A COM-Poisson-type generalization of the negative binomial distribution

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\textbf{ABSTRACT}

This paper introduces a generalization of the negative binomial (NB) distribution in analogy with the COM-Poisson distribution. Many well-known distributions are particular and limiting distributions. The proposed distribution belongs to the modified power series, generalized hypergeometric and exponential families, and also arises as weighted NB and COM-Poisson distributions. Probability and moment recurrence formulae, and probabilistic and reliability properties have been derived. With the flexibility to model under-, equi- and over-dispersion, and its various interesting properties, this NB generalization will be a useful model for count data. An application to empirical modeling is illustrated with a real data set.

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\textbf{KEYWORDS}

COM-Poisson; Empirical modeling; Equi- and over-dispersion; Exponential families; Generalized hypergeometric; Increasing failure rate; Index of dispersion; Log-concavity; Modified power series; Reliability; Stochastic ordering; Under- Unimodality; Weighted distribution.

\textbf{MATHEMATICS SUBJECT CLASSIFICATION}

62E15; 62F03; 62N05

1. Introduction

The Poisson distribution is often used to model count frequency data, but the equality of its mean and variance (equi-dispersion) makes it too restrictive for many applications. This is because unequal mean and variance are usually encountered in observed data. Under-dispersion (variance less than the mean) occurred relatively less in applications. Examples of under-dispersed distributions are the Charlier series distribution and its generalizations (Sugita et al., 2011; Ong et al., 2012). Over-dispersion is more prevalent and this has led researchers to consider alternative approaches to generalize or extend the Poisson model. For example, in the method of mixtures (see Gupta and Ong, 2005), the Poisson parameter (mean) is allowed to vary as a random variable resulting in a mixed Poisson distribution, which has variance greater than the mean (over-dispersion). If this is the gamma random variable, then the negative binomial (NB) distribution (Greenwood and Yule, 1920) is obtained.

Models for count data that can cater for under-, equi- and over-dispersion have also attracted the attention of researchers. One such model is the COM-Poisson distribution (Conway and Maxwell, 1962; Shmueli et al., 2005), which has seen a recent revival of interest.
by enabling us to fit distributions that are not exactly NB or COM-Poisson or Bernoulli, but lie on a continuum (in the range of $\alpha$) between these distributions. The proposed distribution belongs to the modified power series and generalized hypergeometric families, and also arises as weighted NB and COM-Poisson distributions. Similar to the COM-Poisson and GPDs, this COM-Poisson-type NB distribution is also able to model under-, equi- and over-dispersion. It is shown that the COM-Poisson-type NB distribution has many interesting probabilistic properties and is a better empirical model than the COM-Poisson and GPDs.

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**References**


distributions in terms of log-likelihood, $y^2$ and AIC values. Based on the difference in AIC values given in Burnham and Anderson (2002, p. 70), there is considerable empirical support for the COM-NB when compared with the NB COM-Poisson, and generalized Poisson distributions.

Table 1 [here]

7. Conclusion

A generalization of the negative binomial distribution has been proposed in analogy with the COM-Poisson distribution. Many well-known distributions are particular cases of the COM-Poisson type negative binomial distribution while the binomial and COM-Poisson distributions are obtained as limiting distributions. The parameter $\alpha$ of the COM-Poisson type negative binomial distribution adds to its flexibility by enabling us to fit distributions that are not exactly negative binomial or COM-Poisson or Bernoulli, but lie on a continuum (in the range of $\alpha$) between these distributions. The proposed distribution belongs to the modified power series and generalized hypergeometric families, and also arises as weighted negative binomial and COM-Poisson distributions. Like the COM-Poisson and generalized Poisson distributions, the COM-Poisson type negative binomial distribution is also able to model under, equi- and over dispersion. It is shown that the COM-Poisson type negative binomial distribution has many interesting probabilistic properties and is a better empirical model than the COM-Poisson and generalized Poisson distributions.

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