Fracture characterization of ceria partially stabilized zirconia using the GMTSN criterion

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ABSTRACT

Ceria stabilized tetragonal zirconia polycrystal (CeO2-TZP) is one of the zirconia-based materials which is reported to have high fracture toughness and strong resistance to low temperature aging degradation (LTAD). Owing to the brittle behavior of CeO2-TZP, its fracture properties can be estimated using the concept of linear elastic fracture mechanics (LEFM). In this paper, the generalized maximum tangential strain (GMTSN) criterion is applied to predict the fracture initiation angles and the onset of fractures of CeO2-TZP disk specimens under mixed mode I/II conditions. It is found that the inclusion of T-term, the first non-singular term in strain solution, in the GMTSN criterion would yield significantly improved predictions of the experimental data obtained for CeO2-TZP disk specimens and reported in the literature.

1. Introduction

In recent years, there have been a growing interest in using ceramic materials for engineering purposes, thanks to their special characteristics like high hardness and resistance against abrasion, corrosion, electrical flow, and temperature [1]. High chemical stability and strong chemical bonding are among other properties of ceramics which make them more brittle than metallic and/or organic materials [2]. The brittleness along with low fracture toughness and poor reliability are among the major concerns when ceramic components are exposed to brittle fracture [1]. These drawbacks would consequently result in the limited applications of ceramic materials, specially in the biomedical field [3]. In order to overcome these issues, the main development in the field of high-tech ceramics in the past two decades were focused on both strengthening and toughening, which are vitally important to produce load-bearing and flaw-tolerant products [4].

To deal with ceramic materials, one should be aware of their different categorization and particular features. Clapp and Clark [5] presented a versatile classification for ceramic materials based on their chemical bonds. This classification consists of: silicate ceramics (mixed bond), oxide ceramics (predominantly ionic bond): single oxide and oxide combinations, glass ceramics, composites with ceramic matrix and finally materials containing metallic bonding. One of the well-known examples of single-oxide ceramics is zirconia (ZrO2), which to date has attracted a lot of attention as an engineering material. Zirconia has the inherent properties of hardness, wear resistance, low coefficient of friction, low thermal conductivity and high melting temperature [6]. According to these desired characteristics, zirconia is thus utilized for producing several medical and engineering components like prostheses [3,4], machine parts, high temperature insulating parts, engine components, probes, sensors, knives, sliding plates, etc. [5] and is expected to be used more in high-tech utilities in the future.
One of the major issues regarding ceramic materials, including zirconia, is insufficient fracture toughness; for instance, single phase ceramics like alumina or silicon carbide have a fracture toughness of only 2–4 MPa \(\sqrt{m}\) [7]. A practical solution to this problem is dispersion of reinforcement elements such as fibres, whiskers, rods, platelets, metal particles or secondary ceramic phases spread in the brittle matrix. These elements provide the ceramics with extra toughness by micromechanisms such as crack bridging, crack deflection, microcracking or as an interesting property of zirconia toughness enhancement by transformation toughening [8]. The mechanism of transformation toughening is to increase the fracture toughness of a material through a phase transformation process, from tetragonal to monoclinic, which occurs at the tip of an advancing crack [8–16]. This monoclinic phase of pure zirconia could be found at room temperature [17], while the tetragonal phase is attained when the temperature is elevated to 1170°. At the same time, we could also have stable tetragonal phase at room temperature by adding a stabilizer such as \(Y_2O_3\), CeO\(_2\), MgO or CaO [18–20]. Afterwards, to have transformation toughening occurred through tetragonal-to-monoclinic phase transformation, one should subject zirconia to tensile stress which results in a volume increase of 3–5%. In this manner, the compressive stress at the vicinity of crack tip increases and further crack growth would eventually be prevented [6].

