Delay-dependent $H_\infty$ filtering for complex dynamical networks with time-varying delays in nonlinear function and network couplings

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**Abstract**

This paper investigates the $H_\infty$ filtering problem for a class of continuous-time complex dynamical networks with time-varying delays in nonlinear function and network couplings. The aim of the addressed problem is to design a $H_\infty$ filter against the exogenous disturbances, such that the filtering error system of complex dynamical networks is asymptotically stable and guarantees the desired $H_\infty$ performance attenuation level. Based on the Lyapunov stability theory, suitable Lyapunov–Krasovskii functional is constructed in terms of Kronecker product, furthermore, new delay-dependent sufficient stability conditions are derived in terms of linear matrix inequalities by using reciprocal convex combination approach to obtain less conservative results. Finally, numerical examples are exploited to demonstrate the effectiveness of the proposed theoretical results. © 2015 Elsevier B.V. All rights reserved.

**1. Introduction**

The study of complex networks handles with the network whose structure is irregular, complex and networks with highly interconnected nodes, which evolves dynamically with respect to time [1]. Nowadays, extensive research work is focused on complex dynamical networks (CDNs) due to its wider applications in airport networks [2], computer networks [3], biological networks [4], communication networks [5], etc. These systems exhibit complicated dynamics which are represented by a set of interconnected nodes, edges and coupling strength. Therefore, researchers have paid great attention to analyze the dynamical behavior of CDNs. The techniques of synchronization and state estimation for CDNs can be found in [6–9]. For singular CDNs both synchronization and state estimation problem have been investigated in [10]. Recently, authors in [11] have derived the sufficient conditions for adaptive synchronization of CDNs with non-delay and variable delay couplings via pinning control to achieve minimum number of pinning nodes. Robust exponential stabilization and adaptive synchronization for uncertain CDNs with time-delay in the coupling nodes have been discussed in [12,13]. Synchronization happening between two or more coupled networks is known as "outer synchronization". Generalized outer synchronization between two uncertain networks with or without time delay has been proposed in [14].

In real complex network systems, such as in the progress of brain nervous activity, time delay occurs during the information transmission between nerve cells because of the limited speed of signal transmission as well as in the network traffic congestion systems. Thus, the
presence of time delays (coupling delays) in CDNs is unavoidable. It leads often as a source of instability and poor performance of system behaviors, for instance, see [15–18]. The dynamical behavior of CDNs with non-identical nodes are much more complicated than with identical nodes. To deal with different dimensional nodes, authors in [19] have studied the synchronization criteria for time-delayed coupling CDNs with different dimensional nodes by using the decentralized dynamic compensation controllers.

Nowadays, various control methods such as adaptive control, sample data control, impulsive control, and \( H_{\infty} \) control have been explored to realize the synchronization phenomena of CDNs, for details see [11,13,19] and references therein. The sample data controller has been applied in [20,21] to synchronize the CDNs with Markovian jump parameters by considering the time-varying delays in the network. Impulsive control technology introduced in [22,23] is an efficient method to deal with dynamical systems which cannot ensure continuous disturbance. Very recently, the problem of impulsive control and synchronization of CDNs have been investigated in [24]. By using single impulsive controller, sufficient conditions have been obtained in [25] for pinning synchronization of delayed undirected CDNs. Observer based controller design for complex systems has been an interesting topic in control theory. In practical, a non-fragile observer based robust controller for fractional order CDNs has been designed in [26] by using indirect Lyapunov approach. By introducing fractional-order Lyapunov stability theorem, pinning controller for synchronization of the directed and undirected complex networks has been reported in [27].

It is important to note that the external disturbances are ubiquitous in nature as well as in complex networks. In this case, the notion of \( H_{\infty} \) theory has been determined to reduce the effect of exogenous disturbances and to quantify them within the prescribed level [28,29]. Based on decentralized observer, the problem of robust \( H_{\infty} \) observer-based control for synchronization of a class of CDNs has been investigated in [30]. The novel concept of \( H_{\infty} \) synchronization and state estimation for CDNs with mixed delays have been introduced in [31,32] to quantify against the exogenous disturbance of the complex networks. The problem of filtering or state estimation has been widely applied in the fields of signal processing, image processing and control applications. Among various filtering methods, Kalman filtering deals with minimizing the variance of the state estimation error for a given measurement noise. However, in most practical applications, the statistical assumptions on the external noise signals cannot be known exactly. To overcome this limitation, \( H_{\infty} \) filtering technique has been introduced to deal with unavoidable parameter shifts and external disturbances [33,34] Nowadays, the \( H_{\infty} \) filtering technology has been applied recently to various dynamical time-delay systems such as networked control systems [23,35], neutral systems [36], singular systems [37], and T–S fuzzy systems [38,39]. Meantime, the delay-dependent filtering analysis for neural networks has been investigated in [40,41] by employing some inequality techniques to reduce the conservatism.

In the literature discussions above, most of the results have been concerned with synchronization and state estimation of CDNs. However, CDNs have its wider applications in science and engineering. Up to now, only limited works have been done with respect to \( H_{\infty} \) filter design for CDNs. Recently, the robust \( H_{\infty} \) filtering design has been investigated in [42] for a class of discrete-time complex networks which has stochastic packet dropouts and time delays combined with disturbance inputs. The lack of research analysis is probably due to difficulty in designing suitable filter parameters. In order to shorten such a gap, in this paper, suitable full-order filter is designed for continuous-time CDNs with time-varying delays.

On the other hand, it has been well known that there exist various approaches to reduce the conservatism of delay-dependent stability results for time-delay systems. The reduction of conservatism means that to increase the feasibility region. In this paper, we have utilized the reciprocal convex combination approach [43] to derive the sufficient conditions for the problem of delay-dependent \( H_{\infty} \) filtering for CDNs. To the best of authors knowledge, \( H_{\infty} \) filtering analysis for CDNs by using reciprocal convex approach has not been investigated yet. Motivated by the above discussions, this paper is aimed to study the problem of \( H_{\infty} \) filtering for continuous-time CDNs with time-varying delays in the coupling nodes. By constructing suitable Lyapunov–Krasovskii functional in terms of Kronecker product, sufficient stability conditions are derived in terms of linear matrix inequalities (LMIs) to guarantee the existence of the designed \( H_{\infty} \) filters. The main contributions of this paper lie in the following aspects: (i) For the first time, suitable full-order \( H_{\infty} \) filters are designed for each node for continuous-time CDNs with time-varying delays in nonlinear function and network couplings. (ii) The properties of Kronecker product are employed to derive the stability conditions in a more compact form. (iii) In order to reduce the possible conservatism of the result, reciprocal convex combination approach is utilized. (iv) To illustrate the applicability of the proposed results, Barabasi–Albert (BA) scale-free network model is considered.

