Experimental investigation to compare the modulus of rupture in high strength self compacting concrete deep beams and high strength concrete normal beams

Mohammad Mohammadhassani *, Mohd. Zamin Jumaat, Mohammed Jameel

Department of Civil Engineering, University of Malaya, Malaysia

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A B S T R A C T

This paper investigates and compares the cracking moment and modulus of rupture in deep and normal beams. Eight high-strength-self-compacting concrete (HSSCC) deep beams and nine high-strength concrete (HSC) normal beams are casted and tested. The results from ACI 318-95 and CSA-94 codes show the discrepancies in values of modulus of rupture for HSSCC deep and normal beams. These differences are due to non-consideration of load transfer mechanism in deep beams and high strength of concrete. Based on the present study ACI 318-95 is recommended for calculating modulus-of-rupture in HSSCC deep beams whereas CSA-94 is suggested for HSC normal beams.

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1. Introduction

Development of HSC dates back to the early 1930s, when concrete compressive strengths above 97 MPa were achieved. With recent development in concrete sciences, HSC is defined as concrete with strength more than 41.1 MPa. With the advent of super-plasticizers and other additives, HSC has now become an economical solution for the construction of bridges and tall buildings. Strength requirement for commercial concrete has been increasing over the years. Concrete with strength up to 124 MPa have been successfully used in some high-rise buildings e.g. the Water Tower Place (Chicago, USA) and the East Huntington, W.V., bridge over the (Ohio River, USA).

Manufacturing of HSC involves making optimal use of the basic materials that constitute normal-strength concrete (NSC). The strength of the aggregate, the optimum size of the aggregate, the bond between the cement paste and the aggregate, and the surface characteristics of the aggregate influence the ultimate strength of HSC. It would be difficult to produce HSC mixtures more than 70 MPa without using admixture and additives. The properties of HSC have been studied with different additives by many researchers [1,2] using different material. Khanzadi and Behnood [1] had stated that using copper slag aggregate instead of limestone aggregate showed an increase of about 10–15% of compressive strength and 10–18% in tensile strength of HSC. Using various types of material with different proportions, many grades of HSC with varying mechanical and durability properties can be produced. For instance researcher such as Elahi et al. [2] stated that using silica fume improves strength development better than other supplementary cementitious materials.

However, most of the current design codes were developed using experimental results based on NSC; therefore existing codes limits the maximum concrete strength based on it. These limitations reflect the lack of research development rather than the inability of the material to perform its capacity. Thus adequate investigation of its behaviour in structural members is highly desirable. It is well known that concrete is weak in tension and strong in compression. Therefore, its primary purpose in a reinforced concrete structural member is to sustain compressive forces, while steel reinforcement is used to sustain tensile forces. The concrete protects the steel from corrosion effects of the environment. As concrete is used to sustain compressive forces, it is essential that its strength and deformational response under such conditions are determined. In this regard, one of the main parameters that needs to be investigated is the modulus of rupture, due to inherent drawback of concrete in tension. The other important aspect is the cracking moment. It is the moment that is required for the first crack to appear in extreme tension fibre. The importance of modulus of rupture and cracking moment is, at this point the steel reinforcement in the beam is exposed to the environment and possible corrosion. The application of modulus of rupture is most useful for structures such as concrete dams and nuclear reactor vessels, for which the safety concerns are particularly vital and extremely high.

There are many investigations on cracks in concrete structures [3–8] and many researchers studied the modulus of rupture of normal beams experimentally and analytically for high and low
strength concrete [9–18]. The general finding of these aforementioned investigators is that the modulus of rupture decreases with increasing beam size. This is due to the stress and strain distribution in plane section which remains planar after bending. In special cases where the shear deformation is dominant such as in concrete deep beams, experimental work to determine the modulus of rupture is not found. The behaviour of deep beams is unknown compared to normal beams; this creates more problems in addition to the problems already faced due to its material behaviour.