Today, there are several zirconia–based materials and composites among which the largest group includes partially stabilized zirconia, or more specifically, tetragonal zirconia polycrystals (TZP) doped with the most popular stabilizing agents, namely yttrium (Y) and cerium (Ce). Ceria stabilized tetragonal zirconia polycrystals (CeO\(_2\)-TZP) are believed to have high fracture toughness and remarkable durability against hydrothermal degradation (aging) [21–28]. However, due to its larger grain size and higher tendency to stress induced phase transformation, CeO\(_2\)-TZP is reported to have inferior strength and hardness properties when compared to yttria stabilized tetragonal zirconia polycrystals (Y-TZP) [4,26]. As an example, CeO\(_2\)-TZP containing 8–12 mol% CeO\(_2\) [29] manifests a relatively high fracture toughness of 10–20 MPa\(\sqrt{m}\), but at the same time exhibits a moderate hardness of 8 GPa and a modest strength of 600–800 MPa, even for the most excellent content of 12 mol% CeO\(_2\). These properties are severely affected by the grain size and the percentage of CeO\(_2\) in a way that even a small drop in the CeO\(_2\) content would result in the reduced stability of the retained tetragonal phase [30,31] and therefore the propensity to further crack growth would be increased.

Manufacturing, machining processes and mechanical/thermal loads are the most possible reasons behind crack nucleation in ceramic materials. Moreover, the preexisting pores and flaws would act as stress concentrators which might lead to more vulnerability to further crack growth. To analyze the strength of ceramic materials against crack propagation, a fundamental parameter known as fracture toughness is defined [1]. To be more specific, mode I fracture toughness (\(K_I\)) is the most popular measure for quantifying the fracture resistance of structural ceramics and is reported to have profound impact on developing tougher materials [32].

The aim of the present study is to scrutinize the mixed-mode fracture properties of CeO\(_2\)-TZP using a strain based fracture model called the generalized maximum tangential strain (GMTSN) criterion [33]. The analytical data are presented and validated using the experimental data reported for disk specimens [32,34]. It is shown that the GMTSN criterion, which includes the effect of nonsingular strain term, i.e. T-term, as well as singular terms, exhibits significantly improved agreement with the mixed-mode experimental data obtained for CeO\(_2\)-TZP. In the upcoming section, a review of the GMTSN criterion is presented.

2. The GMTSN criterion

Brittle fracture is the most frequent mode of failure in ceramic materials. To deal with this type of failure, the concept of linear elastic fracture mechanics (LEFM) was proposed, which mainly takes account of the specimens having inconsiderable amount of plastic deformations around the crack tip.

In the context of LEFM, the elastic stresses near the crack tip for mixed mode I/II loading can be written as a series expansion given by Williams [35] as

\[
\sigma_r = \frac{1}{\sqrt{2\pi\tau}} \cos(\theta/2) \left( K_I (1 + \sin^2(\theta/2)) + K_{II} \left( \frac{3}{2} \sin\theta - 2\tan(\theta/2) \right) \right) + T \cos^2\theta + O(r^{1/2})
\]

(1)

\[
\sigma_{\theta} = \frac{1}{\sqrt{2\pi\tau}} \cos(\theta/2) \left( K_I \cos^2(\theta/2) - \frac{3}{2} K_{II} \sin\theta \right) + T \sin^2\theta + O(r^{1/2})
\]

(2)

\[
\sigma_{\theta} = \frac{1}{\sqrt{2\pi\tau}} \cos(\theta/2) \left( K_I \sin\theta + K_{II} (3\cos\theta - 1) \right) - T \sin\theta \cos\theta + O(r^{1/2})
\]

(3)

where \(r\) and \(\theta\) are the components of polar coordinates and \(\sigma_r\), \(\sigma_{\theta}\) and \(\sigma_{\theta}\) are the crack-tip stresses expressed in the polar co-ordinates. \(K_I\) and \(K_{II}\) are the mode I and mode II stress intensity factors, respectively, and finally \(T\) which is a constant term parallel to the crack flanks, is called the T-Stress. The higher order terms \(O(r^{1/2})\) could be considered to be negligible when \(r\) approaches zero near the crack tip.

