Research paper

The nonlinear elastic and viscoelastic passive properties of left ventricular papillary muscle of a Guinea pig heart

M.A. Hassan\textsuperscript{a,b,}\textsuperscript{*}, M. Hamdi\textsuperscript{a,b}, A. Noma\textsuperscript{c}

\textsuperscript{a}Department of Engineering Design and Manufacturing, Faculty of Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia
\textsuperscript{b}Center of Advanced Manufacturing & Material processing, Faculty of Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia
\textsuperscript{c}Department of Physiology and Biophysics, Graduate School of Medicine, Kyoto University, 606-8051 Kyoto, Japan

\begin{abstract}

The mechanical behavior of the heart muscle tissues is the central problem in finite element simulation of the heart contraction, excitation propagation and development of an artificial heart. Nonlinear elastic and viscoelastic passive material properties of the left ventricular papillary muscle of a guinea pig heart were determined based on in-vitro precise uniaxial and relaxation tests. The nonlinear elastic behavior was modeled by a hypoelastic model and different hyperelastic strain energy functions such as Ogden and Mooney–Rivlin. Nonlinear least square fitting and constrained optimization were conducted under MATLAB and MSC.MARC in order to obtain the model material parameters. The experimental tensile data was used to get the nonlinear elastic mechanical behavior of the heart muscle. However, stress relaxation data was used to determine the relaxation behavior as well as viscosity of the tissues. Viscohyperelastic behavior was constructed by a multiplicative decomposition of a standard Ogden strain energy function, $W$, for instantaneous deformation and a relaxation function, $R(t)$, in a Prony series form. The study reveals that hypoelastic and hyperelastic (Ogden) models fit the tissue mechanical behaviors well and can be safely used for heart mechanics simulation. Since the characteristic relaxation time (900 s) of heart muscle tissues is very large compared with the actual time of heart beating cycle (800 ms), the effect of viscosity can be reasonably ignored. The amount and type of experimental data has a strong effect on the Ogden parameters. The in vitro passive mechanical properties are good initial values to start running the biosimulation codes for heart mechanics. However, an optimization algorithm is developed, based on clinical intact heart measurements, to estimate and re-correct the material parameters in order to get the in vivo mechanical properties, needed for very accurate bio-simulation and for the development of new materials for the artificial heart.

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\end{abstract}

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1. Introduction

Finite element analysis is a powerful tool for constructing a virtually living beating heart and studying heart mechanics (Dokos et al., 2000). For instance, from the FE-analysis, we can obtain huge data about cardiac motion, especially stress and strain of the heart that are two of the most important determinants of many cardiac physiological and pathophysiological functions. These functions include: the pumping performance of the ventricles; the oxygen demand of the myocardium; the distribution of the coronary blood flow; the vulnerability of the regions to ischemia and infarction diseases; and the risk of arrhythmia (Le Guennec et al., 1990).

There are four fundamental requirements or steps to derive and formulate the equation of motion of the heart. These are (1) kinematics relations, (2) stress equilibrium equations, (3) constitutive relations and (4) boundary conditions (Rao, 2004). Mechanical properties and constitutive relations (the experimental relationship between stress and strain) are the central prerequisite demand in modeling heart mechanics using the FE-method.

In general, biological tissues exhibit time dependence when subjected to relaxation test and hysteresis when subjected to cyclic loading (Katsnelson et al., 2011; Rossmann, 2010; Screen, 2010). Previous studies have a long controversy focused on the hypoelastic, hyperelastic, or viscoelastic behavior of the heart muscle, and which constitutive relation should be confidently used in the heart FE-simulation. Strain energy functions like Mooney–Rivlin and Ogden models are used for hyperelasticity (Sacks, 1999; Humphrey et al., 1990) while viscohyperelastic strain energy functions are used for viscohyperelasticity by Katsnelson et al. (2004). The nonlinear elastic (reversible) model in which the Eulerian or exponential stress–strain relationship was assumed to describe the passive response of the tissue’s material by nonlinear elastic relations known as the hypoelastic behavior (Green and Naghdi, 1965; Atluri, 1984).