Deep beams are widely used structural elements with comparatively small ratios of span (L) to depth. Deep beams have various structural applications, e.g., transfer girders in tall buildings, pile caps, foundations, offshore structures and others. The differentiation of normal beam and deep beams is unclear. For example, the ACI code [19], CEB-FIP model code [20] and CIRIA Guide 2 [21] use the span/depth ratio limit to define RC deep beams; and the Canadian code [22] employs the concept of shear span/depth ratio. Also, the design concept of deep beams is not same as normal beams. In normal beams the plane sections remain planar before and after bending. But in deep beams, due to a different load transferring mechanism and domination of shear deformation, this concept is not applicable. The failure of these elements is more in shear mode rather than flexural. The stress and strain distribution in deep beams is nonlinear in contrast to normal beams. Also, many of the equations for normal beams are not applicable for deep beams as there is more than one neutral axis in deep beams compared to the single neutral axis in normal beams [23].

Many investigations have been published over past two decades on design and behaviour of deep beams [24–29], but very little work has been reported on deep beam bending behaviour and the first crack occurrence. Since crack limitation factor is used as a serviceability requirement for concrete elements, it is important to investigate the appearance of first crack in HSC deep beams. However, many practicing engineers and designers face difficulties in choosing an appropriate average tensile concrete strain to apply in designing deep beams and have expressed reservations about using the existing provisions. Also, the information on first crack occurrence is a key requirement for finite element softwares.

1.1. Research significance

The aim of present study is to determine the modulus of rupture and first bending crack for both normal and deep beams which are very useful in finite element solutions. Further to propose possible code provisions for deep beams, to identify the natural progression of crack which is the limitation of concrete behaviour and its serviceability. This paper presents useful data on the effective parameters of HSC deep beams to determine the possibility of the cracking moment of deep beams and comparing it with the corresponding data regarding normal beams. Eight HSSCC deep beams and nine HSC normal beams with different tensile bar percentages are casted and tested. The beams are tested gradually till failure occurs. The load corresponding to first flexural crack is recorded for each beam. The corresponding flexural moment and modulus of rupture are calculated and compared with existing design codes.

2. Materials and methods

2.1. Test program

Eight HSSCC deep beams have been designed and casted, corresponding to tensile bar percentage ($\rho$) of $0.1\rho_o$, $0.2\rho_o$, $0.3\rho_o$, $0.4\rho_o$, $0.5\rho_o$, $0.6\rho_o$ and $0.7\rho_o$. The beam lengths, depths and thicknesses are kept constant while varying the tensile reinforcements. Two groups of HSC normal beams with low and high tensile bar percentages are chosen. Based on ACI code provisions, five beams with low tensile bar percentages ($\rho_{min}$, $0.2\rho_o$, $0.3\rho_o$, $0.4\rho_o$, $0.5\rho_o$) and four beams with high tensile bar percentages ($0.75\rho_o$, $0.85\rho_o$, $0.9\rho_o$, $1.2\rho_o$) are designed and casted. All the beams are loaded until failure occurred.

### Table 1

| Characteristic cube strength | 75 MPa |
| Aggregate type            | Crushed granite and natural sand |
| Cement type               | Ordinary Portland cement |
| Slump of concrete         | More than 650 mm |
| Coarse aggregate content | $553 \text{ kg/m}^3$ |
| Fine aggregate content   | $887 \text{ kg/m}^3$ |
| Water/binder              | 0.25 |
| Silica fume/cement        | 0.1 |

**Fig. 1.** Workability of HSSCC.

**Fig. 2.** Casting arrangement of deep beam.

2.2. Materials

2.2.1. High strength self compacting concrete deep beams

The material used for the present experimental study is HSSCC. Self-compacting concrete is a highly flowable, non-segregating concrete that can spread into a mould and fill the formwork and encapsulate the reinforcement without any need for consolidation. Because of density of reinforcing bars in deep beams, the vibration of concrete is not an easy task. Therefore, the use of self compacting concrete is highly desirable. Moreover, self compacting concrete requires no additional compaction and therefore relatively less labour is required. The HSSCC mix design is given in Table 1 and the other required details are discussed in [29]. In the mix design local aggregate of maximum 20 mm diameter, ordinary Portland cement, natural river sand, microsilica and super plasticizer are used.

