The Application of Hotelling’s $T^2$ Control Chart in an Automotive Stamped Parts Manufacturing Plant

Muzalwana Abdul Talib & Susila Munisamy

Quality has been a longstanding issue among the small and medium-sized industries (SMIs) automotive parts manufacturing and supply companies in Malaysia. It is no longer a compromise to suppliers or else it will pose as a major threat to the national automotive industry. There have been pressures for the local SMI suppliers to improve the overall quality of parts supplied to the Malaysian pioneer car manufacturer (Wan Mohamed, Cha, Chin & Ayeb 2006; Mohd Najib 2006; Mohd Nor 2006). Previous studies on the Malaysian automotive industry have shown that, to a certain extent, the local automotive parts suppliers have applied a number of the statistical process control tools as their quality initiatives (Jafri, Sha’ri, & Ismail 2007; Lim 2003; Mohd Nizam, Rosmariza, Jaharah, Zulkifli, Ahmad Rasdan & Suriani 2009; Salimah 2001). At the existing rate, the progression in the field of automotive industry focuses on the application of the basic tools like check sheet, cause-and-sheet diagram, histogram, Pareto diagram and control charts. Nonetheless, many of these SMIs automotive parts suppliers are still reported to be bogged down with the longstanding issue of low quality products (Leete 2007). A recent study reveals as high as 56.4 percent of the SMIs automotive supplier admit facing problems related to poor quality of their own products (Hassan, Mohd Asri, Abdul Aziz, Aziz, Katsumi, Watanabe et al. 2006).

The application of statistical methods is still deficient in addressing the quality improvement among the local parts suppliers. More pressing is the lack of study in multivariate approach for quality monitoring in the
Malaysian automotive industry. Previous studies show lack of evidence of the application of multivariate control charting technique among the automotive parts suppliers in Malaysia (Noviyarsi & Sha’ri 2004). Lack of knowledge about the reward of the advanced technique or shortage of the right kind of manpower to learn and use this technique could be the reason for not applying the multivariate techniques. Multivariate $T^2$ control chart is one of the appropriate approaches to monitor the process quality of automotive stamped part production.

As we learn best by doing, this chapter aims to delve into the application of multivariate statistical improvement techniques in a local automotive body and parts manufacturing company. The objective of the chapter is to develop quality monitoring approach with the deployment of multivariate control chart for automotive stamped parts manufacturing. The Hotelling’s $T^2$ control charting technique is most applicable when a multivariate control charting technique is applied for the first time to monitor the quality variables. Generally, in such cases, the population parameters of the quality variables are unknown (Cheng, Away & Hassan 2004). Hotelling’s $T^2$ control chart was originally developed for the cases with unknown population parameters and hence, is highly recommended for retrospective analysis (Alt & Smith 1988; Lowry & Montgomery 1995).

This chapter seeks in part, to provide the reader with an approach towards quality improvement with the retrospective Hotelling’s $T^2$ control charting scheme for an automotive stamped part manufactured by a local company.

**Literature Review**

The fundamental principal for quality improvement is to “reduce variation” for variations are the adversary to quality (Montgomery 2005 cites Deming 1986). Concomitantly, the general objective of applying statistical methods in industries is to study variations in manufacturing process, systematically eliminate or reduce the variations in order to enhance quality (Makrymichalos, Antony, Antony & Kumar 2005). Variations or variability exists as successive observations of a process do not produce exactly the same result. In the past, variability was not investigated until it built up costs as scraps and rework or loss of business through customer dissatisfaction (Morrison 2000). Customers of today and tomorrow value products that have consistent performance and this can be achieved by systematically eliminating variation in processes and hence the quality of end products will be improved (Makrymichalos et al. 2005). Although some amount of variation is impossible to avoid, variation monitoring is still important at least to keep it to a minimum.
From statistical point of view, manufacturing processes contain two types of variation: common cause and special cause (Montgomery 2005). Common cause variation is the natural inherent variation in the process output when all input variables remain stable, i.e., independent and identically distributed. Special cause variation represents any increase in product variability above the level of common cause variation. Studies on automotive metal stamped part reported that dimensional data is used to characterise stamping variation in short term as the common cause variation and in long term as the special cause variation (Hammett, Baron & Smith 2000; Rolfe, Frayman, Webb & Hodgson 2003). The studies have taken place for the past two decades both in manufacturing of sheet stamped panels and assembling work of stamped panels (Ceglarek 2000; Ceglarek & Shi 1996; Guzman, Hammett, & Herrin 2004; Guzman & Hammett 2003; Hammett et al. 2000; Hammett, Baron, & Majeske 1997; Majeske & Hammett 2003; Rolfe et al. 2003; Silva, Baptista, & Martins 2004; Wang 1995; Yang 1996).

