A model for a production–repair system under a time-varying demand process

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In this study, we consider a production system that satisfies a continuous time-varying demand for a finished product over a known and finite planning horizon by supplying either new products or repaired used products. Our model assumes that new products are fabricated from a single type of raw material procured from external suppliers, while used products are collected from the customers and then repaired to an ‘as new’ condition before being sold again. However, we assume that there is no further collection of used products during the period when they are being repaired or shipped. The problem is to determine a joint policy for raw materials procurement, new products production and used products repair such that the total relevant cost of the system is minimized. We propose a numerical solution procedure based on Microsoft Excel Solver’s nonlinear mathematical programming function. Then, the procedure is illustrated with a numerical example.

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1. Introduction

In many production systems, the producer procures raw materials from external suppliers and then processes them into finished products. When raw materials are used in production, their ordering quantities are dependent on the production batch size of the finished product. Therefore, it is often desirable to consider the batch size of the finished product and the ordering quantities of the associated raw materials together by treating production and procurement in a single model. Omar and Smith (2002) have developed such a model that operated under continuous supply to satisfy a linearly increasing time-varying demand process. They consider the lot for a lot case and propose two heuristic policies in addition to an optimal policy.

Besides fabricating the finished product from raw materials, it may be possible to reuse used products collected from the customers. Reuse of products and materials is not a new phenomenon. Metal scrap brokers, wastepaper recycling and deposit systems for soft drink bottles are all examples that have been around for a long time. In these cases, recovery of the used products is economically more attractive than disposal. Furthermore, in the recent past, the growth of environmental concerns has given ‘reuse’ increasing attention (Fleischmann et al., 1997). Today, environmental costs during the whole life cycle of industrial products already play an important role in the calculation of total production costs (Spengler et al., 1997).

In literature, extensive study has been devoted to reuse models under the assumption of constant demand. A deterministic EOQ-type model was first proposed by Schrady (1967). He assumes constant demand and return rates, and fixed lead times for external orders and internal recovery. The costs considered are fixed setup costs for orders and recovery and linear holding costs for serviceable and recoverable stocks. More recently, several authors have proposed extensions to Schrady’s model. Nahmias and Rivera (1979) generalize this model for the
case of finite recycling/repair rate. Mabini et al. (1992) consider stockout service level constraints and a multi-item system where items share the same repair facility. For these extended models, numerical solution methods are proposed. Koh et al. (2002) have proposed control policies for a joint EOQ and EPQ model where two cases are investigated: multiple order setups for a single recycling setup and vice versa. Teunter (2004) has presented a general model where stationary demand is satisfied by new products or recovered used products, which are assumed to be ‘as good as new’. He considers policies that alternate one production setup with multiple recovery setups, and one recovery setup with multiple production setups. Konstantaras and Papachristos (2008) have proposed an exact method that leads to the optimal policy of Teunter’s 2004 model.

In the above models, all returned items are reusable. Richter (1996a,b, 1997), Richter and Dobos (1999), and Dobos and Richter (1999, 2000) have investigated a waste disposal model, i.e., not all returned items are reusable, where the return rate is also a decision variable. They give the optimal number of remanufacturing and production batches depending on the return rate. Richter, in his 1997 paper, has examined the optimal inventory holding policy if the waste disposal (return) rate is a decision variable, with the result that the optimal policy has an extremal property: either reuse all items without disposal, or dispose of all items and produce new ones. Teunter (2001) has offered a model where not all items can be remanufactured, i.e., for a known return rate, the decision maker decides the reuse of returned items. He assumes that the inventory holding cost parameter for the manufactured items is higher than that for the remanufactured items because the remanufacturing costs are lower than the manufacturing costs. Dobos and Richter (2003), assuming that there is only one recycling batch and one production batch, have looked for the cost minimal marginal recycle and return rates. The result of this paper is that it is optimal either to produce, or to recycle all returned items. This result supports the optimal policy proposed in the paper by Richter (1997). Later on, Dobos and Richter (2004) have generalized their earlier work to the case of arbitrary batch numbers. Recently, Jaber and El Saadany (2008) proposed a production, remanufacture and waste disposal model that considers different demand rates for new and remanufactured items, and allowed lost sales situations to occur.

