Two-sided Bayesian and frequentist tolerance intervals: general asymptotic results with applications

Dharini Pathmanathan\textsuperscript{a}, Rahul Mukerjee\textsuperscript{b}\textsuperscript{*} and S.H. Ong\textsuperscript{a}

\textsuperscript{a}Institute of Mathematical Sciences, University of Malaya, 50603 Kuala Lumpur, Malaysia;
\textsuperscript{b} Indian Institute of Management Calcutta, Joka, Diamond Harbour Road, Kolkata 700 104, India

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It is well known that the construction of two-sided tolerance intervals is far more challenging than that of their one-sided counterparts. In a general framework of parametric models, we derive asymptotic results leading to explicit formulae for two-sided Bayesian and frequentist tolerance intervals. In the process, probability matching priors for such intervals are characterized and their role in finding frequentist tolerance intervals via a Bayesian route is indicated. Furthermore, in situations where matching priors are hard to obtain, we develop purely frequentist tolerance intervals as well. The findings are applied to real data. Simulation studies are seen to lend support to the asymptotic results in finite samples.

Keywords: higher order; Jeffreys’ prior; probability matching prior; shrinkage argument

1. Introduction

Statistical tolerance intervals and their use, notably in quality management with industrial, clinical and environmental applications, have a long history; see [12] for an excellent and up-to-date survey, covering both theory and applications, and further references. Earlier accounts are available in [1,3]. A useful R package for obtaining tolerance intervals was recently given in [4]. A $\beta$-content level-$\gamma$ tolerance interval, based on sample data, aims at containing at least a proportion $\beta$ of the underlying population with posterior credibility level or frequentist confidence level $\gamma$, depending on whether the set-up is Bayesian or frequentist, respectively. A tolerance interval can be one- or two-sided, and the present article is focused on the latter.

To motivate the ideas, consider the following data, originally from [6] and quoted later by many others, representing the number of million revolutions before failure for each of 23 ball bearings:

\begin{tabular}{cccccccc}
  17.88 & 28.92 & 33.00 & 41.52 & 42.12 & 45.60 & 48.48 & 51.84 & 51.96 & 54.12 \\
  55.56 & 67.80 & 68.64 & 68.64 & 68.88 & 84.12 & 93.12 & 98.64 & 105.12 & 105.84 \\
  127.92 & 128.04 & 173.40 & & & & & & & \\
\end{tabular}

*Corresponding author. Email: rmuk0902@gmail.com

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