In automotive manufacturing, the quality objective relates to the aspects of design and conformance. Quality of design is the predetermined standards or specific design requirements on the stamped part set by the parent company. Quality of conformance is met if the finished panel fits the specifications and tolerances set for by the design. A number of studies on automotive transports learnt that the major problems in achieving this objective is due to variation or deviations in stamped components (Hammett et al. 2000; Rolfe et al. 2003) Variations in sheet metal stamped parts and component deviating from their nominal specifications larger than allowed by part design tolerances are typically characterised by dimensional mean shifts. Ostensibly, when automotive stamped panels are of large part to part variation, good fits cannot be achieved. Continuous effort is needed to improve the dimensional accuracy of stamped components following the principle that “perfect parts make perfect assemblies”. With reduced variation between nominal specifications and process variation, the underlined critical quality objectives of automotive manufacturing to achieve dimensional integrity and functionality between stamped body components and assembled vehicle body and parts (Guzman & Hammett 2003).

A powerful statistical method to improve industrial processes is statistical process control (SPC) (Woodall 2000). The philosophy of SPC is to use data to continually improve quality by collecting, organising and interpreting the wide variety of information available. By tracking the dimensions of automotive stamped components, SPC can help to measure, understand and control the variables that affect the manufacturing
processes. The major statistical tool used to do this is the control chart. Control chart method helps to evaluate if a process is in a state of statistical control. This is a state where the process is deemed to be stable and predictable. Control charts can also detect any unnatural patterns of variation in process that may lead to out-of-statistical control conditions. The standard control chart, $X$-bar chart or Shewhart control chart is coined after its founder, Walter Shewhart who first introduced it in the early 1920s. The standard $X$-bar chart has been widely applied in many industries due to its versatility and simplicity for use (Jensen, Jones-Farmer, Champ, & Woodall 2006). Within the automotive manufacturing industry, the standard control chart has been applied over the last two decades. Control charting methods have been recommended by the Automotive Industries Association Group (AIAG) in its reference manual Statistical Process Control (SPC) for automotive suppliers in the USA (Majeske & Hammett 2003; *Statistical Process Control (SPC) Reference Manual* 2005).

The central idea of control charts is to know the level of common cause (or chance cause) variation in a process and make it possible to statistically define the control limits. When the limits are exceeded, it is further suggested that one should investigate whether assignable cause (or special) is present (Messina, 1987). The other two types of control charts widely researched on are Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts. While the standard chart applies the data from a sample or subgroup at a time, CUSUM and EWMA utilise current and past information to construct control charts. The standard Shewhart $X$-bar chart, CUSUM and EWMA are all univariate in nature as there is only one quality characteristic to be controlled by each control chart.

Hotelling’s $T^2$ control chart is customarily used as the control chart for multivariate statistical process control analysis. The potentials of multivariate approach to quality monitoring to the automotive parts manufacturers should not go unnoticed. This is because the nature of manufacturing settings comes in form of multi-dimension of products and multifaceted industrial processes. More importantly, the control processes in industrial setting are inherently multivariate and correlated to one another (Mason & Young 2001). If two or more quality characteristics are to be monitored independently in a univariate approach, it could be overwhelming and lead to misleading results (Ryan 2000). Practically, the univariate control charts are inadequate to take into account the multivariate nature of industrial processes (Mason & Young 2001), whereas in a multivariate control charting technique, the issue of correlation between variables are taken into account in computing the parameters employed in constructing the multivariate
control chart. These parameters are mean vector and variance-covariance matrix (Woodall & Montgomery 1999). In a nutshell, the univariate control procedures are widely used in industry and form the basic knowledge, but the extension to multivariate is undeniably crucial.