The rest of this paper is organized as follows. A class of continuous-time delayed CDNs consisting of \( N \) coupled nodes is presented and some necessary definition and lemmas are provided in Section 2. In Section 3, sufficient conditions are obtained in the form of LMIs and then the \( H_{\infty} \) filters are designed. Numerical simulations are given in Section 4, to verify the effectiveness of the designed \( H_{\infty} \) filters. Finally, in Section 5 conclusions and future research interests are given.

**Notation:** Throughout this paper, \( \mathbb{R}^n \) denotes the \( n \) dimensional Euclidean space and \( \mathbb{R}^{m \times n} \) is the space of \( m \times n \) real matrices. \( L_2[0, \infty) \) represents the space of square integrable vector functions over \( [0, \infty) \). The Kronecker product of matrices \( Q \in \mathbb{R}^{m \times n} \) and \( R \in \mathbb{R}^{p \times q} \) is a matrix in \( \mathbb{R}^{mp \times nq} \) and denoted as \( Q \otimes R \). We use \( \text{diag} \{ \cdots \} \) as a block-diagonal matrix. \( A > 0 \) (\(< 0\)) means \( A \) is a symmetric positive (negative) definite matrix, \( A^{-1} \) denotes the inverse of matrix \( A \). \( A^T \) denotes the transpose of matrix \( A \) and \( I \) is the identity matrix with compatible dimension.
2. Problem formulation and preliminaries

Consider the following delayed CDNs consisting of $N$ coupled nodes of the form

\[
\begin{align*}
\dot{x}_i(t) &= A x_i(t)+B_1 f(x_i(t))+B_2 f(x_i(t-\tau(t)))+\sum_{j=1}^{N} W_{ij} f_i(x_j(t)) \\
&\quad + \sum_{j=1}^{N} g_{ij} \tilde{x}_j(t-\tau(t)) + D_i \nu(t), \\
y_i(t) &= C_i x_i(t)+D_2 \omega_i(t), \\
z_i(t) &= E x_i(t), \\
x_i(t) &= \phi_i(t), \quad \forall t \in [-\tau,0],
\end{align*}
\]

where $x_i(t) \in \mathbb{R}^n$ is the state vector of the $i$th node at time $t$; $y_i(t)$ and $\omega_i(t)$ are the disturbance inputs which belong to $L_2[0,\infty)$; $y_i(t) \in \mathbb{R}^m$ is the measured output of the $i$th node; $z_i(t) \in \mathbb{R}^l$ is the signal to be estimated; $\phi_i(t)$ is a compatible vector-valued initial function defined on $[-\tau,0]$; $A, B_1, B_2, E$ are known constant matrices with appropriate dimensions; $D_{ij} \in \mathbb{R}^{\nu \times \nu}$, $C_i \in \mathbb{R}^{m\times n}$ and $D_2 \in \mathbb{R}^{m\times \nu}$ are some constant matrices; $F_1$ and $F_2$ are matrices describing the inner-coupling between the subsystems at time $t$ and $t-\tau(t)$ respectively; $W = [w_{ij}]_{N \times N}$ and $G = [g_{ij}]_{N \times N}$ are the outer-coupling configuration matrices representing respectively the coupling strength and the topological structure of complex networks; $\tau(t)$ is the time-varying delay satisfying the following inequality:

\[
0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \mu < \infty.
\]  

Assumption 1. The nonlinear function $f(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^n$ is assumed to be continuous and satisfies $f(0) = 0$. Further the following sector-bounded condition holds:

\[
[f(x) - f(y) - F_1(x-y)]^T [f(x) - f(y) - F_2(x-y)] \leq 0,
\]

where $F_1$ and $F_2$ are known real constant matrices with appropriate dimensions.

The following full-order filter is designed to estimate the signal $z_i(t)$:

\[
\begin{align*}
\dot{\hat{x}}_i(t) &= A \hat{x}_i(t)+B_1 \hat{y}_i(t), \\
\dot{\hat{z}}_i(t) &= E \hat{x}_i(t), \\
\hat{x}_i(t) &= \hat{\phi}_i(t), \quad \forall t \in [-\tau,0],
\end{align*}
\]

where $\hat{x}_i(t)$ is the filter state vector, and $A_{\hat{f}}$ and $B_{\hat{f}}$ are appropriately dimensioned filter matrices to be designed.

With the matrix Kronecker product, the systems (1) and (4) can be rewritten respectively in the following compact form:

\[
\begin{align*}
\dot{x}(t) &= A x(t)+B_1 f(x(t))+B_2 f(x(t-\tau(t)))+(W \otimes F_1)x(t) \\
&\quad +(G \otimes F_2)(x(t-\tau(t))+D_i \nu(t), \\
y(t) &= C x(t)+D_2 \omega(t), \\
z(t) &= E x(t), \\
x(t) &= \phi(t), \quad \forall t \in [-\tau,0],
\end{align*}
\]

and

\[
\begin{align*}
\dot{\hat{x}}(t) &= A \hat{x}(t)+B_1 \hat{y}(t), \\
\dot{\hat{z}}(t) &= E \hat{x}(t), \\
\hat{x}(t) &= \hat{\phi}(t), \quad \forall t \in [-\tau,0],
\end{align*}
\]

where

\[
A = I \otimes A, \quad B_1 = I \otimes B_1, \quad B_2 = I \otimes B_2, \quad \tilde{E} = I \otimes E, \\
C = \text{diag}(C_1, C_2, \ldots, C_N), \quad D_1 = \text{diag}(D_{11}, D_{12}, \ldots, D_{1N}), \\
D_2 = \text{diag}(D_{21}, D_{22}, \ldots, D_{2N}), \quad \tau(t) = [\tau_1(t) \tau_2(t) \cdots \tau_N(t)]^T, \\
y(t) = [y_1(t) y_2(t) \cdots y_N(t)]^T, \\
A_f = \text{diag}(A_{f1}, A_{f2}, \ldots, A_{fN}), \quad B_f = \text{diag}(B_{f1}, B_{f2}, \ldots, B_{fN}), \\
\nu(t) = [\nu_1(t) \nu_2(t) \cdots \nu_N(t)]^T, \quad \omega(t) = [\omega_1(t) \omega_2(t) \cdots \omega_N(t)]^T, \\
f(\cdot) = [f_1(\cdot) f_2(\cdot) \cdots f_N(\cdot)]^T.
\]

