the energy losses of solar neutrinos in their propagation from center of the sun.

- \( P(\nu_e \rightarrow \nu_e) \) is described by the adiabatic survival probability

\[
P(\nu_e \rightarrow \nu_e) \approx \sin^2 \theta \quad [\text{Parke}]
\]

where \( \theta \) is the mixing angle.
2. Energy Losses of Solar Neutrinos

- If we borrow the mixing angle from the quark sector, $P(\nu_e \rightarrow \nu_e) = 0.2315$.
- The electron density distribution, $N_e$ is taken from a standard solar model [Abu Kassim].
2. Energy Losses of Solar Neutrinos

A more general form of the probability function takes into account the non-adiabatic correction probability [Parke].

\[ P\left(\nu_e \rightarrow \nu_e\right) \approx \sin^2 \theta + P_c \cos 2\theta \]

\[ P_c = e^{-\frac{\gamma_A}{2}} \]

\[ \gamma_A = \frac{1.66 \times 10^8 \Delta m^2 \sin^2 2\theta}{\cos 2\theta E_{\nu_e}} \]
2. Energy Losses of Solar Neutrinos

• Final energy of the neutrino $E^f_\nu$ emerging from the surface of the sun

$$E^f_\nu = \int_0^{R_{\text{sun}}} \frac{dE_\nu}{dR} dR$$

• Total energy loss, $\Delta E_\nu = E^f_\nu - E^i_\nu$, $E^i_\nu$ is the initial neutrino energy.
2. Energy Losses of Solar Neutrinos

![Diagram showing the energy loss of solar neutrinos as a function of neutrino energy.](image)
2. Energy Losses of Solar Neutrinos
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- For $\Delta E_\nu \approx 1$ MeV, $\frac{N_e}{N_A} \sim 10^7$
- Center of the sun, $\frac{N_e}{N_A} \sim 10^2$
- Will not solve solar neutrino deficiency
- $E_\nu(^8B): 0 \rightarrow 14$ MeV
- $^{37}Cl$ detector threshold = 0.814 MeV
- Super Kamiokande = 5.0 MeV
3. Neutrino oscillations in the Big Bang Model

- The cosmic microwave background radiation (CBR) provides evidence that the Universe must have gone through a hot dense phase at earlier times.
- If the cosmic background radiation is the remnant of the radiation from the early Universe, its temperature will provide us with important information about the Universe.
3. Neutrino oscillations in the Big Bang Model

- [Rolfs and W. S. Rodney].
3. Neutrino oscillations in the Big Bang Model

- Reaction rate of neutrinos is larger than the expansion rate of the universe.
- In terms of a temperature variable $T$

$$\Delta E_\nu = \int_{T^i}^{T^f} \frac{dE_\nu}{dT} dT$$

- We integrate from $T^i = 10^{12}$ K to the decoupling temperature of $\nu_e$ with matter $T^f \approx 2.659 \times 10^{10}$ K.
3. Neutrino oscillations in the Big Bang Model

\[ P(\nu_e \rightarrow \nu_e) = 1 \]
3. Neutrino oscillations in the Big Bang Model

• During the decoupling of neutrinos-matter one can imagine that the energy loss for the neutrinos as a gain in the energy of the electrons. Then from the annihilation of the electrons into photons ($\gamma$),

$$e^- + e^+ \rightarrow \gamma + \gamma$$

• We have that the ‘extra’ energy gain by the electrons will be transferred to the photons.

$$\Delta E_v \rightarrow \Delta T_v$$
3. Neutrino oscillations in the Big Bang Model

\[ \Delta \left( \frac{T_{\gamma}}{T_{\gamma}^i} \right) \]

\[ T (K) \]

\[ P (\nu_e \rightarrow \nu_e) = 1 \]
3. Neutrino oscillations in the Big Bang Model
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\[ \Delta \left( \frac{T_{\gamma}}{T_{\gamma_i}} \right) \]

\[ P \left( \nu_e \rightarrow \nu_e \right) \]

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### Table 2: Tabulation of \( \Delta \left( \frac{T_T}{T_T^i} \right) \) at \( T_{DC^e} \)

<table>
<thead>
<tr>
<th>( P(\nu_e \rightarrow \nu_e) )</th>
<th>( \nu_e \rightarrow \nu_\mu )</th>
<th>( \nu_e \rightarrow \nu_\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>4.38E-7</td>
<td>2.04E-8</td>
</tr>
<tr>
<td>0.2</td>
<td>9.36E-7</td>
<td>6.02E-7</td>
</tr>
<tr>
<td>0.4</td>
<td>1.43E-6</td>
<td>1.18E-6</td>
</tr>
<tr>
<td>0.6</td>
<td>1.93E-6</td>
<td>1.77E-6</td>
</tr>
<tr>
<td>0.8</td>
<td>2.43E-6</td>
<td>2.35E-6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.93E-6</td>
<td>2.93E-6</td>
</tr>
<tr>
<td>Average</td>
<td>1.68E-6</td>
<td>1.47E-6</td>
</tr>
</tbody>
</table>
3. Neutrino oscillations in the Big Bang Model

- **Average of** \( \Delta \left( \frac{T_{\gamma}}{T_{\gamma}} \right) \sim 10^{-6} \).

- **Comparable to the present anisotropy of the CBR which is** \( 10^{-5} \)?
References

Thank You