The desired workability of HSSCC has been achieved avoiding segregation and bleeding. Fig. 1 shows the workability of the HSSCC. To prevent segregation workability is kept within the range of 550–740 mm. For each beam, nine cubes (100 mm x 100 mm x 100 mm) and three cylinders (150 mm diameter, 300 mm height) are casted as control specimens. Cubes are tested for measuring strength at 7 days, 28 days, and the age of loading. The cylin-
Table 2
Specification of tested deep beams.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f_c$ (MPa)</th>
<th>$\rho$ (%)</th>
<th>As (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD1</td>
<td>91.5</td>
<td>0.219</td>
<td>191</td>
</tr>
<tr>
<td>BD2</td>
<td>91.5</td>
<td>0.269</td>
<td>236</td>
</tr>
<tr>
<td>BD3</td>
<td>91.1</td>
<td>0.410</td>
<td>383</td>
</tr>
<tr>
<td>BD4</td>
<td>93.72</td>
<td>0.604</td>
<td>558</td>
</tr>
<tr>
<td>BD5</td>
<td>79.1</td>
<td>0.809</td>
<td>760</td>
</tr>
<tr>
<td>BD6</td>
<td>87.5</td>
<td>0.938</td>
<td>854</td>
</tr>
<tr>
<td>BD7</td>
<td>82.24</td>
<td>1.05</td>
<td>964</td>
</tr>
<tr>
<td>BD8</td>
<td>97.2</td>
<td>1.26</td>
<td>1165</td>
</tr>
</tbody>
</table>

Table 3
Bar specifications.

<table>
<thead>
<tr>
<th>Diameter of bars (mm)</th>
<th>$f_y$ (MPa)</th>
<th>$f_u$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi 9$</td>
<td>353.0</td>
<td>446.0</td>
</tr>
<tr>
<td>$\phi 10$</td>
<td>614.4</td>
<td>666.0</td>
</tr>
<tr>
<td>$\phi 12$</td>
<td>621.6</td>
<td>678.4</td>
</tr>
<tr>
<td>$\phi 16$</td>
<td>566.3</td>
<td>656.0</td>
</tr>
</tbody>
</table>

Table 4
Bar schedule specification of deep beams.

<table>
<thead>
<tr>
<th>Beam no.</th>
<th>Tensile bar</th>
<th>$d$ (mm)</th>
<th>$a/d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD1</td>
<td>3 $\phi 9$</td>
<td>435.5</td>
<td>0.920</td>
</tr>
<tr>
<td>BD2</td>
<td>3 $\phi 10$</td>
<td>438.5</td>
<td>0.910</td>
</tr>
<tr>
<td>BD3</td>
<td>2 $\phi 10$ + 2 $\phi 12$</td>
<td>469.0</td>
<td>0.850</td>
</tr>
<tr>
<td>BD4</td>
<td>2 $\phi 10$ + 2 $\phi 16$</td>
<td>462.0</td>
<td>0.860</td>
</tr>
<tr>
<td>BD5</td>
<td>2 $\phi 10$ + 3 $\phi 16$</td>
<td>470.0</td>
<td>0.850</td>
</tr>
<tr>
<td>BD6</td>
<td>1 $\phi 8$ + 4 $\phi 16$</td>
<td>455.0</td>
<td>0.880</td>
</tr>
<tr>
<td>BD7</td>
<td>2 $\phi 10$ + 4 $\phi 16$</td>
<td>460.0</td>
<td>0.760</td>
</tr>
<tr>
<td>BD8</td>
<td>2 $\phi 10$ + 5 $\phi 16$</td>
<td>451.0</td>
<td>0.778</td>
</tr>
</tbody>
</table>

2.2.1.1. Steel. Except for 9 mm non-deformed steel bar, the other used reinforcing bars are high tensile deformed bars with the properties mentioned in Table 3. The properties are determined by carrying out tests on samples taken from each supplied batch. In Table 3 $f_y$ and $f_u$ are the yield and ultimate stress of the bars respectively.