The maximum tangential strain (MTSN) criterion proposed by Chang [36] in 1981 is a strain-based fracture criterion which only takes the singular strain terms into account. In 2001, Ayatollahi and Abbasi [33] suggested the GMTSN criterion which not only considers the singular strains, but also includes the effect of the first nonsingular term, T-strain. They found that the inclusion of T-strain could significantly influence both the fracture initiation angle and the onset of fracture, thus providing better agreement with experimental results. In 2015, Mirmayar [37] made use of a similar criterion to predict the fracture behavior of PMMA specimens. Then in 2017, Hua et al. [38] also employed the mentioned criterion to assess the crack initiation angle and onset of fracture of central cracked Brazilian disks made of rocks and graphite. More recently, Dai et al. [39] applied this GMTSN criterion to evaluate the consistency between the fracture toughness of their newly proposed rock specimen and ISRM-suggested ones.
To proceed with the GMTSN criterion in plane stress condition, the tangential strain near the crack tip is required to be calculated using the Hooke’s law

$$\varepsilon_{\theta\theta} = \frac{1}{E} (\sigma_{\theta\theta} - \nu \sigma_r)$$  \hspace{1cm} (4)

in which $\varepsilon_{\theta\theta}$, $E$ and $\nu$ are the tangential strain, Young’s modulus and Poisson’s ratio, respectively. By substitution of $\sigma_r$ and $\sigma_{\theta\theta}$ from Eqs. (1) and (2) into Eq. (4), one obtains $\varepsilon_{\theta\theta}$ as follows

$$\varepsilon_{\theta\theta} = \frac{1}{4E\sqrt{2\pi r}} \left[ ((3-5\nu)\cos(\theta/2) + (1+\nu)\cos(3\theta/2)) K_I + ((5\nu-3)\sin(\theta/2) - 3(1+\nu)\sin(3\theta/2)) K_{II} \right]$$

$$- \frac{1}{2E} ((1+\nu)\cos(2\theta) + (\nu-1)) T + O(r^2)$$

$$\hspace{1cm} (5)$$

According to the GMTSN criterion, crack growth initiates radially from the crack tip, at an angle $\theta_0$, where the tangential strain is maximum. In addition, the onset of crack growth occurs when the tangential strain ($\varepsilon_{\theta\theta}$) along $\theta_0$ at a critical distance $r_c$ from the crack tip, reaches a critical value $\varepsilon_{\theta\theta_c}$. It should be noted that $\varepsilon_{\theta\theta_c}$ and $r_c$ are material constants and they are independent of the specimen geometry or loading configurations. Neglecting the higher order terms, the fracture initiation angle of the GMTSN criterion is determined as follows

$$\frac{\partial\varepsilon_{\theta\theta}}{\partial \theta} \bigg|_{\theta=\theta_0} = 0 \hspace{0.5cm} \text{and} \hspace{0.5cm} \frac{\partial^2\varepsilon_{\theta\theta}}{\partial \theta^2} \bigg|_{\theta=\theta_0} < 0 \hspace{0.5cm} \text{First derivative}$$

$$+ ((5\nu-3)\cos(\theta_0/2) - 9(1+\nu)\cos(3\theta_0/2)) K_{II} + 8(1+\nu)\sin(2\theta_0) \sqrt{2\pi r_c} T = 0$$

$$\hspace{1cm} (6)$$

Defining the effective stress intensity factor as $K_{eff} = \sqrt{K_I^2 + K_{II}^2}$, the biaxiality ratio as $B = T_{\sqrt{2\pi r}} / K_{eff}$ and $\alpha = \sqrt{2r_c/\alpha}$, one could substitute $\sqrt{2\pi r_c} T$ with $B\alpha K_{eff}$ in Eq. (6). Then, by dividing the whole expression to $K_{II}$ and defining the mixity parameter as $M_e = 2\pi \tan^{-1}(K_I/K_{II})$ which varies from 0 (pure mode II) to 1 (pure mode I), the final expression of crack initiation angle would be simplified as

$$\varepsilon_{\theta\theta} = \frac{1}{4E\sqrt{2\pi r}} \left[ ((5\nu-3)\sin(\theta_0/2) - 3(1+\nu)\sin(3\theta_0/2)) K_I ight]$$

$$+ ((5\nu-3)\cos(\theta_0/2) - 9(1+\nu)\cos(3\theta_0/2)) K_{II} + 8(1+\nu)\sin(2\theta_0) B\alpha \hspace{1cm} (7)$$