By introducing new vector $e(t) = [x^T(t) \tilde{z}_i(t)^T]$ and $\overline{z}(t) = z(t) - \tilde{z}(t)$, the augmented filtering error system is obtained as follows:

\[
\begin{align*}
\dot{e}(t) &= -\overline{X}_1 e(t) + \overline{X}_2 \overline{\omega}(t-\tau(t)) \pm B_f f(\overline{\omega}(t-\tau(t)))+T_e \sigma(t), \\
\overline{z}(t) &= L_e e(t), \quad e(t) = \overline{\phi}(t), \quad \forall t \in [-\tau,0],
\end{align*}
\]

where $\overline{\phi}(t) = [\phi^T(t) \dot{\phi}(t)^T]^T$ and

\[
\overline{X}_1 = \begin{bmatrix} \tilde{A} + W \otimes \Gamma_1 & 0 \\ B_f C & A_f \end{bmatrix}, \quad \overline{X}_2 = \begin{bmatrix} G \otimes \Gamma_2 \\ 0 \end{bmatrix},
\]

\[
\overline{B}_1 = \begin{bmatrix} \tilde{B}_1 \\ 0 \end{bmatrix}, \quad \overline{B}_2 = \begin{bmatrix} \tilde{B}_2 \\ 0 \end{bmatrix},
\]

\[
T_e = \begin{bmatrix} D_1 & 0 \\ 0 & B_f D_2 \end{bmatrix}, \quad T_e = \begin{bmatrix} \tilde{E} & -\tilde{E} \end{bmatrix},
\]

\[
K = [1 \ 0], \quad \sigma(t) = [\nu^T(t) \omega^T(t)]^T.
\]

Remark 1. Many phenomena in nature can be modeled as a network of interconnected dynamical nodes. Examples of such networks include brain structures, social networks, global economic markets, internet, and airport networks. These networks have more than one link between two nodes and each of these links has its own property. Airport networks [2] are a kind of complex systems that display significant fluctuations. Such kind of systems can be modeled as a class of CDNs with exogenous disturbance inputs.

Definition 2.1. The filtering error system (7) is said to be asymptotically stable with a performance $\gamma_\infty$, if it is asymptotically stable with $\sigma(t) = 0$ and, under zero initial condition, there exists $\gamma > 0$, such that $\| \overline{Z}(t) \|_2 \leq \gamma$ for any non-zero $\sigma(t) \in L_2[0,\infty)$.

Lemma 2.1 (Park et al. [43]). Let $f_1, f_2, \ldots, f_N: \mathbb{R}^m \rightarrow \mathbb{R}$ have positive values in an open subset $D$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of $f_i$ over $D$ satisfies

\[
\min_{\sum \alpha_i = 1} \left\{ \sum \alpha_i f_i(t) \right\} \geq \sum \max_{\|x\| \leq D} \sum \alpha_i g_{ij}(t),
\]

where $g_{ij}(t) = f_i(t) + \sum \max_{\|x\| \leq D} \sum \alpha_j g_{ij}(t)$.
subject to
\[
\begin{align*}
\{ g_{ij}: \mathbb{R}^m \rightarrow \mathbb{R}, \quad g_{ij}(t) \triangleq g_{ij}(t), \quad [f_i(t) \quad g_{ij}(t) \quad f_j(t)] \geq 0 \}\end{align*}
\]

Lemma 2.2 (Gu [44]). For any constant matrix \( M \in \mathbb{R}^{n \times n} \), \( M = M^T > 0 \), scalar \( h > 0 \), such that the following integrations are well defined, then
\[
-h \int_{t-h}^t x(t)^T M x(s) ds \leq -\left( \int_{t-h}^t x(t)^T ds \right) M \left( \int_{t-h}^t x(s) ds \right).
\]

Lemma 2.3 (Li [9]). By the definition of the Kronecker product, the following properties hold:
\[
(\alpha A) \otimes B = A \otimes (\alpha B),
\]
\[
(A + B) \otimes C = A \otimes C + B \otimes C,
\]
\[
(A \otimes B)(C \otimes D) = (AC) \otimes (BD),
\]
\[
(A \otimes B)^T = A^T \otimes B^T.
\]

Remark 2. It should be noticed that, the \( \mathcal{H}_\infty \) filtering problem for a class of continuous-time CDNs is investigated for the first time in this work and the proposed LMI conditions are given in the form of Kronecker product in a more compact form which can be easily solved by using Matlab LMI toolbox.

3. Main results

In this section, the stability analysis problem for the filtering error system (7) with \( \Pi(t) = 0 \) is investigated by using Lyapunov functional method combining with LMI techniques and then the \( \mathcal{H}_\infty \) performance analysis for the filtering error system (7) with nonzero \( \Pi(t) \) is derived.