2.2.1.2. Deep beam details. All deep beams have a section of 500 mm depth, 200 mm width and 1500 mm length. Table 4 shows the deep beam reinforcement details. The geometrical parameters of deep beams are shown in Fig. 3.

As shown in Fig. 3, the anchorage of the main tension reinforcements is enhanced by providing 90-degree hooks at the bar ends for preventing bonding failure.

2.2.1.3. Test setup and loading process. All simply supported beams are subjected to two static point loads as shown in Fig. 4. The loads are applied using a hydraulic jack, until failure occurred. Fig. 4 also shows the details of experimental setup adopted for the present study.

The beams are supported on two steel cylinders with diameter 10 cm. After the beam is centred and levelled the steel beam is placed on the test specimen and the load is applied at midpoint with 20 KN intervals until the first flexural crack occurred. The load at which the first flexural/vertical crack occurred is recorded. Periodically during the test, loading is paused to observe cracks. The loads at which cracks occurred are determined by visual inspection. After the appearance of first flexural crack the loading interval is changed to 50 KN until complete failure is noticed. Strains and deflections are recorded at each step of loading using strain gauges and transducers. In loading process, the specimens are vertically aligned to reduce failure due to irregularity of supports. To avoid errors in data collection, the data acquisition system is electronically zeroed prior to testing. The observations are marked on deep beam surface using markers.

2.2.2. High strength concrete normal beams

Nine HSC normal beams having dimensions 2000 × 200 × 300 mm are casted to investigate modulus rupture. Table 5 shows the HSC mix design for normal beams. Details such as tensile strength and flexural strength are discussed in [30,31]. In the mix design local aggregate of maximum 20 mm diameter, ordinary Portland cement, natural river sand, microsilica and super plasticizer is used.

Table 6 shows area of steel ($A_s$), effective depth (d), tensile bar percentage and strength of concrete for HSC normal beams. The concrete strength mentioned in Table 6 is an average strength of three cube samples at age of loading for each beam.

2.2.2.1. Steel properties in HSC normal beams. The properties of steel bars used in HSC normal beams are given in Table 7. The average yield ($f_y$) and ultimate stresses ($f_u$) of bars are determined by carrying out tests on five samples taken from each supplied batch.
Table 5
HSC mix design.

<table>
<thead>
<tr>
<th>Material</th>
<th>Coarse aggregate</th>
<th>Fine aggregate</th>
<th>Water/ cement</th>
<th>Silica fume/ cement</th>
<th>Super plasticizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kg/m³)</td>
<td>723</td>
<td>647</td>
<td>0.32</td>
<td>0.08</td>
<td>11.7</td>
</tr>
</tbody>
</table>

Table 6
Specification of normal beams.

<table>
<thead>
<tr>
<th>Beam no.</th>
<th>As (mm²)</th>
<th>d (mm)</th>
<th>ρ (%)</th>
<th>f_c (kgf/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>308</td>
<td>256</td>
<td>0.61</td>
<td>670</td>
</tr>
<tr>
<td>B2</td>
<td>628</td>
<td>260</td>
<td>1.25</td>
<td>680</td>
</tr>
<tr>
<td>B3</td>
<td>1020</td>
<td>258</td>
<td>2.03</td>
<td>675</td>
</tr>
<tr>
<td>B4</td>
<td>1260</td>
<td>250</td>
<td>2.52</td>
<td>700</td>
</tr>
<tr>
<td>B5</td>
<td>1520</td>
<td>250</td>
<td>3.05</td>
<td>700</td>
</tr>
<tr>
<td>B6</td>
<td>2464</td>
<td>256</td>
<td>4.81</td>
<td>710</td>
</tr>
<tr>
<td>B7</td>
<td>2866</td>
<td>258</td>
<td>5.39</td>
<td>705</td>
</tr>
<tr>
<td>B8</td>
<td>3512</td>
<td>258</td>
<td>6.81</td>
<td>718</td>
</tr>
<tr>
<td>B9</td>
<td>4004</td>
<td>250</td>
<td>8.01</td>
<td>725</td>
</tr>
</tbody>
</table>