Eq. (7) expresses $\theta_0$ (in radians) for different values of $M_e$ and $B\alpha$ and also shows its dependency on the Poisson’s ratio $\nu$. Fig. 1 shows the variation of $\theta_0$ versus the mixity parameter $M_e$ for different values of $B\alpha$ when $\nu = 0.3$ in plane stress condition. It is seen that compared with the case where $T$ or $B\alpha$ is zero, positive values of $B\alpha$ increase the magnitude of $\theta_0$ while negative values of $B\alpha$ decrease $\theta_0$. It should also be noted that for a constant crack length, the value of $B\alpha$ varies with different combinations of mode I and mode II [38]. However, the curves shown in Figs. 1 and 2 have been plotted generally for given constant values of $B\alpha$.

Now that $\theta_0$ is determined, the fracture initiation condition of the GMTSN criterion could be derived as

![Fig. 1. Effect of the T-term on the fracture initiation angle for $\nu = 0.3$ based on the GMTSN criterion.](image-url)
It is more appropriate to write the critical strain \( \varepsilon_{\theta_0} \) in terms of the mode I fracture toughness \( K_{II} \). To do so, one could simplify Eq. (8) for pure mode I, for which \( \theta_0 = 0 \), \( K_{II} = 0 \) and \( K_I = K_{II} \) that is the fracture toughness determined from standard fracture tests. Evaluating Eq. (8) for pure mode I gives

\[
\varepsilon_{\theta_0} = \varepsilon_{\theta_0} - \frac{1}{2E\pi r \sqrt{\nu}} \left[ \left( \frac{3-5v}{2}\cos\left(\frac{\theta_0}{2}\right) + (1 + v)\cos\left(3\theta_0/2\right) \right) K_I + \left( \frac{5v-3}{2}\sin\left(\frac{\theta_0}{2}\right) - 3(1 + v)\sin\left(3\theta_0/2\right) \right) K_{II} \right] 
- \frac{1}{2E} \left[ \left(1 + v\right)\cos\left(2\theta_0\right) + \left(v-1\right) T \right] = \varepsilon_{\theta_0}
\]

Eq. (9) shows that according to the GMTSN criterion, \( K_{II} \) depends on the T-term. To obtain a reference value for fracture toughness, the fracture test can be carried out on cracked specimens for which T-stress is zero. For example, T is zero for a single-edge-notched bend specimen with a crack length to specimen width ratio \( a/W \) of 0.35 or the three-point bend specimen with \( a/W = 0.4 \) [40]. Fracture toughness obtained from such tests is denoted here by \( K_{II}^0 \) and Eq. (9) is then rewritten as

\[
\varepsilon_{\theta_0} = \varepsilon_{\theta_0} - \frac{1}{2E\pi r \sqrt{\nu}} \left[ \left( \frac{3-5v}{2}\cos\left(\frac{\theta_0}{2}\right) + (1 + v)\cos\left(3\theta_0/2\right) \right) K_I + \left( \frac{5v-3}{2}\sin\left(\frac{\theta_0}{2}\right) - 3(1 + v)\sin\left(3\theta_0/2\right) \right) K_{II} \right] 
- \frac{1}{2E} \left[ \left(1 + v\right)\cos\left(2\theta_0\right) + \left(v-1\right) T \right] = \varepsilon_{\theta_0}
\]

According to Eq. (11), fracture occurs when the left hand side of this equation is equal to or greater than \( K_{II}^0 \). Finally, Eq. (11) is solved to find the ratios \( K_I/K_{II}^0 \) and \( K_I/K_{II}^0 \) with respect to \( M_e \) and \( B_\alpha \), and thus the onset of fracture is determined from Eqs. (12) and (13) as