In this study, we consider the reuse of items after a simple repair process from an inventory management point of view. A general framework of this situation is depicted in Fig. 1. The producer satisfies a continuous time-varying demand process for a finished product over a known and finite planning horizon, and collects used products from the customers. For satisfying the demand, he has two options: either he fabricates new products from the raw materials that he procured externally, or he repairs the used products back to an ‘as new’ condition. In the next two sections, we present a model that treats the inventories of the raw materials, the finished product and the used products as interdependent parts of a single system. Our model operates with a predetermined inventory holding policy. In Section 4, we propose a numerical solution procedure based on Microsoft Excel Solver’s nonlinear mathematical programming function. The last section contains a numerical example and some concluding remarks.

2. Model description

The objective of our model is to jointly determine a procurement policy for the raw materials, a production policy for the finished product and a repair policy for the used products that satisfies a continuous time-varying demand process over a known and finite planning horizon, while minimizing the relevant fixed and variable costs. For simplicity, we assume that only one type of raw material (hereafter referred to as raw material 1) is required to fabricate the finished product. We consider multiple production and repair runs per batch period where the frequent production of small lots of new products and the frequent repair of small lots of used products will minimize the serviceable inventory holding cost for the producer. For each batch, the producer fabricates new products in $u$ production runs ($u = 1,2,\ldots$) and repairs used products in $v$ repair runs ($v = 1,2,\ldots$). However, he only orders one lot of raw material 1 for each production run. We assume that the production runs are conducted successively, followed by the repair runs. At the end of $u$ production runs, all units of raw material 1 will be fully processed, and at the end of $v$ repair runs, all the units of the used products will be fully repaired. Moreover, for simplicity, we assume that there is no further collection of used products during the period.

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when used products are repaired or shipped. Before going further, we state the assumptions and nomenclature used in this study. Fig. 2 shows the general pattern of the inventory movement during the \((i+1)\)-th batch when demand is increasing over time and \(u = v = 2\).

### 2.1. Assumptions

- A single product inventory system is considered over a known and finite planning horizon which is \(H\) units of time long.
- The demand rate at time \(t\) is given by the continuous function \(D(t)\).
- The production rate is a known constant \(P\) and \(P > D(t)\) for all \(t\).
- The repair rate is a known constant \(R\) and \(R > D(t)\) for all \(t\).
- The collection rate of the used products, \(r(t)\), is proportional to the demand rate, that is, \(r(t) = \phi D(t)\), \(0 \leq \phi \leq 1\).
- All used products are repaired to an ‘as new’ condition. There is no collection of used products during the repair period.
- Only one type of raw material (raw material 1) is required to fabricate the finished product.
- After an order is placed, raw material 1 is immediately replenished.
- There are \(u\) production runs \((u = 1, 2, \ldots)\) and \(v\) repair runs \((v = 1, 2, \ldots)\) per batch.
- Newly fabricated or repaired products are immediately shipped out.
- Shortages are not allowed during the planning horizon.
- The following cost parameters are considered:
  - \(c_p\), the setup cost of the production run (cost/setup);
  - \(c_r\), the setup cost of the repair run (cost/setup);
  - \(c_1\), the ordering cost of raw material 1 (cost/order);
  - \(h_f\), the inventory holding cost of the finished product (cost/unit/time);
  - \(h_u\), the inventory holding cost of the used products (cost/unit/time);
  - \(h_1\), the inventory holding cost of raw material 1 (cost/unit);
  - \(s_p\), the unit production cost of the finished product (cost/unit);
  - \(s_r\), unit repair cost of the used products (cost/unit).

### 2.2. Nomenclature

- \(q_1\), the quantity of raw material 1 required to produce one unit of the finished product.
- \(n\), the number of batches during the planning horizon \((n = 1, 2, \ldots)\).
- \(t_i\), the total elapsed time up to the start of the \((i+1)\)-th batch’s first production run \((i = 0, 1, \ldots, n-1)\), where \(t_0 = 0\) and \(t_n = H\).
- \(\beta_i\), the total elapsed time up to the start of the \((i+1)\)-th batch’s first repair run.
- \(p_{ij}\), the total elapsed time up to the start of the \(j\)th production run \((j = 1, 2, \ldots, u)\) during the \((i+1)\)-th batch, where \(p_{i1} = t_i\) and \(p_{in+1} = \beta_i\).
- \(\alpha_{ij}\), the total elapsed time up to the end of the \(j\)th production run \((j = 1, 2, \ldots, u)\) during the \((i+1)\)-th batch.
- \(r_{ij}\), the time total elapsed time up to the start of the \(j\)th repair run \((j = 1, 2, \ldots, v)\) during the \((i+1)\)-th batch.