Table 7
Bar specifications.

<table>
<thead>
<tr>
<th>Diameter of bars (mm)</th>
<th>f_y (Mpa)</th>
<th>f_u (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ14</td>
<td>398.1</td>
<td>451.0</td>
</tr>
<tr>
<td>φ18</td>
<td>373.6</td>
<td>412.2</td>
</tr>
<tr>
<td>φ20</td>
<td>401.9</td>
<td>464.8</td>
</tr>
<tr>
<td>φ22</td>
<td>369.7</td>
<td>432.6</td>
</tr>
</tbody>
</table>

2.2.2.2. Normal beam details. Fig. 5 shows a typical reinforcement detail of HSC normal beam. Bars having four different diameters 14 mm, 18 mm, 20 mm and 22 mm are used in design. Table 8 shows reinforcement details for all the normal beams under study. The beams having a cross section of 200 × 300 mm is considered.

2.2.2.3. Test setup and loading process for normal beams. The testing arrangement is shown in Fig. 6. All simply supported beams are subjected to two static point loads. The loads are applied using a hydraulic jack, until failure occurred. Two plates are used to distribute the load and prevent local failure in compression area. The beam is divided into two symmetric halves. Demec gauges in four lines along the depth of beam are mounted to measure the strain variation. The distance kept between each Demec gauge line is 100 mm.

3. Results and discussions

3.1. Cracking moment and modulus of rupture in HSSCC deep beams

With the appearance of the first crack there is a sudden significant change in tensile bar strain. Definition of stress corresponding to the appearance of first crack is important in recognising the change of concrete from linear elastic condition to non-linear plastic condition. The concept of modulus of rupture is based on elastic beam theory. The modulus of rupture is defined as the maximum normal stress in the beam calculated from ultimate bending moment under the assumption that the beam behaves elastically.

As the beam load exceeds the load corresponding to cracking moment P_cr, a hairline crack will appear near the bottom mid-span. By increasing the load, the crack will propagate towards the neutral axis. Additional cracks are observed at the mid-span of the beam towards the supports. The crack mechanism in deep beams and normal beams are different. In deep beam, cracking follows the compression struts that form at ultimate load. In normal beams, cracks occur after the bending of the plane section. Flexural cracks appear first followed by shear cracks at the end of the flexure cracks.

At mid-span the stresses in the beam are primarily flexural tension and compression. These stresses are horizontal and result in vertical cracks. The increasing shear stress towards the support causes the cracks to be more inclined towards the load point; these inclined cracks are known as ‘diagonal tension’ cracks associated with shear. The stress and strain distributions are different along the beams length based on geometry and load variation. Thus the definition of B-region (Beam or Bernoulli) and D-region (Disturbed or Discontinuity) is necessary to recognise for internal force distributions. ACI code defines B and D regions in deep beams based on the St. Venant strain distribution.

The St. Venant’s principle hypothesises states that a localised disturbance such as a concentrated load will dissipate within one beam height from the load point. It is useful to classify portions of the structure as either B-regions or D-regions as shown in Fig. 7. The discussion of these regions is vital in understanding the internal distribution of forces in a reinforced concrete structure.