There are several experimental techniques to assess the mechanical behaviors of the biological materials in vitro (Carter et al., 2001; Hamid et al., 2009a,b; Higa, 2007; Kanyanta and Ivanovic, 2010). Among these are indentation probes, tension, compression and relaxation testers or cyclic shear tests. Ultrasound electrograph (Gao et al., 1996) and magnetic resonance electrograph (Suga et al., 2000) become available techniques for in vivo tests, but in many cases the data obtained are not easily comparable. That is either because the tissue sample sizes employed or testing domain (strain and frequency range) are not comparable, or merely because of few tests conducted on the same tissues. In particular, the Magnetic Resonance Imaging (MRI) and tagging methods mostly provide global information about material properties and deformation of the heart.

In this paper, the authors developed a very precise uniaxial and relaxation test to determine the nonlinear elastic and viscous behaviors of a guinea pig left ventricle myocardium (heart muscle). Nonlinear curve fitting and optimization using the MATLAB tool box and MARC were used to fit our experimental data different material models in order to determine the best model that characterizes the nonlinear elastic and relaxation behavior of the heart tissues. The generalized strain energy function versus time data being used for large strain viscohyperelasticity was generated by using our experimental data to determine the constants of the myocardium free energy function. However, stress relaxation tests at multiple levels of strains were conducted to generate the relaxation function. The results are discussed from the mechanical, biomechanical and physiological points of view. The effect of the amount of experimental information on the model constant was also studied. Finally, the study proposes an optimization algorithm to correct and estimate real material parameters from a simple uniaxial test and clinical intact heart measurements such as the MRI and sonography.

2. Materials and methods

2.1. Apparatus

The experimental apparatus was built on an optical microscope provided with a Charge-Coupled Device (CCD) video camera as shown schematically in Fig. 1. The myocardium (heart muscle) sample in the relaxed state was put on the sample holder and its initial length was measured from the scaled monitor. The sample was attached at one end to a hook from a micrometer of the extensometer for controlling the sample length and extension, and at the other end to a hook from a semiconductor strain gage sensor (F.S.) for force measurement force sensor (Akers AE801; AME, 2.5 N, frequency resonance 1 kHz) with 0.1% resolution was used. The analogue force and displacement signals were amplified and recorded by the thermal array recorder before they were converted to the digital signal, through a data acquisition system, and stored on a P.C. To minimize movement on the hook during extension, the muscle sample was fastened at the two ends with the smallest size of nylon thread which is known commercially as “000 nylon”. The sample measured length was the distance between the two nodes that appeared on the scaled monitor. The electronic stimulator and the digital oscilloscope provided the physiological, biophysical and electrical conditions similar to those of the heart intact. Fig. 2 shows a photograph of the sample, sample holder, extensometer, and force sensor (F.S.).

2.2. Papillary muscle

Guinea pigs (2 kg) were anesthetized with an intravenous injection of sodium pentobarbital, incubated and ventilated. The pericardium was opened, the ascending aorta was cross-clamped, and the heart was arrested with 50–150 ml intracardiac injection of an oxygenated low-sodium high potassium solution. The heart was removed and placed in a beaker containing this solution. The right ventricular free wall and intraventricular septum were excised with scissors, and the left ventricular free wall was isolated. A trabecula papillary muscle was taken out to make samples of the measured length 8 mm and approximate circular cross sectional area of 4 mm². A preload of 0.03 N was applied to measure the slack length of the specimen and the strain was increased gradually till 50% tensile strain.

This protocol was repeated five times for each sample and the force versus stretch ratio was recorded. The relaxation tests were performed by holding a specimen at a constant
Fig. 1 – Schematic diagram of the experimental apparatus for uniaxial and relaxation tests.

Fig. 2 – Photograph of the setup for uniaxial tension and relaxation tests.

tensile strain and measuring the resulting stress as a function of time. Strains were automatically controlled by using the digital extensometer. Seven strain magnitudes, 10%, 20%, 30%, 35%, 40%, 45% and 50% were used for stress relaxation tests. Specimens were tested at each strain magnitude and the material stress relaxation was recorded in each case. This order is one of the standard relaxation tests used to test the viscoelastic mechanical properties of materials like rubber. Although 12 specimens were tested, only the results of the best seven samples were reported in which the sample size and the fiber orientation were assumed constant and parallel to the sample longitudinal axis. All samples were tested at room temperature. Rigor mortis usually develops in the muscle–tissue from 4 to 10 h after death. To avoid change of muscle stiffness due to the rigor mortis effect, the tests were performed in two to three hours including post-mortem time. Time period elapsed between the death and the testing was about 40 min.