Theorem 3.1. For given scalars \( \tau, \mu \) and positive scalars \( \lambda_1, \lambda_2 \), the filtering error system (7) is asymptotically stable with an \( \mathcal{H}_\infty \) performance \( \gamma \), if there exist positive definite matrices \( U_i \) \( (i = 1, 2, \ldots, 5) \) and any matrix \( W_1 \) with appropriate dimension such that the following LMIs hold:
\[
\begin{align*}
\begin{bmatrix}
U_4 & W_1
\end{bmatrix}
\begin{bmatrix}
W_1^T & U_4
\end{bmatrix} \succeq 0,
\end{align*}
\]
\[
\begin{align*}
\Phi &\Omega \begin{bmatrix}
\frac{1}{\tau} Y
\end{bmatrix} \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{14} & \Phi_{15} \\
* & \Phi_{22} & \Phi_{24} & \Phi_{25} \\
* & * & \Phi_{33} & 0 \\
* & * & * & \Phi_{44} & \Phi_{45} \\
* & * & * & * & \Phi_{55}
\end{bmatrix},
\Omega = [\Omega_{16}^T \quad \Omega_{26}^T \quad 0 \quad 0 \quad 0 \quad \Omega_{46} \quad \Omega_{56}],
\end{align*}
\]
\[
Y = [I_2 \quad 0 \quad 0 \quad 0] \begin{bmatrix}
0
\end{bmatrix},
\begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{14} & \Phi_{15} \\
* & \Phi_{22} & \Phi_{24} & \Phi_{25} \\
* & * & \Phi_{33} & 0 \\
* & * & * & \Phi_{44} & \Phi_{45} \\
* & * & * & * & \Phi_{55}
\end{bmatrix},
\begin{bmatrix}
\lambda_1 I_2 \quad -\lambda_2 I_2
\end{bmatrix},
\begin{bmatrix}
\alpha \Omega \Omega^T \\
\end{bmatrix},
\end{align*}
\]
\[
\begin{align*}
Y = [I_2 \quad 0 \quad 0 \quad 0] \begin{bmatrix}
0
\end{bmatrix},
\begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{14} & \Phi_{15} \\
* & \Phi_{22} & \Phi_{24} & \Phi_{25} \\
* & * & \Phi_{33} & 0 \\
* & * & * & \Phi_{44} & \Phi_{45} \\
* & * & * & * & \Phi_{55}
\end{bmatrix},
\begin{bmatrix}
\lambda_1 I_2 \quad -\lambda_2 I_2
\end{bmatrix},
\begin{bmatrix}
\alpha \Omega \Omega^T \\
\end{bmatrix},
\end{align*}
\]
\[
\begin{align*}
\lambda_1 ^* \Omega \begin{bmatrix}
\frac{1}{\tau} Y
\end{bmatrix} \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{14} & \Phi_{15} \\
* & \Phi_{22} & \Phi_{24} & \Phi_{25} \\
* & * & \Phi_{33} & 0 \\
* & * & * & \Phi_{44} & \Phi_{45} \\
* & * & * & * & \Phi_{55}
\end{bmatrix},
\Omega = [\Omega_{16}^T \quad \Omega_{26}^T \quad 0 \quad 0 \quad 0 \quad \Omega_{46} \quad \Omega_{56}],
\end{align*}
\]
\[
\begin{align*}
\lambda_2 ^* \Omega \begin{bmatrix}
\frac{1}{\tau} Y
\end{bmatrix} \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{14} & \Phi_{15} \\
* & \Phi_{22} & \Phi_{24} & \Phi_{25} \\
* & * & \Phi_{33} & 0 \\
* & * & * & \Phi_{44} & \Phi_{45} \\
* & * & * & * & \Phi_{55}
\end{bmatrix},
\Omega = [\Omega_{16}^T \quad \Omega_{26}^T \quad 0 \quad 0 \quad 0 \quad \Omega_{46} \quad \Omega_{56}],
\end{align*}
\]

Proof. Construct the Lyapunov–Krasovskii functional candidate for system (7) as
\[
V(t, e(t)) = \sum_{\pi=1}^4 V_\pi(t, e(t)),
\]
where
\[
V_1(t, e(t)) = e^T(t) U_1 e(t),
\]
\[
V_2(t, e(t)) = \int_{t-\tau(t)}^t e^T(s) K_2^T U_2 e(s) ds,
\]
\[
V_3(t, e(t)) = \int_{t-\mu(t)}^t e^T(s) K_3^T U_3 e(s) ds,
\]
\[
V_4(t, e(t)) = \int_{t-\tau(t)}^t e^T(s) K_4^T U_4 e(s) ds.
\]

Proof. Construct the Lyapunov–Krasovskii functional candidate for system (7) as
\[
V(t, e(t)) = \sum_{\pi=1}^4 V_\pi(t, e(t)),
\]
where
\[
V_1(t, e(t)) = e^T(t) U_1 e(t),
\]
\[
V_2(t, e(t)) = \int_{t-\tau(t)}^t e^T(s) K_2^T U_2 e(s) ds,
\]
\[
V_3(t, e(t)) = \int_{t-\mu(t)}^t e^T(s) K_3^T U_3 e(s) ds,
\]
\[
V_4(t, e(t)) = \int_{t-\tau(t)}^t e^T(s) K_4^T U_4 e(s) ds.
\]
where the inequality in (14) has been derived from Lemma 2.2. Further, the inequality (15) can be reduced from Lemma 2.1 as
\[
\begin{align*}
&\leq \dot{\varepsilon}^T(t)K^T U_d[\dot{K}e(t) - K(t)] \varepsilon(t), \\
&\leq \tau^2 \dot{\varepsilon}^T(t)K^T U_d \dot{K}e(t) - \left[ \begin{array}{c}
K(t) - K(t) \\
K(t) - K(t)
\end{array} \right]^T \left[ \begin{array}{c}
K(t) - K(t) \\
K(t) - K(t)
\end{array} \right] \varepsilon(t), \\
&\leq \left[ \begin{array}{c}
\sqrt{\frac{\mu}{2}}[\dot{K}e(t) - K(t)] \\
\sqrt{\frac{\mu}{2}}[\dot{K}e(t) - K(t)]
\end{array} \right]^T \left[ \begin{array}{c}
\sqrt{\frac{\mu}{2}}[\dot{K}e(t) - K(t)] \\
\sqrt{\frac{\mu}{2}}[\dot{K}e(t) - K(t)]
\end{array} \right] \leq 0,
\end{align*}
\]
where \( \alpha = \frac{\alpha_0}{\tau}, \beta = \frac{\beta_0}{\tau} \). Note that when \( \tau(t) = \tau \) or \( \tau = 0 \), one can obtain \( \dot{\varepsilon}^T(t) - \dot{\varepsilon}^T(t)K^T = 0 \) or \( \dot{\varepsilon}^T(t)K^T = 0 \) respectively. So, the relation (15) still holds.

From Assumption 1, for any positive scalars \( \lambda_1, \lambda_2 \), the following inequalities hold:
\[
\begin{align*}
&-\lambda_1\left[ \frac{\dot{K}e(t)}{f(K(t))} \right]^T \left[ \begin{array}{cc}
\tilde{F}_1 & -\tilde{F}_2 \\
\ast & 2I
\end{array} \right] \left[ \begin{array}{c}
\dot{K}e(t) \\
f(K(t))
\end{array} \right] \geq 0, \\
&-\lambda_2\left[ \frac{\dot{K}e(t) - \dot{K}e(t)}{f(K(t))} \right]^T \left[ \begin{array}{cc}
\tilde{F}_1 & -\tilde{F}_2 \\
\ast & 2I
\end{array} \right] \left[ \begin{array}{c}
\dot{K}e(t) \\
f(K(t))
\end{array} \right] \geq 0.
\end{align*}
\]
(17)