![Fig. 2. Inventory movement in a batch for increasing demand when \(u = v = 2\).](image-url)
where \( r_{it} = \beta_i \) and \( r_{i+1t} = t_{j+1} \).

- \( \gamma_{ij} \), the total elapsed time up to the end of the \( j \)th repair run \( (j = 1, 2, \ldots, v) \) during the \((i+1)\)th batch.

### 3. Mathematical formulation

The total relevant cost of the system when there are \( n \) batches is given by

\[
TRC(n)_{u,v} = n[uc + c_1 + v_c] + H_P + H_R + H_1 + S_P + S_R,
\]

where \( H_P \), \( H_R \) and \( H_1 \) are the total inventory holding cost throughout the planning horizon, respectively, for the finished product, the used products and raw material 1, while \( S_P \) is the total production cost and \( S_R \) is the total repair cost.

First, we consider the finished product inventory during the \((i+1)\)th batch. Since finding the optimal production times, \( p_{ij} \) \( (j = 1, 2, \ldots, u) \), and the optimal repair times, \( r_{ij} \) \( (j = 1, 2, \ldots, v) \), within a batch period is an optimization problem itself, then for simplicity, we fix both the production and repair times to occur at constant intervals within the production and repair periods, respectively, that is

\[
p_{ij} = t_i + (j - 1)X_i, \quad j = 1, 2, \ldots, u.
\]

Using Eq. (2), it follows that

\[
x_{ij} = t_i + (j - 1)X_i + \frac{1}{R} \int_{t_i + (j-1)X_i}^{t_{i+1}X_i} D(t) \, dt, \quad j = 1, 2, \ldots, u.
\]

Then, we consider the \( j \)th production run \( (j = 1, 2, \ldots, u) \). Since the production during the uptime period must satisfy the demand during both the uptime and downtime periods, we have

\[
x_{ij} = p_{ij} + 1P \int_{p_{ij}}^{p_{i+1}X_i} D(t) \, dt, \quad j = 1, 2, \ldots, u.
\]

Similarly, we have

\[
x_{ij} = t_i + (j - 1)Y_i + \frac{1}{R} \int_{t_i + (j-1)Y_i}^{t_{i+1}Y_i} D(t) \, dt, \quad j = 1, 2, \ldots, v.
\]

Now, the inventory level of the finished product at time \( t \) during the \( j \)th production uptime period \( (j = 1, 2, \ldots, u) \), \( I_{1j}(t) \), is given by the remainder of the production from time \( p_{ij} \) to time \( t \) after meeting the demand during the same period, that is

\[
I_{1j}(t) = P(t - p_{ij}) - \int_{p_{ij}}^{t} D(w) \, dw, \quad p_{ij} \leq t \leq x_{ij}, \quad j = 1, 2, \ldots, u.
\]

While the inventory level at time \( t \) during the \( j \)th production downtime period, \( I_{2j}(t) \), is given by the demand yet to be met from time \( t \) to time \( p_{ij+1} \), that is

\[
I_{2j}(t) = \int_{t}^{p_{ij+1}} D(w) \, dw, \quad x_{ij} \leq t \leq p_{ij+1}, \quad j = 1, 2, \ldots, u.
\]

Similarly, the inventory level at time \( t \) during the \( j \)th repair uptime and downtime periods \( (j = 1, 2, \ldots, v) \), \( I_{3j}(t) \) and \( I_{4j}(t) \), are respectively, given by

\[
I_{3j}(t) = R(t - r_{ij}) - \int_{r_{ij}}^{t} D(w) \, dw, \quad r_{ij} \leq t \leq x_{ij}, \quad j = 1, 2, \ldots, v.
\]

Now, the total inventory holding cost of the finished product throughout the planning horizon, \( H_P \), is given by

\[
H_P = h_P \sum_{i=1}^{n-1} \sum_{j=1}^{u} \left[ \int_{p_{ij}}^{p_{i+1}X_i} I_{1j}(t) \, dt + \int_{p_{ij}}^{p_{i+1}X_i} I_{2j}(t) \, dt \right] + \sum_{j=1}^{v} \left[ \int_{r_{ij}}^{t} I_{3j}(t) \, dt + \int_{r_{ij}}^{t} I_{4j}(t) \, dt \right].
\]