**Methodology**

This case study research is conducted in a Malaysian-based automotive part and manufacturing company, Company M Sdn. Bhd. Company M produces various automotive parts such as car roof, bonnet, fender, door and body panels. Company M also specialises in the design, engineering and manufacturing of dies and moulds used in producing the automotive parts. The selected automotive part for this quality study is Reinforced Rear Floor Side Member (Part No. PW835688), an inner panel reinforcing the rear or back floor located on right side of the car.

**Population and Sampling Design**

The population of interest for this study is the set of variables that describe the quality characteristics of the panel. The quality characteristics are the position of flange surface dimensions. The variables characterise the geometrical dimension of surface panel measured by the deviations from their specification limits. Initially, ten critical-to-quality (CTQs) points are selected by Company M quality team based on expert judgment. These measured points are SP1, SP3, SP9, SP11, SP12, SP17, SP26, SP28, SP30, and SP32.

The sample design of this study was originally based on rational sub-grouping where 140 panels are sampled from four different production runs. The size of subgroup for each production run is based on a simple guideline provided by ANSI/ASQC 21.9. For a production size of 800 panels for each run, the suggested number of panels to be collected from a production shift is 35. Using the typical subgroup size of 5, 7 subgroups are collected from each production shift, \( \frac{35}{5} \). The duration of each production shift is used to determine the sampling frequency. With an average of 2.5 hour or 150 minutes per run, a subgroup of 5 consecutive panels is sampled every 21 minutes \( \frac{105}{7} \) of the production time.
Data Measurement and Collection

Measuring work for quality variables of the stamped panels take place manually using hard checking fixtures and measuring equipment since there is no automatic measuring equipment such as Coordinated Measuring Machine (CMMs) available. Caliper and taper gauge are the two measuring equipments used to measure mean deviations in surface variables.

Data Preparation

Data collected are screened and cleaned for three potential problems, namely, presence of collinearity, autocorrelation and outliers. To detect presence of collinearities and outliers, the multivariate method of Principal Component Analysis (PCA) is applied. To check on collinearities, eigenstructures of PCA on correlation matrix are utilised to compute condition index (Mason & Young 2001). At this stage, outlier detection is simply based on box plots for each individual quality point measurement. The outlier detection technique based on multivariate $T^2$ statistics will be used in the later stage.

Results of the data screening of the collected data (not shown here) indicated that there is also no collinearities exist in the covariance matrix of the quality data. Thus, there is no effect of serious multicollinearity and that no variables are required to be removed from the dataset. The level of autocorrelation is also not significant. It is assumed that the data set is not affected by autocorrelation, hence, the effect of autocorrelation is taken into account in designing the control chart scheme.

The set of screened data contributes to the development of reference sample. Figure 8.1 illustrates the processes involved in identifying problems in the data set. The final stage takes place before establishing the reference sample is validating the process analysis. This stage helps to examine the appropriateness of sampling technique which has been employed earlier in this study. Sampling is an important issue in designing control charts. For control charting purposes, the basic idea of sampling is that sampling should be done so that it is likely for any changes in the process to occur between samples and not within samples (Reynolds & Stoumbos 2004). Data sets should be collected as such variations occur within subgroups will only reflect the common variations. On the other hand, any significant variations occur between subgroups should reflect special causes (Sefik 1998). In other word, the process changes should not occur while the subgroups of panels are being collected as such sampling points of rational subgroups should not coincide with any possible process disruptions.
In this study the analysis made on the automotive stamping process variability aims to examine the appropriateness of selecting rational subgrouping. For this purpose, variance component analysis technique is applied. Component of variance measures the extent of how much variance is attributable between the different factor levels and variability within the factor levels or simply the random error. It includes how much of that variability is attributable between production batches, between subsamples and how much is assignable for variations within the particular quality variable. Generally, component of variance is expressed in percentage form.

The analysis results show large percentage of the variation in surface is attributable to the random error or error within the individual observation where the ‘within-individual’ measurement variation accounts for more than 90 percent of the total variation and only a minimal of 7-9 percent is assigned to the between-run variation. Table 8.1 exhibits the summary of the analysis. The implication of the sizeable ‘part-to-part variation’ is that control charting scheme based on individual observations is more appropriate for automotive stamping in comparison to the method of rational sub-grouping.