2.3. Strain analysis

Because the specimen was fastened at the two ends with nylon thread, the specimen deformation was non-homogeneous near the nodes. It was assumed that the fiber direction is constant and parallel to the sample longitudinal axis and the deformation at the center of the specimen-like cylinder was homogeneous. A CCD camera and scaled monitor, AD/DA capture card was adjusted to the system to measure the displacement at the center of the specimen. The data acquisition algorithm was developed to simultaneously record the load from the force transducer and the images taken by the CCD camera at a controllable rate. The image processing toolbox under the MATLAB software was used to measure the local strain or stretch ratio experienced by the specimen. A reference image was taken from the sample at rest. The software toolbox compares captured images with the reference image and measures the number of pixels between reference points (sample diameter at rest) and their new locations (current diameter during testing). The image processing converts the measurement from number of pixels to a standard unit of measurement.

2.4. Theory

2.4.1. Heart mechanical cycle

The blood pressure versus the blood volume in the left ventricle cavity in the heart cycle is shown schematically in Fig. 3(a). The diagram has many advantages, but the four phases of the cardiac pressure cycle become very useful, especially for the FE-simulation, if they are illustrated as a function of time as shown schematically in Fig. 3(b). In these two figures, the mitral valve closes at A and the left ventricle undergoes isovolumic contraction; muscles generate rapid contractile force while the blood volume is constant; therefore the blood pressure is rapidly rising until B. When the left ventricular pressure exceeds the aortic pressure and the aortic valve opens, blood is ejected to the aorta and the left ventricle volume begins to decrease. The aortic valve closes at the end of systole C, because the muscle starts to relax and the intraventricular pressure falls below the aortic pressure. The left ventricular muscles then undergo isovolumic relaxation from C to D. The mitral valve reopens at D when the pressure of the left ventricle is lower than that of the left atrium. Blood pressure in the right ventricle changes in a similar manner over the heart contraction cycle, but its magnitude is about three times smaller than that of the left
ventricle (Hunter and Smaill, 1988). The total time of this cycle is 800 ms and that is why the heart beats at 75/min. The work done by the left ventricle in one cycle is the area enclosed in the loop ABCD. Including the viscous properties (stress relaxation time constants) of the heart muscle in the FE-analysis of heart mechanics cost an exhaustive simulation time. This poses an issue when using biosimulation software packages especially on PCs. And now the question arises; is the viscous property of the heart muscle important for heart mechanics? Is it possible to ignore it in biomechanical simulation of heart motion? The answer will be discussed based on the results of the stress relation tests under different strain levels.

2.4.2. Material models

Hypoeelastic material model

In this model, heart tissues are considered as a nonlinear solid whose elastic modulus is dependent only on strain i.e. the elastic tangent modulus is linear in relation with stress. One way to describe this material behavior is the exponential model given by the empirical equation (1a) where, $\sigma$ is the Cauchy stress, $\epsilon$ is the Green–Lagrange strain, $k$ and $k_s$ are material constants determined from experiment (Freed et al., 2010). The elastic tangent modulus, $E_t$, is calculated from Eq. (1b)

$$\sigma = \frac{k_s}{k} (e^{k \epsilon} - 1)$$  \hspace{1cm} (1a)

$$E_t = k \sigma + k_s.$$  \hspace{1cm} (1b)

Hyperelastic material models

Materials like the heart muscle are described by strain energy functions in order to guarantee that the rigid body motions play no role in the constitutive law. Mathematically, this is achieved by postulating the existence of a strain energy density function, $W$, to be a scalar potential, which depends on the component of the right Cauchy–Green deformation tensor or Green’s strain tensor. Components of the second Piola–Kirchhoff stress tensor are given by the derivatives of $W$ with respect to the components of Green’s strain tensor. For the isotropic hyperelastic material, the strain energy is constant for all orientations of the coordinate axes. Thus the strain energy is an invariant of Green’s strain tensor, $E_{ij}$ and can be expressed as a function of the three principal invariants of $E_{ij}$ as shown in Eqs. (2a), (4) and (5) which are known as Mooney–Rivlin models whereas the Signorini model, the Yeoh form and the Neo–Hookean material model are expressed by Eqs. (6)–(8), respectively (Bower and Allan, 2009).

$$W(E_{ij}) = W(I_1, I_2, I_3) = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3 \left( \frac{1}{I_2^2} - 1 \right) + C_4(I_3 - 1)^2.$$  \hspace{1cm} (2a)