Then, assuming that \( D(w) \) can be integrated with respect to \( w \) over the real line and letting \( J = \int_{t_1}^{t} D(w) \, dw = g(b) - g(a) \), as well as using Eqs. (2)–(9), it follows that

\[
H_P = h_P \sum_{i=1}^{n-1} \left[ X_i \sum_{j=1}^{u} g(t_i + jX_i) - \frac{1}{2P} \sum_{j=1}^{u} \left[ \int_{t_i + (j-1)X_i}^{t_{i+1}X_i} D(t) \, dt \right]^2 \right]
\]

\[
+ Y_i \sum_{j=1}^{v} g(t_i + jY_i) - \frac{1}{2R} \sum_{j=1}^{v} \left[ \int_{t_i + (j-1)Y_i}^{t_{i+1}Y_i} D(t) \, dt \right]^2
\]

\[
- \int_{t_i}^{t_{i+1}} g(t) \, dt \right].
\]

Secondly, we consider the used products inventory. The inventory level of the used products at time \( t \) during the production period, \( I_{u}(t) \), is given by the amount of used products collected from time \( t_i \) to time \( t \), that is

\[
I_{u}(t) = \phi \int_{t_i}^{t} D(w) \, dw, \quad t_i \leq t \leq \beta_i.
\]

Next, referring to the top graph in Fig. 2, the time-weighted inventory holding of the used products during the repair period is given by the sum of the areas of \( v \) triangles and \((v - 1)\) rectangles, where \( B_{ij} \) \( (j = 1, 2, \ldots, v) \) is the area of the corresponding right triangle and \( C_{ij} \) \( (j = 1, 2, \ldots, v - 1) \) is the area of the corresponding rectangle. Since the repair during the \((r_{ij} \leq t \leq r_{ij+1})\) period must satisfy the demand during the \((r_{ij} \leq t \leq r_{ij+1})\) period, where \( j = 1, 2, \ldots, v \), we have

\[
B_{ij} = \frac{R}{2} (\gamma_{ij} - r_{ij})^2 = \frac{1}{2R} \left[ \int_{r_{ij}}^{r_{ij+1}} D(t) \, dt \right]^2, \quad j = 1, 2, \ldots, v.
\]
\[ C_{ij} = R(\alpha_{ij+1}^1 - \alpha_{ij}^1)(\beta_{ij}^1 - \beta_i^1) \]

\[ = jY_j \int_{t_i}^{t_{i+1}} D(t) \, dt, \quad j = 1, 2, \ldots, v - 1. \]  \hfill (14)

Then, the total inventory holding cost of the used products throughout the planning horizon, \( H_R \), is given by

\[ H_R = \sum_{i=0}^{n-1} \left\{ \int_{t_i}^{t_{i+1}} \sum_{j=1}^{v} B_{ij} + \sum_{j=1}^{v-1} C_{ij} \right\}. \]

And then, using Eqs. (3), (13) and (14), it follows that

\[ H_R = \sum_{i=0}^{n-1} \phi \left[ \int_{t_i}^{t_{i+1}} (g(t) - g(t_i^1))(\beta_i^1 - t_i^1) \right] + \sum_{i=0}^{n-1} \frac{1}{2R} \sum_{j=1}^{v} \left[ \int_{t_j^1}^{t_{j+1}^1} D(t) \right]^2 + \sum_{j=1}^{v-1} \left[ \int_{t_j^1}^{t_{j+1}^1} D(t) \right]. \]  \hfill (15)

Thirdly, we consider the raw material 1 inventory. Since there is only one order of raw production and there are \( u \) production runs, then the time-weighted inventory holding during the batch period is given by the sum of \( A_{ij} \) \((j = 1, 2, \ldots, u)\), the area of the right triangles as shown in the bottom graph of Fig. 2. Hence, using Eqs. (2) and (4), the total inventory holding cost of raw material 1 throughout the planning horizon, \( H_1 \), is given by

\[ H_1 = H_1 = \sum_{i=0}^{n-1} \sum_{j=1}^{u} A_{ij} = \frac{h_1 q_1}{2P} \sum_{i=0}^{n-1} \sum_{j=1}^{u} \left[ \int_{t_j^1}^{t_{j+1}^1} D(t) \right]^2. \]  \hfill (16)