The B-regions of a member have internal states of stress distribution that are simply obtained from the sectional forces in bending, shear, etc. In these regions, the strain in reinforcement and concrete is directly proportional to the distance from the neutral axis and the beam behaves elastically.
The D-regions are regions adjacent to the concentrated load points that are affected by abrupt change in load and reactions points or adjacent to abrupt changes in geometry, such as internal holes or changes in cross section. In D-regions, the strain distributions are not linear. For structural members where plane sections are non-planar after bending, the strain distribution is non-linear and therefore the linear approach does not apply. The elastic stress distribution in a D-region is difficult to determine and it changes as cracks progress. Schlaich et al. [32] introduced a design method on plasticity theory using a Strut and Tie Model (STM). The model could be used for both the ultimate and serviceability check. In this model, the internal forces can be calculated using STMs. These models have three components, the strut, the tie, and the node(s). Schlaich and Schafer described three strut configurations that should adequately cover all cases of compression stress fields:

- Prismatic stress fields (Fig. 8a).
- The bottle shaped stress field which develops considerable transverse stress (Fig. 8b).
- Fan shaped stress fields which does not develop transverse stresses (Fig. 8c).

Based on the Schlaich and Schafer method, the fan and bottle shaped stress fields are normally found in the D-regions while the prismatic stress field is found in the B-regions. The theory of linear elastic behaviour is applicable in B-regions and it is non-applicable in D-regions. The mid span area between the two point loads undergo linear prismatic stress and therefore it is a B-region. Some investigators, such as Schlaich et al. [32] and Schlaich and Schafer [33], state that the structure will crack due to the elastic stresses, and if the structure is reinforced according to these stresses, cracking can be minimised. Fig. 9 shows evidence of vertical cracks due to horizontal strain distribution in pure flexure zone of beam BD1.

Before cracking, the concrete is nearly elastic in tension and largely unaffected by the presence of reinforcement. The experimental cracking moments $M_{cr,exp}$ is computed with the corresponding moments based on Eq. (1):

$$M_{cr} = \frac{f_t - f_y}{y_t}$$

Fig. 6. Details of testing arrangement.

Fig. 7. B-regions and D-regions.

Fig. 8. Common types of strut 9 (a) prism, (b) bottle and (c) fan.
where $f_r$ is the modulus of rupture; $I_y$ is the moment of inertia of gross concrete section and $y$ is the distance of extreme tension fibre from neutral axis.

![Fig. 9. Crack progressions in BD1.](image)

![Fig. 10. The (a) shear diagram and (b) moment diagram of tested deep beams.](image)

![Fig. 11. The stress and strain distribution of beams BD1, BD2 and BD3 at the occurrence of the first crack in mid-span.](image)

![Fig. 12. The stress and strain distribution of beams BD4, BD5 and BD6 at the occurrence of the first crack in mid-span.](image)

### Table 9

Corresponding load and distance of the extreme compression fibre from neutral axis.

<table>
<thead>
<tr>
<th>Tested beams</th>
<th>Load corresponding to first crack (kN)</th>
<th>Percentage of first crack load to ultimate load</th>
<th>Distance of the extreme compression fibre from the N.A (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD1</td>
<td>215.8</td>
<td>0.42</td>
<td>330</td>
</tr>
<tr>
<td>BD2</td>
<td>187.3</td>
<td>0.25</td>
<td>180</td>
</tr>
<tr>
<td>BD3</td>
<td>242.0</td>
<td>0.30</td>
<td>280</td>
</tr>
<tr>
<td>BD4</td>
<td>285.0</td>
<td>0.42</td>
<td>360</td>
</tr>
<tr>
<td>BD5</td>
<td>239.0</td>
<td>0.36</td>
<td>250</td>
</tr>
<tr>
<td>BD6</td>
<td>307.0</td>
<td>0.25</td>
<td>270</td>
</tr>
<tr>
<td>BD7</td>
<td>198.0</td>
<td>0.13</td>
<td>250</td>
</tr>
<tr>
<td>BD8</td>
<td>325.0</td>
<td>0.17</td>
<td>190</td>
</tr>
</tbody>
</table>

### Table 10

Corresponding experimental moment and the modulus of rupture of deep beams.