\[
K_{II} = \frac{4\left(1-v\right)\cos\left(M_e\pi/2\right)}{\left( \frac{3-5v}{2}\cos\left(\frac{\theta_0}{2}\right) + (1 + v)\cos\left(3\theta_0/2\right) \right) \sin\left(M_e\pi/2\right) + \left( \frac{5v-3}{2}\sin\left(\frac{\theta_0}{2}\right) - 3(1 + v)\sin\left(3\theta_0/2\right) \right) \cos\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_e\pi/2\right) + 4\left(1 + v\right)\cos^2\theta_0\sin\left(M_e\pi/2\right) \sin\left(M_
(horizontal axis), fracture toughness is increased by increasing $T$. Opposite occurs for pure mode II (vertical axis) where fracture toughness is lower for specimens having positive $T$. There is an intermediate loading condition between mode I and mode II where the effect of $T$-term changes. The transition points in these curves depend on the magnitude of $B\alpha$; for example, for $B\alpha = -0.2$, the $T$-term effect changes when $K_I/K_{II}^0$ is about 0.77, while for $B\alpha = 0.2$ this change happens when $K_I/K_{II}^0$ is approximately 0.93.

3. Fracture characterization of CeO$_2$-TZP

Fracture properties of CeO$_2$-TZP could be analyzed either from the experimental or theoretical points of view:

3.1. The experimental standpoint

Singh and Shetty [32,34] conducted a series of fracture tests on precracked disk specimens made of CeO$_2$-TZP for three different grain sizes including 1.6, 3.6 and 6.7 $\mu$m, for which the properties are shown in Table 1. Based on Singh and Shetty [34] explanations, there are a few steps to be taken to execute fracture tests on ceramic materials. Here a summary of these preparation and testing steps for CeO$_2$-TZP disk specimens with grain size 3.6 $\mu$m is reviewed:

(a) Fabrication of the chevron notched disk specimens

At first, ZrO$_2$ powder with 12 mol% CeO$_2$ was milled with ZrO$_2$ milling media for 48 h. Then, the dried powder was sieved through a 65-$\mu$m screen, die pressed at 41.4 MPa to form the disks, and subsequently isostatically pressed at 207 MPa. Afterwards, the CeO$_2$-TZP disks were presintered at 750–800 °C and then chevron notched using high-speed steel cutting blades 25 mm in diameter and 0.2 mm in thickness. Eventually, the final sintering of the ceramic specimens was done at 1550 °C for 2 h. Fig. 3 shows a schematic of a prepared ceramic disk.

(b) Precracking and testing of the disk specimens

After the preparation of disk specimens, precracking was done by loading the disks in mode I (i.e., along a diameter through the chevron notch) to about 95% of the critical load. The specimens were subjected to this load for 2–6 min. and then unloaded. The precracks in the disk specimens were found to be stable for the crack length $a \leq a_1$, while unstable crack growth was observed for crack length $a > a_1$. Finally, the precracking was accomplished to obtain a crack length $a$ close to $a_1$ (but not greater than $a_1$).

In order to perform the mixed-mode fracture tests, the disk specimen should be reoriented in a way that the direction of diametral compression aligns with the angle $\alpha$ with respect to the preexisting notch, as shown in Fig. 4. In this configuration, the crack inclination angle $\alpha$ could be varied from 0° (i.e. pure mode I) to about 23° (i.e. pure mode II) to conduct different mixed-mode fracture tests.

(c) Fracture surface of CeO$_2$-TZP

After performing the experimental tests on CeO$_2$-TZP disk specimens, fracture surfaces could be analyzed using fractographic

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Table 1
Geometrical properties of the disk specimens made of CeO$_2$-TZP.

<table>
<thead>
<tr>
<th>Grain size ($\mu$m)</th>
<th>$R$ (mm)</th>
<th>$t$ (mm)</th>
<th>$a_0$ (mm)</th>
<th>$a_1$ (mm)</th>
<th>$a \approx a_0 + 0.9 \times (a_1 - a_0)$</th>
<th>$a/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6 [32]</td>
<td>15.5</td>
<td>2.2</td>
<td>2.6</td>
<td>6</td>
<td>5.66</td>
<td>0.36</td>
</tr>
<tr>
<td>3.6 [34]</td>
<td>11.18</td>
<td>2.54</td>
<td>2.65</td>
<td>5.38</td>
<td>5.10</td>
<td>0.46</td>
</tr>
<tr>
<td>6.7 [32]</td>
<td>15.5</td>
<td>2.2</td>
<td>2.6</td>
<td>6</td>
<td>5.66</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Fig. 3. Geometrical features of the chevron notched disk specimen.
techniques. According to Singh and Shetty [34], the intergranular fracture is the predominant failure mode when the specimens are subjected to pure mode I loading. On the contrary, the mode of failure under pure mode II loading is reported to be preponderantly transgranular and thus grain interlocking and abrasion would arise, which are believed to be major reasons for increased mode II fracture toughness.