And fourthly, since all production runs in a batch must fully meet the demand during the production period and all repair runs as well during the repair period, then the total production cost, \( S_p \), and the total repair cost, \( S_R \), are respectively, given by

\[ S_p = s_p \sum_{i=0}^{n-1} \left\{ \int_{t_i}^{t_{i+1}} D(t) \, dt \right\}. \]

\[ S_R = s_p \sum_{i=0}^{n-1} \left\{ \int_{t_i}^{t_{i+1}} D(t) \, dt \right\}. \]  \hfill (18)

The problem is to minimize Eq. (1) by seeking the optimal integer values of \( n, u, v \), as well as the optimal real values of \( t_i^1 \) for that set of \((n, u, v)\), subject to the following constraints:

\[ \beta_i^1 < t_i^1, \quad t_{n-1}^1 = 0, \quad n = H. \]

Before proceeding to the next section, we present a lemma.

**Lemma 1.** For any demand function \( D(t) \) that can be integrated over the interval \([t_i, t_{i+1}]\), \( \beta_i^1 \) is a function of \((t_i, t_{i+1})\), where \( i = 0, 1, \ldots, n - 1 \).

**Proof.** Since the used products collected during the production period must fully satisfy the demand during the repair period, we have

\[ \phi \int_{t_i}^{t_{i+1}} D(t) \, dt = \int_{t_i}^{t_{i+1}} D(t) \, dt. \]  \hfill (19)

Since \( 0 < \phi < 1 \), then for Eq. (19) to hold, \( \beta_i^1 < t_i^1 \) must be true. Finally, it is easily observable that \( \beta_i^1 \) is a function of \((t_i, t_{i+1})\). \hfill \( \Box \)

4. Solution procedure

We employ Microsoft Excel’s Built-In Solver to solve the minimization problem. Since the problem is a non-linear one, the Solver uses the generalized reduced gradient method, as implemented in the GRG2 code by Lasdon et al. (1978), to solve it. The details of using Solver can be found in Fylstra et al. (1998).

As aforementioned, there are three integer decision variables \((n, u, v)\) in this problem in addition to the real decision variables \( t_i \). Therefore, to deal with this mixture of discrete and continuous variables, we first fix \((n, u, v)\) and then optimize \( t_i \) using Excel Solver. Next, we change \((n, u, v)\) to improve the total relevant cost. This is done sequentially: We first fix \( n \) and \( u \) and then seek \( v = V \) from a range of \( v \) such that the conditions \( TRC(V)_{n,u} < TRC(V)_{n,1} \) and \( TRC(V)_{n,u} < TRC(V)_{1,u} \) are met. Next, we fix only \( n \) and then, by applying the previous step over a range of \( u \), we seek \( u = U \) such that the conditions \( TRC(U)_{n,v} < TRC(U - 1)_{n,v} \) and \( TRC(U)_{n,v} < TRC(U + 1)_{n,v} \) are met. Finally, by applying the previous steps over a range of \( n \), we seek \( N = N \) such that the conditions \( TRC(N)_{U,V} < TRC(N - 1)_{U,V} \) and \( TRC(N)_{U,V} < TRC(N + 1)_{U,V} \) are met this time.

We note that observations of the optimal total relevant cost function for each \( v \) given \( n \) and \( u \), for each given \( n \) and \( v \), and for each \( n \) given \( v \) and \( u \), show that they are all monotonic decreasing functions of \( v, u, n, \) respectively, up to certain points, respectively, \( V, U, N \) and \( n \). Beyond that, they are all monotonic increasing functions. Now, given \( n, u, v, \) and \( n \), let the optimal values of \( t_i \) be represented by \( t_i^* \).