Considering Eqs. (10)–(13) and (15)–(17), we obtain
\[
\begin{align*}
&V(t, e(t)) \leq \dot{V}(t, e(t)) - \lambda_1\left[ \frac{\dot{K}e(t)}{f(K(t))} \right]^T \left[ \begin{array}{cc}
\tilde{F}_1 & -\tilde{F}_2 \\
\ast & 2I
\end{array} \right] \left[ \begin{array}{c}
\dot{K}e(t) \\
f(K(t))
\end{array} \right] \\
&\quad - \lambda_2\left[ \frac{\dot{K}e(t) - \dot{K}e(t)}{f(K(t))} \right]^T \left[ \begin{array}{cc}
\tilde{F}_1 & -\tilde{F}_2 \\
\ast & 2I
\end{array} \right] \left[ \begin{array}{c}
\dot{K}e(t) \\
f(K(t))
\end{array} \right] \leq \dot{\varepsilon}^T(t)\Phi \Omega^T \dot{\varepsilon}(t),
\end{align*}
\]
(18)

where
\[
\dot{\varepsilon}^T(t) = [\eta^T(t) \varepsilon(t) \varepsilon(t)], \quad \Omega_{66} = \tau^2 T_1 K^T U_d K T_1,
\]
\[
\eta^T(t) = [\dot{e}^T(t) \dot{e}^T(t) - \dot{e}^T(t) K^T f'(K(t)) e'(K(t) - \dot{K}(t))].
\]

Now, we prove that for the case of \( \varepsilon(t) = 0 \), the filtering error system (7) is asymptotically stable. If \( \varepsilon(t) = 0 \), from (18), we obtain
\[
\dot{V}(t, e(t)) \leq \dot{\eta}^T(t) \Phi \eta(t).
\]
(19)

Hence, one can conclude from (9) that \( \Phi < 0 \), which implies \( \dot{V}(t, e(t)) < 0 \). Thus, the filtering error system (7) with \( \varepsilon(t) = 0 \) is asymptotically stable.

For \( \mathcal{H}_\infty \) performance analysis for filtering error system (7) with non-zero \( \varepsilon(t) \), we define the following performance index:
\[
J_T = \int_0^T \left[ \frac{1}{N} \dot{\varepsilon}(t) \varepsilon(t) - \gamma^2 \bar{\eta}^T(t) \bar{\eta}(t) \right] dt.
\]
(20)

The performance index (20) can be rewritten as
\[
J_T = \int_0^T \left[ \frac{1}{N} \dot{\varepsilon}(t) \varepsilon(t) - \gamma^2 \bar{\eta}^T(t) \bar{\eta}(t) + V_1(t) e(t) \right] dt - \int_0^T V_1(t) e(t) dt \\
\leq \int_0^T \left[ \Phi^T(t) \Theta \Phi(t) \right] dt - V(T, e(T)) \\
\leq \int_0^T \left[ \Phi^T(t) \Theta \Phi(t) \right] dt
\]

where \( \Theta = \left[ \Phi^T(t) \Omega^T \right] \). By using Schur complement [45], (9) is equivalent to \( \Theta < 0 \) and consequently, if (8) and (9) hold, then \( J_T < 0, \forall T > 0 \). That is
\[
\int_0^\infty \left[ \frac{1}{N} \dot{\varepsilon}(t) \varepsilon(t) - \gamma^2 \bar{\eta}^T(t) \bar{\eta}(t) \right] dt < 0,
\]
which means that \( \frac{1}{N} \varepsilon(t)^2 \leq \gamma^2 \bar{\eta}^T(t) \bar{\eta}(t) \) for all nonzero \( \bar{\eta}(t) \). Therefore, the filtering error system (7) is asymptotically stable with the \( \mathcal{H}_\infty \) performance \( \gamma \). This completes the proof. \( \square \)

Remark 3. Lemmas 2.1 and 2.2 are applied to the corresponding terms in the Lyapunov–Krasovskii functional \( V_1(t, e(t)) \) to achieve less conservative results with fewer decision variables. In addition, a more generalized sector-like condition is assumed to well describe the nonlinear functions in the network.

The following theorem is derived to design the parameters of the desired filters defined in (4) by using the sufficient conditions established in Theorem 3.1.

**Theorem 3.2.** Given scalars \( \tau, \mu \) and positive scalars \( \lambda_1, \lambda_2 \), the filtering error system (7) is asymptotically stable with an \( \mathcal{H}_\infty \) performance \( \gamma \), if there exist positive definite matrices \( P_i (i = 1, 2, \ldots, 5) \), matrices \( \Theta_{ij}, \Theta_{ij} (i = 1, 2, \ldots, N) \), and any matrix \( W_1 \) with appropriate dimension such that the following LMIs hold:
\[
\begin{bmatrix}
I \otimes P_5 & I \otimes W_1 \\
(l \otimes W_1)^T & I \otimes P_5
\end{bmatrix} \geq 0,
\]
\[
\begin{bmatrix}
\Phi & \Omega & \Omega_{66} \\
\ast & \ast & -\gamma^2 I \\
\ast & \ast & \ast
\end{bmatrix} < 0,
\]
(22)

where
\[
\begin{bmatrix}
\varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} \\
\varphi_{22} & 0 & 0 & 0 \\
\varphi_{33} & \varphi_{34} & \varphi_{35} & \varphi_{36} \\
\varphi_{44} & 0 & 0 \\
\varphi_{55} & \varphi_{56} \\
\varphi_{66}
\end{bmatrix}
\]
\[
\begin{bmatrix}
\varphi_{11}^T & \varphi_{12}^T & \varphi_{13}^T & \varphi_{14}^T & \varphi_{15}^T & \varphi_{16}^T \\
\ast & 0 & 0 \\
\ast & \ast & \ast \\
\ast & \ast & \ast \\
\ast & \ast & \ast \\
\ast & \ast & \ast
\end{bmatrix}
\]

Now, we prove that for the case of \( \varepsilon(t) = 0 \), the filtering error system (7) is asymptotically stable. If \( \varepsilon(t) = 0 \), from (18), we obtain
\[
\dot{V}(t, e(t)) \leq \dot{\eta}^T(t) \Phi \eta(t).
\]
(19)

Hence, one can conclude from (9) that \( \Phi < 0 \), which implies \( \dot{V}(t, e(t)) < 0 \). Thus, the filtering error system (7) with \( \varepsilon(t) = 0 \) is asymptotically stable.
The method has been employed in [38] to tackle the time-stability results, there exist various advanced methods for delays in the sensor networks. In [40] slack matrix method (8) and (9), we obtain (21) and (22), in which the parameter, free-weighting matrix method has been employed provides less conservative results due to the increased efforts have been made to further reduce the conservatism inequalities for cross products terms. Recently, greater the filtering error system (7) has been obtained through the following corollary.