The findings on large part-to-part or within variation in quality variables of automotive stamped parts actually support several earlier
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studies on automotive stamping variation (Rolfe et al. 2003). The findings further support several studies claiming that the appropriate control charting procedures for automotive stamped parts manufacturing are individual control charts for such observations (Hammett et al. 2000). The process analysis justifies the choice of retrospective $T^2$ control chart but the design scheme should be made based on individual observations.

### Table 8.1: Variance component of Surface (%)

<table>
<thead>
<tr>
<th></th>
<th>Between Run</th>
<th>Between Subsample</th>
<th>Within</th>
</tr>
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<tbody>
<tr>
<td>SP1</td>
<td>12.7</td>
<td>29.8</td>
<td>7.5</td>
</tr>
<tr>
<td>SP3</td>
<td>4.4</td>
<td>3.7</td>
<td>91.9</td>
</tr>
<tr>
<td>SP9</td>
<td>1.1</td>
<td>10.2</td>
<td>88.6</td>
</tr>
<tr>
<td>SP11</td>
<td>2.3</td>
<td>15.6</td>
<td>82.2</td>
</tr>
<tr>
<td>SP12</td>
<td>3.6</td>
<td>10.6</td>
<td>85.7</td>
</tr>
<tr>
<td>SP26</td>
<td>17.4</td>
<td>27.5</td>
<td>55.0</td>
</tr>
<tr>
<td>SP28</td>
<td>4.4</td>
<td>17.7</td>
<td>77.8</td>
</tr>
<tr>
<td>SP30</td>
<td>2.9</td>
<td>16.9</td>
<td>80.3</td>
</tr>
<tr>
<td>SP32</td>
<td>2.6</td>
<td>23.7</td>
<td>73.7</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>5.71</strong></td>
<td><strong>17.30</strong></td>
<td><strong>76.97</strong></td>
</tr>
</tbody>
</table>

The application of Hotelling’s $T^2$ control chart can be divided into two different phases of operation, i.e. retrospective or Phase I and prospective or Phase II operations. Under retrospective or Phase I operation, Hotelling’s

### Retrospective Hotelling $T^2$ Control Chart

The Hotelling’s $T^2$ control chart is employed to monitor the mean of geometrical dimension of a selected automotive stamped part. The main reasons opting for the Hotelling’s $T^2$ control charting techniques are due to, firstly, unknown population parameters of the automotive stamped data (Alt & Smith 1988; Cheng et al. 2004; Lowry & Montgomery 1995) and secondly, Hotelling’s $T^2$ control chart is most suitable for the cases where mean shifts in the mean vector are not small (Cheng et al. 2004; Woodall 2000). Previous empirical studies have shown that automotive stamping process bound to produce large mean shifts in the quality variables of its stamped parts (Hammett et al. 2000). Evidently, a number of past studies claim that control charting procedure for individual observations is suitable for automotive panel stamping process (Hammett et al. 2000).
The Application of Hotelling’s $T^2$ Control Chart

$T^2$ chart is used to identify outliers in the historical data set, mean shifts in the new subgroups of data and other distributional deviations from in-control distributions. While Phase II operations detect shifts in process when new observations are drawn (Mason & Young 2001). The retrospective phase can be further divided into two stages, Stage I and Stage II. Figure 8.2 illustrates the phases and stages involved in the Hotelling’s $T^2$ control charting scheme.

![Figure 8.2: Phases and stages involved in Hotelling’s $T^2$ control charting scheme](image)

The distinction between the two stages is crucial and must be made clearly, particularly, when the parameters are unknown and have to be estimated. (Champ, Jones-Farmer, & Rigdon 2005). In this chapter, the Hotelling’s $T^2$ chart is only focused on the retrospective or Phase I analysis since the population parameters are unknown (Alt & Smith 1988; Lowry & Montgomery 1995). For readings on the prospective or Phase II control charting, a list of reference is available in a review of literature by Willis, Jones-Farmer, Charles, & William (2006). In the retrospective procedure, three major steps are involved. The three steps are outlier deletion, variable selection and parameter estimation (Mason & Young 2001; Yang & Trewn 2004).