Then, \( TRC(n)_{u,v} \) is given by

\[ TRC(n)_{u,v} = \sum_{i=0}^{n-1} \sum_{j=1}^{u} A_{ij} = \frac{h_1 q_1}{2P} \sum_{i=0}^{n-1} \sum_{j=1}^{u} \left[ \int_{t_j^1}^{t_{j+1}^1} D(t) \right]^2 + \sum_{j=1}^{v-1} \left[ \int_{t_j^1}^{t_{j+1}^1} D(t) \right]. \]  \hfill (15)

Then, it is easy to see that \( v = V_0 \) is an upper bound until which \( TRC(V)_{n,u} \) must be computed to validate that \( TRC(V)_{n,u} < TRC(V)_{n,u} \) for any \( v > V \), where \( v = V_0 \) is the first solution to the inequality

\[ n[u(c_p + c_1)] + \min_{x_{n-1}^{t_{n-1}^1}} \left\{ x \int_{0}^{H} D(t) \, dt \right\} > TRC(V)_{n,u}. \]  \hfill (20)

Moreover, \( u = U_0 \) is an upper bound until which \( TRC(U)_{n,v} \) must be computed to validate that \( TRC(U)_{n,v} < TRC(U)_{n,v} \) for any \( u > U \), where \( u = U_0 \) is the first solution to the inequality

\[ n[u(c_p + c_1)] + \min_{x_{n-1}^{t_{n-1}^1}} \left\{ x \int_{0}^{H} D(t) \, dt \right\} > TRC(U)_{n,v}. \]  \hfill (21)

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and $n = N_0$ is an upper bound until which $TRC^*(n)_{u,v}$ must be computed to validate that $TRC^*(N)_{u,v} < TRC^*(n)_{u,v}$ for any $n > N$, where $n = N_0$ is the first solution to the inequality
\[ n[v+c_R+u(c_P+c_I)] + \min_{x>0} \left\{ x \int_0^t D(t) \, dt \right\} > TRC^*(N)_{u,v}. \]  
(22)

Finally, the computer algorithm of the optimal procedure is outlined below:

1. Let $n = 1$.
2. Let $u = 1$.
3a. Let $v = 1$.
3b. Compute the total relevant cost, $TRC(n)_{u,v}$. Note that this is already minimized.
4a. Increase $v$ by 1.
4b. Compute the optimal total relevant cost, $TRC^*(v)_{u,v}$, using Excel Solver.
4c. If $TRC^*(v)_{u,v} > TRC^*(v-1)_{u,v}$, stop. Let $V = v - 1$. Return $TRC^*(V)_{u,v}$ as $TRC^*(n)$.
4d. If $TRC^*(v)_{u,v} < TRC^*(v-1)_{u,v}$, repeat Step 4.
5a. Compute the integer $V_0$ such that the inequality in (20) is solved.
5b. Validate that $TRC^*(v)_{u,v}$ from $v = V + 1$ to $v = V_0$ are all greater than $TRC^*(V)_{u,v}$.
6a. Increase $u$ by 1. Repeat Steps 3–5 to compute the new $TRC^*(u)$.
6b. If $TRC^*(u)_{n} > TRC^*(u-1)_{n}$, stop. Let $U = u - 1$. Return $TRC^*(U)_{n}$ as $TRC^*(n)$.
6c. If $TRC^*(u)_{n} < TRC^*(u-1)_{n}$, repeat Step 6.
7a. Compute the integer $U_0$ such that the inequality in (21) is solved.
7b. Validate that $TRC^*(u)_{n,v}$ from $u = U + 1$ to $u = U_0$ are all greater than $TRC^*(U)_{n,v}$.
8a. Increase $n$ by 1. Repeat Steps 2–7 to compute the new $TRC^*(n)$.
8b. If $TRC^*(n) > TRC^*(n-1)$, stop. Let $N = n - 1$. Return $TRC^*(N)$ as the optimal total relevant cost of the system.
8c. If $TRC^*(n) < TRC^*(n-1)$, repeat Step 8.
9a. Compute the integer $N_0$ such that inequality in (22) is solved.
9b. Validate that $TRC^*(n)_{u,v}$ from $n = N + 1$ to $n = N_0$ are all greater than $TRC^*(N)_{u,v}$.