3.2. The theoretical standpoint

Ceramic materials, including CeO₂-TZP, are mostly recognized as brittle materials. According to the previous section, LEFM along with its related criteria, are the underlying concept when dealing with brittle fracture problems. One of the well-known and versatile criteria in this field is known to be the GMTSN criterion, which has recently gained a lot of attention.

For different brittle materials, including ceramics, the GMTSN criterion reviewed in Section 2 can be employed to extract the fracture properties, namely the angle and the onset of fracture growth. To proceed with the crack growth angle, Eq. (6) can be solved to find \( \theta_0 \) in terms of \( K_I, K_{II}, T, \gamma, \) and \( \nu \). On the other hand, Eq. (11) can be modified by dividing \( K_{eff} \) to its both sides and using the dimensionless parameter \( B\alpha = \sqrt{2\pi\gamma T/K_{eff}} \) to find the onset of fracture as

\[
\frac{K_{eff}}{K_{eff}^0} = \frac{4(1-\nu)\sqrt{K_{eff}^2 + K_{II}^2}}{(3-5\nu)\cos(\theta_0/2) + (1+\nu)\cos(3\theta_0/2)K_I + ((5\nu-3)\sin(\theta_0/2)-3(1+\nu)\sin(3\theta_0/2))K_{II} + 4(1+\nu)\cos^2(\theta_0)\sqrt{2\pi\gamma T}}
\]

in which \( K_{eff}/K_{II}^0 \) is the ratio of effective stress intensity factor to the reference value of mode I fracture toughness, and is defined as a function of \( \theta_0, K_I, K_{II}, T, \gamma, \) and \( \nu \). As indicated earlier, mode-I precracking is done on the specimens prior to mixed mode fracture tests, which provides a crack length \( a \) very close to \( a_1 \); this could be interpreted as \( a \approx a_1 \) and thus the fracture parameters \( K_I, K_{II} \) and \( T \) in the cracked Brazilian disk (BD) specimen could approximately be used in Eq. (14) as

\[
K_I = Y_I \cdot \frac{P}{\pi R t} \sqrt{\pi a}
\]

\[
K_{II} = Y_{II} \cdot \frac{P}{\pi R t} \sqrt{\pi a}
\]

\[
T = T^* \cdot \frac{P}{\pi t (R-a)}
\]

where \( Y_I, Y_{II}, \) and \( T^* \) are the normalized forms of \( K_I, K_{II}, \) and \( T \), respectively, and are functions of the crack length ratio \( a/R \) and the crack inclination angle \( \alpha \). The variations of \( Y_I, Y_{II}, \) and \( T^* \) versus \( M_e \) have been derived in [1] for the BD specimen in different crack length ratios \( a/R \). Here the respective curves for \( a/R = 0.36 \) and 0.46 are interpolated (using the original graphs in [1]) and the results are demonstrated in Fig. 5.
Substitution of Eqs. (15)–(17) into Eq. (6) gives the angle $\theta_0$ for the disk specimen in terms of $Y_I$, $Y_{II}$, and $T^*$ as

$$-\nu Y_0 + Y_0 + (5\nu - 3)\sin(3\theta_0/2) - 3(1 + \nu)\sin(3\theta_0/2)Y_I + ((5\nu - 3)\cos(3\theta_0/2) - 9(1 + \nu)\cos(3\theta_0/2))Y_{II} + 8(1 + \nu)\sin(2\theta_0) - \frac{2\nu}{a} R = 0$$

and Eq. (14) can also be modified in the same manner to determine the onset of fracture as

$$K_{eff} = \frac{4(1-\nu)\sqrt{Y_I^2 + Y_{II}^2}}{((3-\nu)\cos(\theta_0/2) + (1 + \nu)\cos(3\theta_0/2))Y_I + ((5\nu - 3)\sin(\theta_0/2) - 3(1 + \nu)\sin(3\theta_0/2))Y_{II} + 4(1 + \nu)\cos^2(\theta_0) - \frac{2\nu}{a} R = 0}$$