**The Hotelling $T^2$ Statistics for Individual Observations**

The multivariate $T^2$ control charting technique is the extension of the Shewhart univariate procedures. In the univariate statistical control, the normal distribution is used to describe the behaviour of a continuous quality characteristic $X$. The exponent of the probability density function can be written as $(x - \mu)(\sigma^2)^{-1} (\mu - \sigma)$. In a multivariate case where there are $p$ quality variables, $X_1, X_2, \ldots, X_p$, the $p$-component vector $\mathbf{X} = [X_1, X_2,$
..., \( X_p \) and the mean vector is given as \( \mu' = [\mu_1, \mu_2, \ldots, \mu_p] \). The variances and covariances of random vector \( X \) are represented in the form of a \( p \times p \) covariance matrix \( \Sigma \) where the main diagonal and off-diagonal elements are the variances and covariances, respectively. Assuming the distribution of \( p \) quality variables are \( p \)-variate normal, \( N_p (\mu, S) \), the generalized form of the squared distance from \( X \) to \( m \) is the Hotelling’s \( T^2 \) statistics given as:

\[
T^2 = (X - \mu)' \Sigma^{-1} (X - \mu)
\]  

(1)

In a retrospective operation, the Hotelling’s \( T^2 \) statistic can be used to identify outliers in the historical data set, to identify the mean shifts in the new groups of data and other distributional deviations from in-control distributions (Mason, Chou & Young 2009). The commonly used univariate control charting technique for individual observations are control charts for individuals (\( X \)) and for moving ranges (\( MR \)) (Chou, Mason & Young 2006). For multivariate individual observations, evidence from past empirical research shows all the observations are pooled to estimate the mean vector and covariance matrix in computing the Hotelling’s \( T^2 \) statistic (Huwang, Yeh, & Wu, 2007, cited Jackson, 1985; Tracy, Young & Mason, 1992; Wierda, 1994; Lowry & Montgomery, 1995). For the \( i \)th sample vector contains observations on each of the \( p \) variables \( X_{i1}, X_{i2}, \ldots, X_{ip} \) hence, the sample mean vector is defined as,

\[
\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i
\]  

(2)

and the sample covariance matrix is

\[
S = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})'
\]  

(3)

The Hotelling \( T^2 \) statistic measures the significance shifts from the out-of-control mean vector, \( \mu_s \) to the nominal mean vector \( \mu_0 \) as such the larger the \( T^2 \) value, the more distant is the observation from the mean vector. This concept is in line with testing the null hypothesis \( H_0: \mu = \mu_0 \) vs. \( H_1: \mu \neq \mu_0 \), the null hypothesis is to be rejected if

\[
(X_i - \mu)' \Sigma^{-1} (X_i - \mu) > T^2_{p, \alpha}
\]  

(4)

In Equation (4), the covariance matrix is assumed to remain constant as shift in the mean vector are monitored. If \( \mu \) and \( \Sigma \) are both unknown, they are estimated by \( \bar{X} \) as in equation (2) and \( S \) as in equation (3) hence the \( T^2 \) statistic is given as (Tracy et al. 1992);
The Application of Hotelling’s $T^2$ Control Chart

$T^2 = (X - \bar{X})S^{-1}(X - \bar{X})'$  \hfill (5)

The retrospective limits of the $T^2$ control chart in Equation (4) is based on a beta distribution and the UCL of this $T^2$ statistic is given as below (Mason & Young 2001):

$$UCL = \frac{(n - 1)^2}{n} \beta_{\frac{p}{2}, \frac{n-p-1}{2}} \hfill (6)$$

where $\beta_{\frac{p}{2}, \frac{n-p-1}{2}}$ is the upper $\alpha$-th percentile of beta distribution with parameters $p/2$ and $(n - p - 1)/2$ where $p$ is the number of quality variable being monitored and $n$ is the size sample observations in the historical data set.

During the prospective phase, the use of multivariate chart is to test whether the process remains in-control when future observations are sampled. With the sample mean vector $\bar{X}$ and sample covariance matrix $S$, computed in the retrospective phase, the $T^2$ statistic of the prospective $T^2$ control chart is defined as (Mason & Young 2001):

$$T^2 = (x_f - \bar{X})S^{-1}(x_f - \bar{X}) \hfill (7)$$

where $x_f$ is the individual observations of a new sample for $f = 1, 2, 3...$

The $T^2$ statistic is used to monitor future observations in prospective phase based on the upper control limit computed by the following formula (Mason & Young, 2001):

$$UCL = \frac{p(n + 1)(n - 1)}{n(n - p)} F_{\alpha, p, n-p} \hfill (8)$$

and the $T^2$ statistic has a $\left[ \frac{p(n + 1)(n - 1)}{n(n - p)} \right] F_{p, n-p}$ distribution.