<table>
<thead>
<tr>
<th>Deep beam no.</th>
<th>$M_r$ (exp) (N mm)</th>
<th>$f_r$ (exp) (MPa)</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD1</td>
<td>377,65,000</td>
<td>5.98</td>
<td>0.625366</td>
</tr>
<tr>
<td>BD2</td>
<td>327,77,500</td>
<td>2.83</td>
<td>0.296060</td>
</tr>
<tr>
<td>BD3</td>
<td>423,50,000</td>
<td>5.69</td>
<td>0.596339</td>
</tr>
<tr>
<td>BD4</td>
<td>498,75,000</td>
<td>8.62</td>
<td>0.890247</td>
</tr>
<tr>
<td>BD5</td>
<td>478,90,000</td>
<td>5.74</td>
<td>0.644942</td>
</tr>
<tr>
<td>BD6</td>
<td>537,25,000</td>
<td>6.96</td>
<td>0.744350</td>
</tr>
<tr>
<td>BD7</td>
<td>346,50,000</td>
<td>4.16</td>
<td>0.458504</td>
</tr>
<tr>
<td>BD8</td>
<td>568,75,000</td>
<td>5.19</td>
<td>0.526118</td>
</tr>
</tbody>
</table>

### Table 11

Comparison of the various modulus of rupture.

<table>
<thead>
<tr>
<th>Reference and authors</th>
<th>$f_r$ (MPa)</th>
<th>$f_r\sqrt{l} = 70$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-95 [36]</td>
<td>0.62$\sqrt{l}$</td>
<td>5.18</td>
</tr>
<tr>
<td>ACI 363 [37]</td>
<td>0.97$\sqrt{l}$</td>
<td>8.12</td>
</tr>
<tr>
<td>CSA84 [38]</td>
<td>0.30$\sqrt{l}$</td>
<td>2.51</td>
</tr>
<tr>
<td>Ahmad and Shah [35]</td>
<td>0.42$\sqrt{l}$</td>
<td>5.02</td>
</tr>
</tbody>
</table>

**Fig. 10** shows a general shear force and bending moment diagram of all the tested deep beams. The maximum bending moment obtained is 350°/kN mm. Based on Eq. (1), the location of first neutral axis depth (N.A.D) is important for determining the value of $y$. For this purpose, at the emergence of the first crack, the stress-strain distribution at mid-span of beam is analyzed and drawn in Figs. 11 and 12. The mid-span readings are taken from the strain...
gazes for the upper portion and the Demec gauges for the lower portion (refer to Fig. 4).

As observed from Figs. 11 and 12, the stress–strain distribution of deep beams is non-linear even at the occurrence of first crack and there is more than one neutral axis depth. The corresponding loads of the first bending crack and the neutral axis depth is presented in Table 9. It shows that out of eight deep beams tested, six beams had indexes more than 25% and it is supported by the finding of Maco [34]. The beams that had indexes less than 25% are the beams with additional support widths which results in increase in ultimate load capacity and consequently decrease in the ratio.

For a better comparison of modulus of rupture with code provisions, an $x$ coefficient is added to the concrete strength as shown in Eq. (2):

$$f_r = x \sqrt{f_c}$$

Experimental cracking moment corresponding to the first crack and the modulus of rupture are shown in Table 10. Also the value of $x$ calculated from the respective concrete strengths (refer Table 2) using Eq. (2) are presented in Table 10.

From Table 10, $x_{mean} = 0.6$ is calculated. The modulus of rupture for deep beams studied with $a/d$ ratio of 0.8 is shown in Eq. (3).

$$f_r = 0.6 \sqrt{f_c} \text{ MPa}$$

Table 11 shows the comparison of modulus of rupture in HSSCC deep beams with code provisions, Ahmad and Shah [35] and the proposed equation from this study.