The constants $C_3$ and $C_4$ are related to $C_1$ and $C_2$ as;

$$C_3 = \frac{1}{2} C_1 + C_2, \quad C_4 = \frac{C_1(5u - 2) + C_2(11u - 5)}{2(1 - 2u)}.$$  \hspace{1cm} (2b)

Poisson’s ratio is $u = 0.49$, whereas the strain invariants $I_1$, $I_2$ and $I_3$ in terms of the principal stretch ratios, $\lambda_1$, $\lambda_2$ and $\lambda_3$ are defined as;

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad I_2 = \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_1^2 \lambda_3^2,$n  \hspace{1cm} (2b)

$$I_3 = \lambda_1^2 \lambda_2^2 \lambda_3^2.$$  \hspace{1cm} (2b)

In Eq. (2a), $C_1$ and $C_2$ are material constants which must be determined experimentally. For complete incompressibility of the material, $I_3 = 1$. However, Spencer (1980) noted that it is not sufficient to set $I_3 = 1$ in Eq. (2a), since certain derivatives of $W$, tend to infinity in the limiting case of incompressibility. This problem is overcome by introducing the arbitrary constants $C_3$ and $C_4$. Also, in Eq. (2a), it can be noted that the use of $(I_1 - 3)$ and $(I_2 - 3)$ ensures that the strain energy is zero when the strains are zeros. This can be easily explained, because for zero strains the principal stretch ratios $\lambda_1 = \lambda_2 = \lambda_3 = 1$. In such case, Eq. (2b) reduces to $I_1 = I_2 = 3$ and $I_3 = 1.

i. First order Mooney energy function

Since the constants $C_3$ and $C_4$ in Eq. (2a) are dependent, they can be easily determined if the constants $C_1$ and $C_2$ are known. Because heart tissues are nearly incompressible, the principal stretches in the uniaxial test are given by

$$\lambda_1 = \lambda_1, \quad \lambda_2 = \lambda_3 = \frac{1}{\sqrt[3]{\lambda_1}},$$  \hspace{1cm} (3a)

and the logarithmic stress in axial direction is given by

$$\sigma_1 = \sigma_1 = \lambda_1^2 \frac{\partial W}{\partial \lambda_1^2} = 2C_1(\lambda_1^2 - 1) + 4C_2(\lambda_1 - 1);$$  \hspace{1cm} (3b)

and the corresponding engineering stress is given by

$$S_1 = S_1 = \frac{\sigma_1}{\lambda_1^2} = 2C_1(\lambda_1 - 1) + 4C_2 \left( 1 - \frac{1}{\lambda_1^2} \right),$$  \hspace{1cm} (3c)

where, $\lambda_1$ is the stretch ratio in the uniaxial test.

In order to obtain a good fitting with the experimental data, different models are used as shown in the following equations;

ii. Second order Mooney–Rivlin energy function

$$W = C_{10}(I_1 - 3) + C_{20}(I_2 - 3) + C_{11}(I_1 - 3)(I_2 - 3) + C_{20}(I_1 - 3)^2.$$  \hspace{1cm} (4)

Fig. 3 – Schematic diagram for: (a) ventricular pressure–volume loop, (b) ventricular pressure, after (Spencer, 1980).
iii. Third order Mooney–Rivlin energy function
\[ W = C_{10}(l_1 - 3) + C_{01}(l_2 - 3) + C_{11}(l_3 - 3)(l_2 - 3) + C_{20}(l_1 - 3)^2 + C_{30}(l_1 - 3)^3. \]

iv. The Signorini model
\[ W = C_{10}(l_1 - 3) + C_{01}(l_2 - 3) + C_{20}(l_1 - 3)^2. \]

v. The Yeoh form
\[ W = C_{10}(l_1 - 3) + C_{20}(l_1 - 3)^2 + C_{30}(l_1 - 3)^3. \]

vi. Neo–Hookean material model
\[ W = \frac{1}{2} G((l_1^2 + l_2^2 + l_3^2) - 3). \]

vii. Slightly compressible energy function (Ogden model)