Stage I: Development of Reference Sample

Two steps involved in developing a reference sample or the ‘in-controlled’ sample for Hotelling’s $T^2$ control charting scheme are outlier deletion and variable selection (Mason & Young 2001).

Outlier Deletion

Detecting for outliers is so important that failure to discard them will affect the parameter estimation of control chart for the actual monitoring purposes during prospective phase. Statistical purging of unusual observations
in retrospective operation is actually similar to the outlier detection problem. Here, in this study, the $T^2$ statistic approach is applied to detect the multivariate outliers (Mason & Young 2001). $T^2$ statistic is a simple procedure to apply as well as a helpful approach in locating individual outlying observations particularly in repeated use of control charts (Mason & Young 2001).

For a given value of $a$, if an observation vector is found to have $T^2$ value greater than the UCL, it is regarded as an outlier and is discarded from the data set. Other observation vectors whose $T^2$ values are less than or equal to the upper control limit (UCL) remain in the data set. In retrospective operation, the $T^2$ statistics distribution for outlier purging process follows beta distribution as below (Mason & Young 2001)

$$T^2 \sim \frac{(N-1)^2}{N} B_{\frac{p}{2}} \left( \frac{N-p-1}{2} \right)$$

(9)

In the above, $N$ denotes the number of total observations as the design scheme of retrospective $T^2$ control chart in this study is based on individual observations. The corresponding UCL is similar to Equation (6) but the sample size $n$ is now denoted as $N$ given by:

$$T^2 \sim \frac{(N-1)^2}{N} B_{\frac{p}{2}} \left( \frac{N-p-1}{2} \right)$$

(10)

$B_{\frac{p}{2}} \left( \frac{N-p-1}{2} \right)$ is the $(1 - \alpha)$ th quantile of beta distribution $B_{\frac{p}{2}} \left( \frac{N-p-1}{2} \right)$ and $p$ is the number of quality variables. $N$ is the total number of observations in the data set and will be used onwards to denote the sample size of this study.

After deleting the observations found as outliers, estimates of mean vector and covariance matrix are computed based on the remaining set of data. The set of data goes through a second pass with the new mean vector and covariance matrix and the iterative process begins. Any outliers detected are removed and the process is repeated until a homogeneous set of observations is obtained. The homogeneous set of data would contribute to the establishment of reference sample or ‘in-control’ sample.

**Variable Selection**

Variable selection of surface quality variables commence after the outlier deletion process is completed. Selection is made by reducing the geometrical dimension through Factor Analysis technique with Principal Component Analysis (PCA). Factor analysis is a multivariate technique to identify variables belonging to the same factor based on variability. In this case, factor analysis aims to describe the set of quality variables in terms of
a smaller number of factors. For this purpose, SAS JMP Version 8 is used to perform factor analysis with principal components as initial factors. Here, the factor analysis is set for varimax rotation with Kaiser normalisation. Thus, dimensional reduction is performed to select 4 variables which contribute most variation in dimensional measurement of finished stamped part. Initially, the subgroup size \( n \) is 5. So for the sample size \( n \) to be larger than number of quality characteristics \( (n > p) \) (Mason & Young, 2001), the number of characteristics, \( p \) to be selected is 4.

**Finding and Discussion**
Findings of this study are discussed separately under different subsections.

**Development of Reference Sample**
Results of the purging process of 140 individual observations of surface data is explained as follows. With 140 individual surface quality variables \( (m = 140) \), and 9 quality variables after data screening \( (p = 9) \), upper control limit for retrospective individual data is computed by using the formula on the right.

The first attempt or Pass 1 (the term used in study by Mason & Young, 2001) of purging process results in outliers exceeding the upper control limit of 26.02. Figure 8.3 exhibits the Hotelling’s \( T^2 \) control charts executing the outlier purging process with UCL computed at \( \alpha = 0.001 \). The nine outliers produced during Pass 1 include observations \#7, \#20, \#71, \#91, \#95, \#106, \#107, \#108 and \#119. After discarding the nine outliers, the subsequent charting, Pass 2 revise and refine the control limits at \( \alpha = 0.001 \) with the new mean vector and covariance matrix. Any outliers detected are removed and the iterative process begins until a homogeneous set of observations is obtained where no outliers are detected at Pass 5 with the upper control limit of 25.75. Table 8.2 summarises the results of iteration process.

