Count Distribution for Generalized Weibull Duration with Applications

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An extension of the Poisson distribution is derived by considering a stochastic point process where the duration has a generalized Weibull distribution. This distribution is able to represent under, equal and over dispersion, a useful feature in data analysis. The computation of the probabilities and renewal function (expected number of renewals) are examined. Parameter estimation by the method of maximum likelihood is considered with applications of the count distribution to real frequency count data exhibiting under and over dispersion. It is shown that the generalized Weibull count distribution offers much better than the Weibull and gamma duration models.

Keywords: Birth and renewal processes, Inter arrival times, Gamma distribution, Poisson distribution, Renewal function, Over and under dispersion, Laplace transform, Parameter estimation, Goodness-of-fit, Likelihood ratio test.

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1. Introduction

The Poisson distribution is a popular model for the statistical analysis of count data in diverse settings. An important characteristic of this distribution is that the mean equals the variance (equi-dispersion). However, observed data tends to exhibit unequal mean and variance. Therefore, various approaches have been used to extend and modify the Poisson distribution. Examples of such approaches are by generalizing the Poisson process and birth process (Paddy, 1997), through the method of mixtures (Gupta and Ong, 2005) leading to over dispersed distributions (variance greater than the mean) like the negative binomial (NB) (Greenwood and Yule, 1920), Neyman Type A and generalizations of the NB (Gupta and Ong, 2004), and the weighting of the Poisson distribution (Castillo and Perea-Casany,