**Corollary 3.1.** Given scalars $\tau, \mu$ and positive scalars $\lambda_1, \lambda_2$, the filtering error system (7) is asymptotically stable with an $H_\infty$ performance $\gamma$, if there exist positive definite matrices $P_i (i = 1, 2, \ldots, 5)$ and matrices $M_m, N_m (m = 1, 2, \ldots, 5)$, $\mathcal{A}_f$, $\mathcal{B}_f (i = 1, 2, \ldots, N)$ with appropriate dimensions such that the following LMI holds:

$$
\begin{bmatrix}
\mathcal{T} & \mathcal{D}^{(1)} & \mathcal{M}^T & \mathcal{N}^T & \frac{1}{\gamma} F
\end{bmatrix} < 0,
$$

where

$$
\mathcal{F} = \begin{bmatrix}
\mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{A}_{13} & \mathcal{A}_{14} & \mathcal{A}_{15} & \mathcal{A}_{16}
\end{bmatrix},
$$

and

$$
\begin{bmatrix}
\mathcal{D}^{(1)} & \mathcal{D}^{(2)} & \mathcal{M}^T & \mathcal{N}^T & \frac{1}{\gamma} F
\end{bmatrix} < 0.
$$

Moreover, the parameters of the desired filters are given as

$$
A_f = (I \otimes P_2)^{-1} A_F, \quad B_f = (I \otimes P_2)^{-1} B_F,
$$

where $A_F = \text{diag}(A_{F1}, A_{F2}, \ldots, A_{F_N})$ and $B_F = \text{diag}(B_{F1}, B_{F2}, \ldots, B_{FN})$.

**Proof.** Substituting the values of $U_j$ ($j = 1, 2, \ldots, 5$), (7) into (8) and (9), we obtain (21) and (22), in which the parameters are defined as $(I \otimes P_2)A_F = A_F$ and $(I \otimes P_2)B_F = B_F$. Hence, if Eqs. (21) and (22) hold, the filter gain matrices are given by (23). This completes the proof.\(^4\)

**Remark 4.** To develop less conservative delay-dependent stability results, there exist various advanced methods for time-delay systems. For example, the input–output method has been employed in [38] to tackle the time-delays in the sensor networks. In [40] slack matrix method provides less conservative results due to the increased freedom of slack variables. Further, to estimate the upper bound on the derivative of Lyapunov–Krasovskii functional, free-weighting matrix method has been employed in [41,46,47] to circumvent the utilization of bounding inequalities for cross products terms. Recently, greater efforts have been made to further reduce the conservatism issue on time-delay systems. In this aspect, the reciprocal convex combination approach has been widely used in the literature [35,39] to directly relax the integral term of quadratic quantities into the quadratic term of integral quantities together with Jensen's inequality. The reciprocal convex combination lemma provides not only less conservative results but also it includes only less number of decision variables when compared to free-weighting matrix method. We extend the proposed main results by using free-weighting matrix method as employed in [41] to compare the conservativeness of reciprocal convex approach with free-weighting matrix method. Based on this, the corresponding $H_\infty$ performance analysis result for the filtering error system (7) has been obtained through the following corollary.
4. Numerical examples

To illustrate the validity of the proposed $H_{\infty}$ filtering design, three numerical examples with simulation results are provided for the delayed CDNs (1).

Example 4.1. Consider the delayed CDNs system (1) with $N=3$ and its parameters are defined as

\[
A = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 & 0.8 \\ 0.6 & 0.5 \end{bmatrix},
\]

\[
B_2 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},
\]

\[
\Gamma_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0.2 & 0.1 \\ 0.5 & 0.2 \end{bmatrix},
\]

\[
D_{13} = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.1 \end{bmatrix}.
\]

The parameters of the desired filters are defined in (23).

Proof. Choose the Lyapunov–Krasovskii functional for the system (7) as

\[
V(t, e, i) = V_1(t, e, i) + V_2(t, e, i) + V_3(t, e, i),
\]

where $V_1(t, e, i)$ and $V_2(t, e, i)$ are defined in (10) and $V_3(t, e, i)$ is defined as

\[
V_3(t, e, i) = \int_{t-\tau}^{t} \int_{t-\tau}^{t} e^{T} \bar{S} e d\theta d\theta.
\]

Then, by applying the free-weighting matrix method in the derivative of Lyapunov–Krasovskii functional $V(t, e, i)$ and following the same way of method for proving Theorems 3.1 and 3.2, one can obtain the LMI (24) in Corollary 3.1. Thus, the filtering error system (7) is asymptotically stable with $H_{\infty}$ performance $\gamma$. This completes the proof.\[\]

The estimation responses $e_i(t)$ and its estimates $\hat{e}_i(t)$ are illustrated in Fig. 1 and Fig. 2 for Example 4.1.

\[
C_1 = \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.9 & 0.4 \\ 0.3 & 0.8 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \quad D_{23} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}, \quad E = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}.
\]

The coupling matrices and the nonlinear function $f(\cdot)$ are given by

\[
W = \begin{bmatrix} -0.6 & 0.3 & 0.3 \\ 0.3 & -0.6 & 0.3 \end{bmatrix}, \quad G = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & 0.1 & -0.2 \end{bmatrix}, \quad f(x_i(t)) = \begin{bmatrix} 0.1x_{11}(t) \cos^2(x_{11}(t)) - 0.05(x_{11}(t) - x_{12}(t)) \\ -0.05x_{11}(t) - 0.1x_{12}(t) \end{bmatrix}.
\]

According to (3), the relevant parameter matrices can be chosen as

\[
F_1 = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -0.05 & 0.05 \\ 0.05 & 0.05 \end{bmatrix}, \quad F_3 = \begin{bmatrix} -0.05 & 0.05 \\ 0.05 & -0.05 \end{bmatrix}.
\]

For given $\gamma = 0.6, \tau = 0.7$ and $\mu = 0.35$, solving the LMI (24) using Matlab LMI toolbox, the desired filter matrices and the positive scalars $\lambda_1, \lambda_2$ are obtained as

\[
A_{F_1} = \begin{bmatrix} -0.5036 & -0.2360 \\ 0.2327 & -0.5009 \end{bmatrix}, \quad A_{F_2} = \begin{bmatrix} -0.5035 & -0.0457 \\ 0.0425 & -0.5008 \end{bmatrix}, \quad A_{F_3} = \begin{bmatrix} -0.5035 & -0.2065 \\ 0.2032 & -0.5009 \end{bmatrix}, \quad B_{F_1} = \begin{bmatrix} 0.0068 & 0.0030 \\ 0.0059 & 0.0029 \end{bmatrix}, \quad B_{F_2} = \begin{bmatrix} 0.0057 & 0.0023 \\ 0.0027 & 0.0018 \end{bmatrix}, \quad B_{F_3} = \begin{bmatrix} 0.0046 & 0.0033 \\ 0.0083 & 0.0031 \end{bmatrix}.
\]