**Variable Selection Based On Principal Component Analysis**
The output of the principal component analysis of factor analysis produced by JMP8 depicts the eigenvalue for each factor and is presented in Table 8.3. The eigenvalues are used as the rule of thumb to determine the number of factors. Eigenvalues greater than unity suggest the number of factor components accountable for the variation in the data set (Manly 2005).
The other important result from PCA of factor analysis is factor loadings. The factor loadings identify the variable assigned to most variation as explained by the number of factor component. In general, variables with high factor loadings (0.60 or more) indicate that they are related to the factor component (Manly 2005).

Table 8.3 displays three eigenvalues are greater than unity. Using the “rule of thumb”, three factors should be considered for analysis. However, selection is made based on two factors only because the objective of variable selection at this stage is to reduce the number of measure points or variables for control charting purposes. As stated earlier, only the four variables selected. Four variables with high factor loadings are considered while those variables with low loadings are discarded from the selection list. Table 8.3 exhibits two quality variables are selected from each factor based on their relatively high PCA factor loadings. The four selected variables are SP9, SP11, SP26 and SP28.
The Application of Hotelling’s $T^2$ Control Chart

Parameter Estimation

The established reference sample of surface quality features is the outcome of two separate processes of outlier deletion and variable selection. The established reference sample forms a process believed to be stable or ‘in-control’ from which a set of unbiased estimates of the mean vector and covariance matrix are to be estimated. Here, 140 sample observations are initially employed to establish the set of reference sample. During outlier deletion process, a total of 18 observations are discarded. Based on factor analysis applied to the remaining 122 observations, only two variables with highest loadings are chosen from each factor component. Four variables selected to establish the reference sample for the control charting scheme are SP9, SP11, SP26 and SP28. To check the state of ‘statistical control’ of the established reference sample, multivariate Hotelling’s $T^2$ control charting is applied to the four variables at $\alpha = 0.0027$ (Mason & Young 1999; Nedumaran 1997). The first pass produces two out-of-control observations. Further two passes purge another observation before the ‘in-control’ sample of the four variables is obtained. The reference sample is now comprised of 119 observations and the ‘in-control’ Hotelling’s $T^2$ chart with the computed UCL equals to 15.42 is illustrated in Figure 8.4. The output of PCA on covariance from this chart is produced in Table 8.4. Table 8.4 shows two components have more than 80% of variance explained (with

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</table>
Figure 8.4: The Hotelling’s $T^2$ chart of the ‘in-control’ sample of surface data
(Note: UCL is calculated based on $\alpha = 0.0027$)

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Percent</th>
<th>Cum Percent</th>
<th>ChiSquare</th>
<th>DF</th>
<th>Prob&gt;ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.049</td>
<td>64.290</td>
<td>64.290</td>
<td>207.724</td>
<td>9.000</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>0.0169</td>
<td>21.774</td>
<td>86.063</td>
<td>56.494</td>
<td>5.000</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>0.0060</td>
<td>7.722</td>
<td>3.785</td>
<td>1.387</td>
<td>2.000</td>
<td>0.4999</td>
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<tr>
<td>0.0048</td>
<td>6.215</td>
<td>100.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

$\rho < 0.001$). Table 8.5 includes the pooled covariance matrix and mean vector estimated from the established reference sample.

**Stage 2 of Retrospective Hotelling’s $T^2$ Control Chart**

This subsection evaluates the effectiveness of the $T^2$ control charting scheme developed earlier. For Stage 2 of Retrospective Hotelling’s $T^2$ control chart, the $T^2$ is computed based on Equation (7) and the UCL is derived based on an $F$-distribution as in Equation (8), and the mean vector and covariance parameters estimated from the ‘in-control’ reference sample is utilised in computing $T^2$ of new sample observations. For evaluation purposes, 35 new observations are sampled from a separate production run and applied to the $T^2$ control charting scheme at Type I error $\alpha$ equals to 0.0027. Figure 8.5 exhibits the Hotelling’s $T^2$ chart with the UCL computed as 17.89 at Type I error $\alpha = 0.0027$. Table 8.6 shows that two factor components have more than 80% of variance explained ($\rho < 0.001$ and $\rho = 0.0061$, respectively).
The Application of Hotelling’s $T^2$ Control Chart