In this model, the heart tissues are taken to be virtually incompressible and slightly compressible. Such energy function given by Eq. (9) is called the Ogden model (Ogden, 1997).
\[ W = \sum_{n=1}^{N} \psi_n \left[ \frac{\mu_n}{\bar{\lambda}_n^\alpha} \left( \bar{\lambda}_n^\alpha + \bar{\lambda}_n^\beta + \bar{\lambda}_n^\gamma - 3 \right) + 4.5 \lambda \right]^2 \]

where, \( \mu_n \) and \( \alpha_n \) are material constants, \( K \) is the initial bulk modulus, and \( J \) is the volumetric ratio defined by \( J = \bar{\lambda}_1 \bar{\lambda}_2 \bar{\lambda}_3 \) where \( \bar{\lambda}_1, \bar{\lambda}_2 \) and \( \bar{\lambda}_3 \) are the principal stretch ratios. The order of magnitude of the volumetric changes per unit volume should be 0.01 (Spencer, 1980). Usually, the number of terms taken into account in the Ogden models is \( N = 2 \).

The Ogden model is different from the Mooney–Rivlin model in several respects. The Mooney material model is defined with respect to the invariants of the right or left Cauchy–Green strain tensor and implicitly assumes that the material is incompressible. The Ogden formulation is defined with respect to the invariants of the right or left Cauchy–Green strain, and the presence of the bulk modulus implies some compressibility.

In Eq. (9), the second term on the left hand side stands for energy due to the change in volume. The 4.5 scalar is made to ensure that the second order partial differentiation of Eq. (9) with respect to \( J \) should approach 0 when \( J \) approaches 1.

viii. Viscohyperelastic material models

For large strain viscoelastic materials, the strain energy function can be expressed as a Prony series expansion as (MARC, 1991);
\[ \psi(t) = \psi^\infty + \sum_{n=1}^{N} \delta^n W \exp(-t/\lambda^n) \]

where \( \delta^n \) are time dependent scalar multipliers and \( \lambda^n \) are the associated relaxation times. At time zero (or for time process: \( t < \lambda^n \)), the elastic energy of Eq. (10a) reduces to:
\[ \psi(0) = W = \psi^\infty + \sum_{n=1}^{N} \delta^n W \quad \text{or} \quad \psi^\infty = \left[ 1 - \sum_{n=1}^{N} \delta^n \right] W. \]

Then the time dependent energy function is given by the substitution of Eq. (10b) into Eq. (10a) as:
\[ \psi(t) = W \left[ 1 - \sum_{n=1}^{N} \delta^n \left( 1 - \exp(-t/\lambda^n) \right) \right]. \]

Discussion will be restricted in the case \( N = 2 \) for simplicity. Hence,
\[ \psi^\infty = [1 - \delta_1^2] W \]
\[ \psi(t) = W[(1 - \delta_1^2)(1 - \exp(-t/\lambda_1^n)) - \delta_2^2(1 - \exp(-t/\lambda_2^n))] = WR(t) \]

where, \( \delta_1, \delta_2 \) are nondimensional scalar multipliers and \( \lambda_1, \lambda_2 \) are the corresponding relaxation time constants.

The viscohyperelasticity energy function, given by Eq. (10d), is based on a multiplicative decomposition of the Ogden model, \( W \), for instantaneous deformation and a relaxation function in a Prony series \( R(t) \), the terms in square brackets.

### 2.5. Parameters calculation

The experimental tensile test data were substituted into each of the above models and solved by means of nonlinear least square method using the Levenberg–Marquardt nonlinear curve fitting algorithm under the MATLAB optimization toolbox and under the commercial finite element software package MSC.MARC. The energy function, \( \psi(t) \) versus time data being used for viscohyperelasticity was generated by fitting the experimental data provided by standard quasi-static tests, such as tensile to determine the material constants of the free energy function \( (W) \). However, standard relaxation test data were used to obtain scalar multipliers, \( \delta_1, \delta_2 \) and relaxation time constants, \( \lambda_1, \lambda_2 \) the relaxation function \( R(t) \). For different percentage strain levels, the scalar multipliers and relaxation time constants together with the viscosity, \( \mu \), elastic shear modulus \( G \), and elastic modulus, \( E \), were determined and tabulated. Relative error was calculated to assess the quality of the fitting process, because engineering judgment is best to determine the best fit based upon physical and not mathematical reasons.

Poisson’s ratio is fixed at 0.49 and the material is almost incompressible; however there still is the change in volume. In general, for linear elastic materials, the ratio of the change in volume \( \Delta V/V \) is calculated as \( \Delta V/V = 3\sigma_m(1 - 2\nu)/E \), where \( \sigma_m \) is the mean or hydrostatic stress and \( \nu \) is Poisson’s ratio whereas \( E \) is the elastic modulus.