Based on Table 11, the modulus of rupture from ACI 318-95 [36] is the value nearest to that obtained from the proposed modulus of rupture in this study. Thus based on this study, the ACI 318-95 code provision is an acceptable conservative prediction for the modulus of rupture in HSSCC deep beams.

3.2. Cracking moment and modulus of rupture in HSC normal beams

In normal beams, the tension reinforcement and the concrete in compression both behave elastically up to yield of the reinforcement. Based on experimental results, the first crack occurs at the extreme tension fibre at different loads for each of these beams. Fig. 13 presents the corresponding load to the first crack in normal beams.

Table 12 shows the ratio of load corresponding to first crack to ultimate failure load in normal beams. Table 13 shows the comparison between the theoretical cracking moment using ACI codes and the experimental cracking moment from this study for normal beams. Table 14 shows comparison between present study, other researchers and code provisions for normal beams. Fig. 13 and Tables 13 and 14 show results of a thorough evaluation of the modulus of rupture for normal beams.

As per Table 12 the ratio of load corresponding to the first crack to ultimate failure load ranges between 3–25% with a mean of 8.4%. In comparison with the deep beams, this ratio is relatively lower. Cracking moment has been evaluated with ACI code provision using Eq. (1). The results of this theoretical and experimental study are then compared and presented in Table 13. Based on Table 13 and Eq. (2), the modulus of rupture for these normal beams is proposed in Eq. (4).
The comparison between the proposed Eq. (4), the code provisions and other investigators results are presented in Table 15. The value of modulus of rupture for normal HSC beams in the present study agrees with CSA 94 [38]. ACI code is found to give highly conservative results. Therefore CSA 94 is recommended to calculate modulus of rupture for HSC normal beams.

### Table 15

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>$f'_0$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-95 [37]</td>
<td>0.62 $\sqrt{f'_0}$</td>
</tr>
<tr>
<td>ACI 360 [38]</td>
<td>0.97 $\sqrt{f'_0}$</td>
</tr>
<tr>
<td>CSA 94 [39]</td>
<td>0.30 $\sqrt{f'_0}$</td>
</tr>
<tr>
<td>Ahmad and Shah [36]</td>
<td>0.42 $\sqrt{f'_0}$</td>
</tr>
<tr>
<td>Current study</td>
<td>0.33 $\sqrt{f'_0}$</td>
</tr>
</tbody>
</table>

\[ f'_0 = 0.33 \sqrt{f'_0} \text{ (MPa)} \] (4)

The differences in ratios of corresponding load to the first crack with ultimate failure load in deep beams and normal beams are due to the distance in load transfer mechanism in deep beams. In normal beams, flexural cracks appear first followed by shear cracks at the end of the flexure cracks leading to failure mode. In deep beams, part of the load is transmitted directly to the support by compression strut without any contribution to flexural stress (refer Fig. 8). This kind of load transfer mechanism in deep beams commonly leads to shear in the form of splitting failure and raises a need for design concept different to normal beams. Due to this failure mode and the load transfer mechanism from load point to support in deep beams, the first crack will appear at a load much higher than in normal beams.

4. Conclusions

Deep beams show non-linear strain distribution with more than one neutral axis depth before tensile bar yields. The modulus of rupture of HSSC deep beams is different from HSC normal beams. The results show that the modulus of rupture in HSSC deep beams with shear span to depth ratio of less than 0.80 is around 0.6 $\sqrt{f'_0}$, and in normal beams, it is around 0.33 $\sqrt{f'_0}$. The first crack to appear in these two types of beams is a flexural crack. Due to the shear mechanism of load transfer in deep beams, the corresponding load to the occurrence of the first crack is around two times more than in normal beams. Based on the present study ACI 318-95 is recommended for calculating modulus of rupture in HSSC deep beams whereas CSA 94 is suggested for HSC normal beams.

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