5. Numerical example and conclusion

To illustrate the application of the optimal procedure, we present a numerical example with the demand function in the form of $D(t) = 6 + 15t$, and the parametric values of $c_R = 1$, $c_P = 300$, $c_I = 50$, $h_R = 10$, $h_P = 15$, $h_I = 5$, $\phi = 0.7$, $P = 100$, $R = 100$, $H = 5$, $q_1 = 30$, $s_P = 30$, and $s_R = 10$. For varying values of $n$, $u$, and $v$, selected optimal total relevant costs, $TRC^*(n)_{u,v}$, are tabulated in Table 1. The optimal total relevant cost when $n$, $u$, and $v$ are optimal, $TRC^*(N)_{u,v}$, is 7221.59 with $N = 4$, $U = 1$, $V = 3$, and the optimal policy of $t_0 = 0$, $t_1 = 1.5898$, $t_2 = 2.8453$, and $t_3 = 3.9610$. This means, for example, for the second batch, one production run is conducted at $t = 1.5898$ and three repair runs are conducted at (2.3974, 2.5467, 2.696).

In summary, in this paper, we have proposed a model for the inventory management of a production system where a continuous time-varying demand for a finished product can be satisfied by newly fabricated products or by repaired products. For simplicity, we assume that only one type of raw material is required to fabricate the finished product, and there is no collection of used

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Table 1: Optimal total relevant costs for varying $n$, $u$, and $v$.

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products during the repair period. Moreover, we assume that the system procures raw materials, fabricates finished products and repair used products in multiple lots during each batch throughout the planning horizon. For the optimal joint policy, the continuous variables are computed using Microsoft Excel Solver while an exhaustive search procedure is used for the integer variables $n, u,$ and $v$. We also presented a numerical example to illustrate the optimal procedure. Finally, we conclude the paper by noting that our model may be extended to the following cases. One is the case where used products are collected during the repair period. Another case is the removal of the assumption of constant intervals between multiple production and repair times in a batch.

**Appendix**

We rewrite Eq. (10) as Eq. (11) here. Using Eqs. (2)–(5) and \( \int_0^z D(w) dw = g(b) - g(a) \), we have

\[
\int_{p_{ij}}^{x_{ij}} I_{1j}(t) dt = \int_{p_{ij}}^{x_{ij}} [P(t - p_{ij}) - g(t) + g(p_{ij})] dt
\]

\[
= \frac{P}{2} (x_{ij} - p_{ij})^2 - \int_{p_{ij}}^{x_{ij}} g(t) dt + g(p_{ij})(x_{ij} - p_{ij}) \]

\[
= \frac{1}{2P} \left[ \int_{t_{j,i+1}-1}^{t_{j,i+1}} D(t) dt \right]^2 - \int_{p_{ij}}^{x_{ij}} g(t) dt
\]

\[
+ \frac{g(t_{j} + j - 1)X_{i}}{P} \int_{t_{j,i+1}-1}^{t_{j,i+1}} D(t) dt
\]

\[
\int_{p_{ij}}^{x_{ij}} I_{1j}(t) dt + \int_{p_{ij}}^{x_{ij}} I_{2j}(t) dt = \int_{p_{ij}}^{x_{ij}} g(t_{j} + jX_{i}) - \frac{1}{P} \int_{t_{j,i+1}-1}^{t_{j,i+1}} D(t) dt
\]

\[
- \int_{p_{ij}}^{x_{ij}} g(t) dt
\]

\[
\int_{p_{ij}}^{x_{ij}} I_{1j}(t) dt + \int_{p_{ij}}^{x_{ij}} I_{2j}(t) dt = \int_{p_{ij}}^{x_{ij}} g(t_{j} + jX_{i}) - \frac{1}{P} \int_{t_{j,i+1}-1}^{t_{j,i+1}} D(t) dt
\]

\[
- \int_{p_{ij}}^{x_{ij}} g(t) dt
\]

Hence, it follows that

\[
\int_{p_{ij}}^{x_{ij}} I_{1j}(t) dt + \int_{p_{ij}}^{x_{ij}} I_{2j}(t) dt = \int_{p_{ij}}^{x_{ij}} g(t_{j} + jX_{i}) - \frac{1}{P} \int_{t_{j,i+1}-1}^{t_{j,i+1}} D(t) dt
\]

\[
- \int_{p_{ij}}^{x_{ij}} g(t) dt
\]

Finally, we have

\[
\int_{p_{ij}}^{x_{ij}} I_{1j}(t) dt + \int_{p_{ij}}^{x_{ij}} I_{2j}(t) dt = \int_{p_{ij}}^{x_{ij}} g(t) dt.
\]

\[
(26)
\]

References


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