For given values for \( \sigma_m \) and \( E \) with the ratio of volume change \( \Delta V/V = 0 \) (incompressible material), the equation \( \Delta V/V = 3\sigma_m(1 - 2\nu)/E \) yields to \( (1 - 2\nu) = 0 \) and results in \( \nu = 0.5 \). But if the ratio of volume change \( \Delta V/V \) is assumed to be 0.01, Poisson’s ratio will be 0.49. The choice of Poisson’s ratio depends on the ratio of the change in volume (volumetric strain). The Ogden model is a slightly compressible material model in which the ratio of volume change is made 0.01 and consequently Poisson’s ratio should be 0.49.

### 3. Results and discussion

#### 3.1. Experimental

Fig. 4, shows the Cauchy or true stress versus stretch ratio as obtained from the uniaxial test for seven samples taken from the left ventricle of the Guinea pig heart. In order to prevent irreversible damage (plastic strain), the samples were not stretched beyond the stretch ratio of \( \lambda = 1.6 \). The results vary by approximately 5% for stretch lower than 1.25, whereas the variation increases as the stretch ratio increases beyond 1.25. This could be attributed to the fact that the specimens were not cut from exactly the same area of the papillary muscle or it could be due to the physiological variation and the fact that the specimens taken from different hearts could have anatomical difference. The determination of zero
Table 3 summarises the constants of the Ogden model after each strain increment. This technique is very useful in hypoelasticity because polynomial functions are not always sufficient to bring the tissue specimen to geometric flatness and prevent the soft tissue from sagging.

### Table 1 – Summary of mean $K_\gamma$, overall average, standard deviation and significant value. Eq. (1a).

<table>
<thead>
<tr>
<th>Test no.</th>
<th>$K_\gamma$-value (kPa)</th>
<th>K-value (constant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.89</td>
<td>2.61</td>
</tr>
<tr>
<td>2</td>
<td>6.68</td>
<td>1.79</td>
</tr>
<tr>
<td>3</td>
<td>9.37</td>
<td>9.40</td>
</tr>
<tr>
<td>4</td>
<td>9.38</td>
<td>10.01</td>
</tr>
<tr>
<td>5</td>
<td>9.29</td>
<td>7.32</td>
</tr>
<tr>
<td>6</td>
<td>10.80</td>
<td>19.49</td>
</tr>
<tr>
<td>7</td>
<td>10.83</td>
<td>24.60</td>
</tr>
<tr>
<td>Average</td>
<td>9.03</td>
<td>10.76</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.55</td>
<td>7.83</td>
</tr>
<tr>
<td>P-value (t-test)</td>
<td>0.0001</td>
<td>0.0152</td>
</tr>
</tbody>
</table>

### 3.3. Hyperelastic properties

Fig. 6(b)–(h) show an example of the curve fitting of the tensile to the hyperelastic energy functions. Table 2 lists the relative errors for least square fitting of uniaxial data with the material models. It is clear that the Ogden material model gives the best fit; while the fits of the other six hyperelastic models are invariably poor. Therefore, the Ogden material model was chosen to represent the hyperelastic behavior of the papillary muscle and to study the effects of strain levels on material constants of the heart. The fitting process was constrained for always getting positive constants, because sometimes constants often turned negative and therefore, physically are not meaningful. This phenomenon is a numerical hardship, i.e. negative constants are just values obtained from good fitting and not the fundamental material behavior. Table 3 summarises the constants of the Ogden model together with the tissue bulk modulus after fitting the seven tensile data.

To implement these properties in finite element simulation code, the elasticity matrix (matrix relating stress with strain) should be updated during the simulation process. Since the components of the elasticity matrix are functions of Poisson’s ratio, the elastic modulus and the elastic anisotropy of the tissues are functions of the strains. For instance, the tangent modulus, $E_t$ of the hypoelastic model must be updated using Eq. (1b) after each strain increment. This technique was implemented in the author’s FE C-code as well as in the MARC FE-package.