For a specific mode mixity $M_e$, the angle $\theta_0$ is calculated from Eq. (18) and then is replaced into Eq. (19) to find the ratio $K_{eff}/K_0^0$ for the disk specimen. In the next section, these equations are employed to find the fracture properties of CeO$_2$-TZP disk specimens analytically.

4. Results and discussion

According to the previous section, $r_c$ and $\nu$ are among the key parameters for determining the angle and the onset of fracture from the GMTSN criterion. The critical distance $r_c$ represents the radius of process zone around the crack tip inside which the material is damaged owing to localized large strains. For ceramic materials, the process zone is usually referred to an area containing numerous microcracks [1]. The more the applied load is increased, the denser the microcracks ahead of the initial crack will be, until the level that the process zone is fully saturated and then brittle fracture commences. As stated in some research papers (such as [41]), direct relations have been observed between the size of critical distance $r_c$ and the material’s grain size for some brittle or quasi-brittle materials. Ayatollahi and Aliha [1] found an empirical relation between the value of $r_c$ and the grain size of ceramic materials which reveals the size of $r_c$ to be approximately 100 times the average grain size of the studied ceramics. Based on their finding, for three grain sizes 1.6, 3.6 and 6.7 $\mu$m of CeO$_2$-TZP, the values of $r_c$ are about 0.16, 0.36 and 0.67 mm, respectively.

The other important parameter for fracture studies based on the GMTSN criterion is the Poisson’s ratio $\nu$. As suggested by Singh and Shetty [32,34], who performed several fracture experiments on ceramics, the Poisson’s ratio of CeO$_2$-TZP specimens could be taken approximately equal to 0.22; accordingly, the same value is also used in the calculations of the present paper.

Now that $r_c$ and $\nu$ are known for CeO$_2$-TZP specimens, one can determine $\theta_0$ and $K_{eff}/K_0^0$ from Eqs. (18) and (19) using Fig. 5 which demonstrates the variation of fracture parameters $Y_I$, $Y_{II}$, and $T^*$ with respect to $M_e$. In the following figures, the analytical fracture data based on the GMTSN criterion are plotted along with the experimental results reported by Singh and Shetty [32,34]. Figs. 6–8 show the fracture data for the disk specimens made of CeO$_2$-TZP with grain sizes 1.6, 3.6 and 6.7 $\mu$m, respectively. In each
The fracture graph of the GMTSN criterion is separated from the ones of classical criteria, namely the MTS and the MTSN criteria. The reason behind this separation is the difference in the vertical axes of the mentioned graphs; in other words, the fracture graphs related to the GMTSN criterion are plotted as $K_{eff}/K_{If}$ versus $M_e$, while for the MTS and the MTSN criteria, $K_{eff}/K_{If}$ is presented versus $M_e$. Converting the vertical axis from $K_{eff}/K_{If}$ to $K_{eff}/K_{If0}$ will cause the experimental data to be shifted up/down by a coefficient which is calculated as follows. Firstly, the equality $\sqrt{2\pi\epsilon} T = B_1 K_{eff}$ should be used in Eq. (8) and the resulting expression must then be evaluated for pure mode I condition in which $B = B_1$ (i.e. the biaxiality ratio for pure mode I) and $K_{eff} = K_{If}$. Secondly, $B_1 a$ should be substituted by $\sqrt{2\pi\epsilon} T_{I0}/K_{I0}$ in which $T_{I0}$ and $K_{I0}$ are the values of T-term and mode I stress intensity factor at $M_e = 1$ (i.e. pure mode I); this leads to

$$\frac{K_{If}}{E_{If}^{\pi r(1-v^2)}} = \epsilon_0$$

(20)