$\lambda_1 = 43.6673, \quad \lambda_2 = 13.1806$.\[\]
Example 4.1. and the rest of the parameters are same as defined in Example 4.2. Consider the delayed CDNs system (1) with presented in Tables 1 and 3 respectively. The coupling matrices are given by By choosing the initial conditions $x_i(0) = [-0.4 0.4]^T$, $\dot{x}_i(0) = [0.2 -0.4]^T$, $\dot{x}_j(0) = [0.6 -0.5]^T$, $x_i(0) = [-0.5 0.4]^T$, $\dot{x}_j(0) = [0.5 -0.7]^T$ and the disturbance inputs $\nu(t) = \sin(t)e^{-2t}$, $\omega(t) = \sin(t)e^{-4t}$, the simulation results for Example 4.1 are shown in Figs. 1 and 2. Fig. 1 shows the state trajectories $x_i(t)$, its estimates $\hat{x}_i(t)$ and Fig. 2 shows the estimation errors $e_i(t)$ for $i = 1, 2, 3$. Further, for different values of $\gamma$ and $\mu$, the maximum allowable upper bound (MAUB) values of $\tau$ are presented in Tables 1 and 3 respectively.

**Example 4.2.** Consider the delayed CDNs system (1) with $N=5$ and its parameters are defined as

$$D_{14} = \begin{bmatrix} 0.2 & 0.3 \\ 0.7 & 0.3 \end{bmatrix}, \quad D_{15} = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.1 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 0.5 & 0.3 \\ 0.6 & 0.1 \end{bmatrix},$$

$$C_5 = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 0.3 \end{bmatrix}, \quad D_{24} = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.5 \end{bmatrix}, \quad D_{25} = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.2 \end{bmatrix}.$$  

The coupling matrices are given by

$$W = \begin{bmatrix} -0.8 & 0.3 & 0.3 & 0.1 & 0.1 \\ 0.3 & -0.8 & 0.3 & 0.1 & 0.1 \\ 0.3 & 0.1 & -0.8 & 0.3 & 0.1 \\ 0.1 & 0.3 & 0.1 & -0.8 & 0.3 \\ 0.1 & 0.3 & 0.1 & 0.3 & -0.8 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.5 & 0.2 & 0.1 & 0.1 & 0.1 \\ 0.2 & -0.5 & 0.1 & 0.1 & 0.1 \\ 0.2 & 0.1 & -0.5 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 & -0.5 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0.1 & -0.5 \end{bmatrix}$$

and the rest of the parameters are same as defined in Example 4.1.

By choosing the initial conditions $x_i(0) = [-0.4 0.4]^T$, $\dot{x}_i(0) = [0.2 -0.4]^T$, $\dot{x}_j(0) = [0.6 -0.5]^T$, $x_i(0) = [-0.5 0.4]^T$, $\dot{x}_j(0) = [0.5 -0.7]^T$ and the disturbance inputs $\nu(t) = \sin(t)e^{-2t}$, $\omega(t) = \sin(t)e^{-4t}$, the simulation results for Example 4.1 are shown in Figs. 1 and 2. Fig. 1 shows the state trajectories $x_i(t)$, its estimates $\hat{x}_i(t)$ and Fig. 2 shows the estimation errors $e_i(t)$ for $i = 1, 2, 3$. Further, for different values of $\gamma$ and $\mu$, the maximum allowable upper bound (MAUB) values of $\tau$ are presented in Tables 1 and 3 respectively.

Table 1

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\tau$</th>
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<tbody>
<tr>
<td>0.6</td>
<td>0.7835</td>
</tr>
<tr>
<td>0.7</td>
<td>2.1184</td>
</tr>
<tr>
<td>0.8</td>
<td>3.0830</td>
</tr>
<tr>
<td>0.9</td>
<td>3.6093</td>
</tr>
<tr>
<td>1</td>
<td>3.8806</td>
</tr>
<tr>
<td>1.1</td>
<td>4.0371</td>
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</table>

Table 2

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.4589</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8692</td>
</tr>
<tr>
<td>0.9</td>
<td>1.1902</td>
</tr>
<tr>
<td>1</td>
<td>1.4659</td>
</tr>
<tr>
<td>1.1</td>
<td>1.6740</td>
</tr>
<tr>
<td>1.2</td>
<td>1.8187</td>
</tr>
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</table>

Table 3

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<th>$\mu$</th>
<th>$\tau$</th>
</tr>
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<tbody>
<tr>
<td>0.35</td>
<td>0.6061</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5921</td>
</tr>
<tr>
<td>1</td>
<td>0.5921</td>
</tr>
<tr>
<td>10</td>
<td>0.5921</td>
</tr>
<tr>
<td>100</td>
<td>0.5921</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4589</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4589</td>
</tr>
<tr>
<td>1</td>
<td>0.4589</td>
</tr>
<tr>
<td>10</td>
<td>0.4589</td>
</tr>
<tr>
<td>100</td>
<td>0.4589</td>
</tr>
<tr>
<td>0.6</td>
<td>0.2078</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2078</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2078</td>
</tr>
<tr>
<td>0.9</td>
<td>0.2078</td>
</tr>
<tr>
<td>1</td>
<td>0.2078</td>
</tr>
</tbody>
</table>

For given $\gamma=0.7$, $\tau=0.4$ and $\mu=0.2$, solving the LMIIs in Theorem 3.2 using Matlab LMI toolbox, the desired filter matrices and the positive scalars are obtained as

$A_{F1} = \begin{bmatrix} -0.5010 & -0.6034 \\ 0.0626 & -0.5002 \end{bmatrix}$

$A_{F2} = \begin{bmatrix} -0.5010 & -0.0346 \\ 0.0338 & -0.5002 \end{bmatrix}$

Fig. 3. The state trajectories $x_i(t)$ and its estimates $\hat{x}_i(t)$ ($i = 1, \ldots, 5$) for Example 4.2.

Fig. 4. The estimation error responses $e_i(t)$ ($i = 1, \ldots, 5$) for Example 4.2.

Fig. 5. The structure of BA scale-free complex networks with five nodes.
Example 4.3.

Remark 6. It can be analyzed from Table 4 that the maximum value of $\gamma$ obtained from Theorem 3.2 via reciprocal convex approach is 0.7385, whereas the maximum value of $\gamma$ obtained from Corollary 3.1 via free-weighting matrix method is 0.6061. This shows that the proposed delay-dependent stability conditions provide less conservative results. Similarly, from Table 4, it can be analyzed that reciprocal convex approach is less conservative than the free-weighting matrix method.