### 3.4. Parameters $\delta$ and $\lambda$ of the relaxation function

The viscohyperelastic energy function is given by Eq. (10d), which is based on a multiplicative decomposition of the Ogden strain energy function, $W$, for instantaneous deformation and a relaxation function $R(t)$, in a Prony series for $N = 2 R(t)$ stated as;

$$R(t) = [1 - \delta^1(1 - \exp(-t/\lambda^1)) - \delta^2(1 - \exp(-t/\lambda^2))]. \quad (11)$$

The four parameters $\lambda^1$, $\lambda^2$, $\delta^1$ and $\delta^2$ are obtained and listed in Table 4. According to the experiments, the relaxation function $R(t)$ was assigned two relaxation terms; one ($\lambda^1$, $\delta^1$) for fast relaxation and the other ($\lambda^2$, $\delta^2$) for slow relaxation.
Fig. 6 – (a)–(h) show an example of the fitting of the experiment with eight martial models.

Table 2 – Relative errors for least square fitting of uniaxial data with the material models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Neo–Hookean</th>
<th>Mooney (2)</th>
<th>Mooney (3)</th>
<th>Signiorini</th>
<th>Second order</th>
<th>Third order</th>
<th>Yeoh</th>
<th>Odgen (2)</th>
<th>Ogden (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative error</td>
<td>6.35</td>
<td>2.71</td>
<td>0.54</td>
<td>0.44</td>
<td>0.08</td>
<td>0.02</td>
<td>0.08</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The four parameters increase significantly as the strain level $\varepsilon$ increases from 10% up to 50%. The averages values of $\lambda^1$, $\lambda^2$, $\delta^1$ and $\delta^2$ for the seven samples are 0.1523, 0.3443, 1.7417 and 52.1607, respectively.
3.5. Viscosity and shear modulus

Based on the Maxwell element model, the stress at any time, \( t \) required to maintain the strain constant is given by \( \sigma = \sigma_0 e^{-t/\tau} \) which was used for curve fitting of the relaxation test data. The overall relaxation time constant \( \tau \) which is equal to \( \mu/G \), was determined. The viscosity coefficient \( \mu \) and the elastic shear modulus \( G \) were calculated and listed in Table 5. At the beginning of the relaxation (0–7 s), the average relaxation time obtained (\( \tau_{\text{ave}} = 666.2521 \) s) is very large compared with the heart cycle time (800 ms) which means that the heart muscle contracts very fast before any significant relaxation takes place. Therefore stress relaxation in the heart muscle is very small and could be ignored confidently in the simulation of heart mechanics. For the time (7–90 s), the seven samples demonstrate non-significant decrease in stress versus time and a significant decrease in the rate of stress relaxation with increasing strain were observed. The average relaxation time for data range (7–90 s) was 56.2774 s while it was 445.2925 s for all data range. At the beginning of relaxation, time range (0–7 s), the samples show slight increase in the stress relaxation rate with increasing strain level. The average value of the muscle viscosity 17 \( N\ s/mm^2 \) indicates a small damping ratio and the muscle tends to behave like a solid rubber material. It should be highlighted that the stress relaxation behavior of skeletal muscle tissue in compression was experimentally investigated by Van Loocke et al. (2008) at various rates and fiber orientations. Their results showed that, above a very small compression rate, the viscoelastic effects play a significant role on muscle mechanical properties. A substantial amount of stress-relaxation occurred when deformation was held constant after a compressive ramp over 45% and 60%.

The significant value \( (p\text{-value}) \) of the t-test were calculated for each parameter and listed in Tables 1 and 3-5. Statistically, when \( p\text{-value} < 0.05 \) it may draw some doubts on precision and uncertainty levels of each parameter. However, the used t-test assumes that the mean value for each parameter is zero (null hypothesis) and the data come from normal distribution which is not practically true. In other words, the material parameters obtained are confidently accepted \( (p \geq 1) \) when the real experimental mean value of each parameter was used regardless of the shape of its distribution curve.

4. Effect of amount of information on the constants of the energy functions

A series of fits (Ogden model; Eq. (9)) were conducted that used progressively more information as the basis for the curve fitting. Mixed biaxial and volumetric compression data taken from the literature (Dokos et al., 2000; Sacks, 1999) is used together with the uniaxial experimental data obtained to assess the constants of Eq. (9). Table 6 summarizes the constants calculated in each case. Adding biaxial data to the uniaxial data had a strong influence on the quality of the fit and changed the constants greatly. However, adding further volumetric compression data has a pronounced effect on the constants calculated because the Ogden model is slightly compressible and therefore it is sensitive to the volumetric compression data.