Thirdly, one should substitute Eqs. (15) and (17) into Eq. (20) and finally equate it with Eq. (10) which yields the relation between $K_{If}$ and $K_{If0}$ as

$$K_{If0}^2 = \frac{1}{(1-v)^3} \left( 1 - \sqrt{\frac{2\pi\epsilon}{a}} \frac{T_{I0}}{Y_{I0}} \frac{R}{R-a} \right) K_{If}$$

(21)

in which $T_{I0}$ and $Y_{I0}$ are the normalized forms of $T_{I0}$ and $K_{I0}$ and are obtained from Fig. 5 at $M_e = 1$. Thus, it is not suitable to demonstrate the fracture curves of the GMTSN criterion together with other classical criteria in a single graph.

It should also be noted that the experimental data for the fracture initiation angle was only reported for the 3.6 μm grain size ceramic [34]. Therefore, the predictions for crack growth angle are only presented for this specific grain size and the results are shown in Fig. 7.

According to the obtained results, the GMTSN criterion is found to have significantly better agreement with the mixed mode experimental data, comparing to the MTS and the MTSN criteria. This improved behavior could be attributed to the consideration of T-term in addition to $K_I$ and $K_{II}$ in the mathematical formulation, which greatly enhances the accuracy of analytical estimations and reduces the error to its minimum.

However, it is observed in almost all of the fracture curves that there is an increase in the value of experimental data close to pure mode II region, which is not covered by any criteria. According to the fractographies of Singh and Shetty [34], the increase in the pure mode II fracture toughness can be mainly due to grain interlocking and abrasion arising from transgranular fracture in pure mode II. Notwithstanding the foregoing, considering the T-term in the mathematical formulation of the GMTSN criterion is shown to have adequately compensated for the increased $K_{If}$.

As a final note, it would be very useful to investigate the fracture properties of other brittle materials like PUR [42] using the same criterion or similar criteria such as the maximum tangential strain energy density criterion.

5. Conclusions

(1) Fracture properties of CeO$_2$-TZP disk specimens were investigated using the generalized maximum tangential strain criterion first proposed by Ayatollahi and Abbasi. The considerable influence of T-term on both the direction of fracture initiation and the onset of fracture was also discussed. In fact, based on the generalized maximum tangential strain criterion, the crack growth angle increases with increasing T and decreases with decreasing T. Moreover, the mode I fracture toughness obtained from the GMTSN criterion becomes larger for positive values of T, while the opposite occurs for pure mode II.

Fig. 6. Mixed mode fracture of CeO$_2$-TZP with grain size 1.6 μm tested by Singh and Shetty [32] using the precracked disk specimen: (A) fracture curve based on the GMTSN criterion (B) fracture curves based on the MTS and the MTSN criteria.
Fig. 7. Mixed mode fracture of CeO$_2$-TZP with grain size 3.6 μm tested by Singh and Shetty [34] using the precracked disk specimen: (A) fracture initiation angles (B) fracture curve based on the GMTSN criterion (C) fracture curves based on the MTS and the MTSN criteria.

Fig. 8. Mixed mode fracture of CeO$_2$-TZP with grain size 6.7 μm tested by Singh and Shetty [32] using the precracked disk specimen: (A) fracture curve based on the GMTSN criterion (B) fracture curves based on the MTS and the MTSN criteria.
The normalization process for presenting the fracture curves is better to be done with a reference value which is not dependent on the geometry or loading configuration. Therefore, the parameter $K_0^f$ was employed to present the results of the GMTSN criterion.

According to the change from $K_0^f$ to $K_0^p$ for the GMTSN criterion, the experimental data should also be shifted up/down. Thus, it is more appropriate to plot the fracture curves of the GMTSN criterion separated from the classical criteria.

Scrutinizing three sets of fracture data for CeO$_2$-TZP disk specimens of grain sizes 1.6, 3.6 and 6.7 μm approves the improved correlation with the experimental data when the GMTSN criterion is utilized rather than the MTS and the MTSN criteria, which is mainly attributed to the inclusion of T-term in addition to the singular terms $K_I$ and $K_{II}$.

References