Example 4.3. Many real-world complex networks can be successfully modeled as scale-free networks [48,49]. In the sequel, we consider a scale-free network consisting of five dynamical nodes and the coupling topology are randomly generated by a BA scale-free model from Fig. 5, where each node is a continuous-time dynamical system (1) and its parameters are defined as

$$A = \begin{bmatrix} -0.9 & 0.26 \\ 0.31 & -0.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.25 \\ 0.35 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.14 \\ 0.45 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.12 \\ 0.16 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.19 \\ 0.13 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.13 \\ 0.16 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 0.55 \\ 0.6 \end{bmatrix}, \quad C_5 = \begin{bmatrix} 0.14 \\ 0.25 \end{bmatrix}.$$

$$D_{11} = \begin{bmatrix} 0.13 \\ 0.11 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0.2 \\ 0.25 \end{bmatrix},$$

$$D_{13} = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}, \quad D_{14} = \begin{bmatrix} 0.25 \\ 0.27 \end{bmatrix},$$

$$D_{15} = \begin{bmatrix} 0.6 \\ 0.45 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix},$$

$$D_{22} = \begin{bmatrix} 0.55 \\ 0.3 \end{bmatrix}, \quad D_{23} = \begin{bmatrix} 0.35 \\ 0.13 \end{bmatrix},$$

$$D_{24} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \quad D_{25} = \begin{bmatrix} 0.6 \\ 0.45 \end{bmatrix}.$$
The coupling matrices are given by
\[
W = G = \begin{bmatrix}
-2 & 1 & 0 & 0 & 1 \\
1 & -3 & 1 & 0 & 1 \\
0 & 1 & -2 & 1 & 0 \\
0 & 0 & 1 & -2 & 1 \\
1 & 1 & 0 & 1 & -3
\end{bmatrix},
\]
and the nonlinear function is assumed to be same as defined in Example 4.1.

For given \( \gamma = 0.7 \), \( \tau = 0.4 \) and \( \mu = 0.6 \), solving the LMIs in Theorem 3.2 using Matlab LMI toolbox, the desired filter matrices and the positive scalars are obtained as
\[
A_{F1} = \begin{bmatrix}
-0.5080 & 3.3823 \\
-3.3740 & -0.5027
\end{bmatrix}, \quad A_{F2} = \begin{bmatrix}
-0.5080 & 2.5119 \\
-2.5036 & -0.5027
\end{bmatrix}, \\
A_{F3} = \begin{bmatrix}
-0.5082 & 3.4571 \\
-3.4490 & -0.5028
\end{bmatrix}, \quad A_{F4} = \begin{bmatrix}
-0.5080 & 1.2384 \\
-1.2301 & -0.5027
\end{bmatrix}, \\
A_{F5} = \begin{bmatrix}
-0.5080 & 5.1832 \\
-5.1750 & -0.5028
\end{bmatrix}, \quad B_{F1} = \begin{bmatrix}
0.0084 & -0.0045 \\
0.0116 & -0.00074
\end{bmatrix}, \\
B_{F2} = \begin{bmatrix}
0.0134 & -0.0140 \\
0.0009 & 0.0083
\end{bmatrix}, \quad B_{F3} = \begin{bmatrix}
0.0095 & -0.0043 \\
0.0138 & -0.0137
\end{bmatrix}, \\
B_{F4} = \begin{bmatrix}
0.0167 & 0.0121 \\
0.0297 & -0.0180
\end{bmatrix}, \quad B_{F5} = \begin{bmatrix}
0.0972 & -0.1780 \\
-0.0362 & 0.0842
\end{bmatrix}.
\]
\( \lambda_1 = 8.5372, \quad \lambda_2 = 4.2713. \)

By choosing the initial conditions \( x_1(0) = [-0.5, 0.1]^T, \ x_2(t) = [0.5, 0.5]^T, \ x_3(t) = [-0.2, 0.5]^T, \ x_4(t) = [0.8 - 0.2]^T, \ x_5(t) = [-0.6, 0.3]^T, \ x_6(t) = [4.1 - 0.6]^T, \ x_7(t) = [-0.9, 0.5]^T, \ x_8(t) = [0.6 - 0.9]^T, \ x_9(t) = [-1.1, 0.4]^T, \ x_{10}(t) = [0.7, -1.1]^T \) and the disturbance inputs as \( \nu_i(t) = \sin(t)e^{-2t}, \ \omega_i(t) = \sin(t)e^{-0.9t} \), the simulation results for Example 4.3 are shown in Figs. 6 and 7. Fig. 6 shows the state trajectories \( x_i(t) \), its estimates \( \hat{x}_i(t) \) and Fig. 7 shows the estimation errors \( e_i(t) \) for \( i = 1, \ldots, 5 \).

**Remark 7.** The numerical complexity of the derived results is closely related to number of decision variables and the dimension of the LMIs involved. It can be calculated that the number of decision variables involved in Theorem 3.2 via reciprocal convex approach is \( \frac{1}{2}(5n(n + 1)) + n^2(1 + 2N) + 2 \), where \( n \) represents the size of the system matrices and \( N \) represents the number of nodes. In order to compare the computational complexity with other existing methods, the number of variables involved via free-weighting matrix method is calculated as \( \frac{1}{2}4n(n + 1) + n^2(10 + 2N) + 2 \), which leads to increase the number of decision variables.

**Remark 8.** To illustrate the advantage of employing reciprocal convex approach with respect to computational complexity, Table 5 lists the number of decision variables involved for various values of \( n \) and \( N \). It can be observed that for \( n = 2, N = 3 \), the number of variables by reciprocal convex approach is 45 and by free-weighting matrix method it is 78, which increases the computation time while increasing the number of decision variables. Also, it can be analyzed that when the size of \( n \) gets increases, the number of variables is drastically increasing via free-weighting matrix method and the computation time becomes heavier due to the increase in the dimension of the LMIs. Hence, reciprocal convex combination approach provides less conservative result with much less number of decision variables.

### 5. Conclusions

In this paper, a \( H_{\infty} \) filtering design is proposed for a class of continuous-time CDNs. The system under consideration includes time-varying delays in the coupling nodes and in the nonlinear function. The novel asymptotic stability conditions have been derived for the filtering error system which satisfies the minimum \( H_{\infty} \) performance criteria by utilizing Lyapunov–Krasovskii functional, reciprocal convex combination lemma and the Kronecker product techniques. Then, the desired filter has been established. Finally, numerical examples have been demonstrated to show the effectiveness of the proposed theoretical results. It is well known that in actual systems, complex networks sometimes have finite modes switching from one to another with certain transition rate governed by Markov processes. Hence, in future we will concentrate to extend the proposed main results for filtering analysis of CDNs with Markovian jump parameters and stochastic packet dropouts to develop new \( H_{\infty} \) filter design for CDNs.

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