Table 8.5: Descriptive Statistics and Covariance Matrix of the reference sample

<table>
<thead>
<tr>
<th>Quality Variable</th>
<th>SP9</th>
<th>SP11</th>
<th>SP26</th>
<th>SP28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Observations</td>
<td>119</td>
<td>119</td>
<td>119</td>
<td>119</td>
</tr>
<tr>
<td>Sample Size</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mean</td>
<td>2.60000</td>
<td>2.87563</td>
<td>3.17899</td>
<td>3.35546</td>
</tr>
<tr>
<td>Std Deviation</td>
<td>0.10814</td>
<td>0.09294</td>
<td>0.20538</td>
<td>0.12261</td>
</tr>
<tr>
<td>Cov SP9</td>
<td>0.0117</td>
<td>0.0049</td>
<td>0.0037</td>
<td>-0.0016</td>
</tr>
<tr>
<td>Cov SP11</td>
<td>0.0049</td>
<td>0.0086</td>
<td>0.0024</td>
<td>-0.0020</td>
</tr>
<tr>
<td>Cov SP26</td>
<td>0.0037</td>
<td>0.0024</td>
<td>0.0422</td>
<td>0.0160</td>
</tr>
<tr>
<td>Cov SP28</td>
<td>-0.0016</td>
<td>-0.0020</td>
<td>0.0160</td>
<td>0.0150</td>
</tr>
</tbody>
</table>

Figure 8.5: The Hotelling’s $T^2$ chart of the new set of sample observations

(Note: UCL is calculated based on $\alpha = 0.0027$)

The process stamping is not ‘in-control’. Four outliers are identified and these are observations #6, #16, #21 and #22.

The large erratic behaviour of points could be explained by several production upsets which actually occur during the stamping process from which the new observations are sampled. Such production upset conditions will seriously affect the quality of panel surface. A number of upset conditions during stamping production runs is, however, not unusual for the case of Company $M$. 

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Table 8.6: The Principal Components on Covariance of the new set of observation

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Percent</th>
<th>Cum Percent</th>
<th>ChiSquare</th>
<th>DF</th>
<th>Prob&gt;ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0499</td>
<td>64.290</td>
<td>64.290</td>
<td>59.853</td>
<td>9.000</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>0.0169</td>
<td>21.774</td>
<td>86.063</td>
<td>16.278</td>
<td>5.000</td>
<td>0.0061</td>
</tr>
<tr>
<td>0.0060</td>
<td>7.722</td>
<td>93.785</td>
<td>0.400</td>
<td>2.000</td>
<td>0.8189</td>
</tr>
<tr>
<td>0.0048</td>
<td>6.215</td>
<td>100.000</td>
<td>0.000</td>
<td>0.000</td>
<td>.</td>
</tr>
</tbody>
</table>

**Conclusion and Implication**

Statistical oriented quality control and improvement framework offers a methodical approach towards the quality enhancement of automotive stamping process. While large mean shifts in quality variables of automotive stamped part is common to automotive stamped parts manufacturers, knowledge in statistical quality improvement tool among these manufacturers is new. This study exemplifies the application of retrospective Hotelling’s $T^2$ control charting scheme to monitor the quality of an automotive stamped panel in a local automotive stamped part manufacturing company. By utilising SAS JMP8 software program, the Hotelling’s $T^2$ control charting scheme is applied to the new observations of automotive surface data reveal a statistically ‘out-of-control’ condition of the automotive stamping process. The ‘out-of-control’ condition actually could be explained by several production upset which occur during the particular stamping process. Such condition could affect the quality of panel surface as revealed by the standard Hotelling’s $T^2$ control chart. The Hotelling’s $T^2$ control chart has a powerful multivariate diagnostic feature that effectively detects any ‘out-of-control’ condition. This scheme support any efforts to increase the quality of parts manufactured within Company M. To a broader perspective, this multivariate technique could be applied within the local automotive parts manufacturing industry to spearhead a more effective quality enhancement endeavour so as to increase the competitive edge of the national car.