5. Effect of anisotropy

The consideration of anisotropy leads to a more elaborate mechanical response than for isotropic strain-stiffening materials. (Humphrey et al., 1990) have studied modeling cardiac mechanical properties in three dimensions and found that the ventricular myocardium is 1.5–3 times stiffer in fiber directions than in the cross fiber direction. A difficulty with any constitutive model based only on tensile tissue tests is uncertainty (due to anisotropy) as to how the tensile properties of the isolated specimen of papillary muscle are related to the properties of the intact ventricular wall. In addition, the current does not investigate the response of the tissue under the compression test. Published works on myocardial muscles have shown that the tension compression response is asymmetric as same as the response of skeletal muscles under the tension compression test (Simms et al., 2011). To overcome such a problem, Section 6 proposes an optimization algorithm that aims to avoid exhaustive experiments and have more precise in vivo tissue mechanical properties. The algorithm is based on the correction of material constants through a combination of finite element model of large deformation heart mechanics and clinical Magnetic Resonance Imaging (MRI) tagging measurements.
6. Correction of material constants

Strain energy functions used in the above sections for the passive material properties of the cardiac muscle are based partly on observations of tissue microstructure and partly on the results of the uniaxial and/or biaxial and volumetric testing of tissue samples. There is always some doubt; whether in vitro tests can completely define the in vivo material properties. For these tests on passive tissue in particular, it is impossible during samples preparation to avoid some disruption of the perimysial collagen (very thin tissue covering the heart muscles). A judicious combination of in vitro and in vivo tests is therefore needed to determine the material parameters. A finite element model of large deformation heart mechanics can be used for this purpose with clinical Magnetic Resonance Imaging (MRI) tissue tagging measurements. Another important reason for correcting and deriving material parameters from clinical intact heart measurements is to assist with the diagnosis of heart diseases. Many disease states can be characterized by the underlying tissue properties.

The correction process is based on the nonlinear FE analysis and proceeds as follows:

1. Selection of an initial set of material parameter estimates (hypoelastic or hyperelastic). This choice is based on the parameters estimated from the uniaxial tension tests.
2. Substitution of the current material parameter estimates and predetermined strain field into the stress-strain relations and the use of C++ FE ventricular mechanics model to compute the internal stress field and external loads (nodal forces) required to maintain the equilibrium.
3. Computing a set of error residuals, based on the differences between the experimentally measured external loads and the nodal forces computed using the model.
4. Minimizing these error residuals with respect to the unknown material parameters using a suitable nonlinear optimization technique, such as the Levenberg-Marquardt or the sequential quadratic programming.

The algorithm developed is shown schematically in Fig. 7 and could be applied for each deformed state arising from the FE model resulting in several sets of estimated material parameters with associated sets of error residuals. Overall parameter estimates could be based on simple averages of the individual parameter estimates across the different states of deformation. A better approach may be to combine the sets of error residuals from the various deformed states and minimize this global set of residuals with respect to the unknown material parameters.

7. Conclusions

A very precise test rig that had been established for making the uniaxial and relaxation tests of small samples of left ventricular papillary muscle of the Guinea pig heart and passive in vitro mechanical properties was determined. The muscle material behavior was modeled by hypo-elastic, hyper-elastic and viscohyperelastic models. Material parameters or the constants of the constitutive relation were determined by fitting the experimental data with these models using the MATLAB and MRAC FE-Package optimization tool boxes. Because the period of the heart cycle is 800 ms and the relaxation time measured is very large, the stress relaxation in heart tissues is very small and can be confidently ignored. Therefore, the viscoelastic model is not suitable for the simulation of heart mechanics because of the large computational time required and its proneness to instability and divergence problems. We conclude that the hypoelastic and hyperelastic (Ogden) models represent well the mechanical behaviors of the heart muscle and could be used to regenerate the experimental data and the tissue response under different load types. The uniaxial tension test gives good initial material properties, however more experimental information is needed to model the response of heart material under any loading conditions. In order to avoid exhaustive experiments and obtain precise in vivo tissue mechanical properties, an optimization algorithm was proposed to correct and estimate real material parameters from simple uniaxial tests and clinical intact heart measurements such as the MRI and sonography.
Fig. 7 – A flow chart of the C++ FE-code for the non-linear simulation of heart contraction based on material parameters corrections.

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